The Importance of Timing Attitudes in Consumption-Based Asset Pricing Models*

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Abstract

This paper introduces a new utility kernel for Epstein-Zin-Weil preferences to obtain greater flexibility in setting the intertemporal elasticity of substitution, the relative risk aversion (RRA), *and* the timing attitude compared to their standard implementation. We show that these new preferences resolve a puzzle in the long-run risk model, where consumption growth is too strongly correlated with the price-dividend ratio and the risk-free rate. The proposed preferences also resolve the puzzlingly high RRA in DSGE models, by enabling an otherwise standard New Keynesian model to match the equity premium and the bond premium with a low RRA of 5.

Keywords: Bond premium puzzle, Equity premium puzzle, Early Resolution of Uncertainty, Long-run risk, Risk-free rate puzzle.

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1 Introduction

Following the seminal work of Epstein and Zin (1989) and Weil (1990), a large number of consumption-based models use so-called Epstein-Zin-Weil preferences to explain asset prices (see Bansal and Yaron (2004), Guvenen (2009), Gourio (2012) to name just a few). An important property of these preferences is to disentangle relative risk aversion (RRA) and the intertemporal elasticity of substitution (IES) which otherwise have an inverse relationship when using expected utility. It is also well-known that the separation of the IES and RRA in Epstein-Zin-Weil preferences is achieved by imposing a timing attitude on households, which either prefer early or late resolution of uncertainty. This embedded constraint implies that Epstein-Zin-Weil preferences determine i) the IES, ii) the RRA, and iii) the timing attitude using only two parameters. For instance, to raise RRA and hence separate it from the IES, one has to increase the households' preference for early or late resolution of uncertainty. But, experimental evidence suggests that the timing attitude has an independent effect on decision making beyond what is implied by RRA, and that the timing attitude often is unrelated to RRA (see for instance Chew and Ho (1994) and van Winden et al. (2011)). This raises the question, whether Epstein-Zin-Weil preferences perform well because they separate the IES from RRA or because they imply a timing attitude?

The contribution of the present paper is to address this question by relaxing the embedded constraint in Epstein-Zin-Weil preferences, and to explore whether a more flexible specification of the time attitude helps to explain asset prices. We address these questions by introducing a new and more flexible utility kernel than the standard power-specification adopted in Epstein and Zin (1989) and Weil (1990), because this extension allows us to disentangle the IES, the RRA, *and* the timing attitude. We derive this new utility kernel by accounting for home consumption as in Benhabib et al. (1991) and ratio-habits in market consumption following the work of Abel (1990).[1] The utility contribution from home consumption is given by a power specification, whereas the contribution from habit-adjusted market consumption is determined by $u(\cdot)$, where $u'(\cdot) > 0$ and $u''(\cdot) < 0$. Our preferred specification of $u(\cdot)$ has one additional behavioral parameter τ that increases the speed by which marginal utility in the power kernel decreases for higher consumption. That is when $\tau \to 0$, the presence of home consumption and ratio-habits disappear and we recover the standard power utility kernel, and hence the traditional implementation of Epstein-Zin-Weil preferences. The main benefit of introducing the additional parameter τ is to obtain greater flexibility in setting $u'(\cdot)/u(\cdot)$ and $u''(\cdot)/u'(\cdot)$ compared to the power utility kernel, where a single coefficient determines both ratios. Much attention in the literature has been de-

[1] See also Chew and Epstein (1989) for alternative ways to control the timing attitude.

voted to $u''(\cdot)/u'(\cdot)$, because it controls the IES. The ratio $u'(\cdot)/u(\cdot)$, on the other hand, is often ignored but is the main focus of the present paper, as it determines how households' timing attitude affects RRA. To realize this, consider the case where $u'(\cdot)/u(\cdot)$ is low and $u(\cdot)$ therefore hardly changes. This in turn generates low variability in the value function, which implies that the timing attitude only has a relatively small effect on risk aversion. Indeed, our preferred specification of $u(\cdot)$ has the property that a relatively high value of τ generates a low ratio of $u'(\cdot)/u(\cdot)$ and hence enables strong preferences for early resolution of uncertainty to coincide with low RRA.

We start by studying the asset pricing implications of our new utility kernel in the long-run risk model of Bansal and Yaron (2004). Using a novel analytical second-order approximation, we first show that households' timing attitude has a separate effect on asset prices beyond the IES and RRA, which is consistent with the experimental evidence cited above. To evaluate the empirical support for our new utility kernel, we next estimate the long-run risk model by generalized method of moments (GMM) using unconditional first and second moments. As a useful benchmark, we first verify that the standard long-run risk model matches the level and variability in asset prices with a low RRA of 10. This analysis also confirms the finding of Beeler and Campbell (2012) that consumption growth in this model is too strongly correlated with the price-dividend ratio due to its strong reliance on long-run risk. We further show that this property of the model also makes consumption growth too strongly correlated with the risk-free rate, and these findings therefore question the empirical support for the required degree of long-run risk in the model. An important empirical finding in the present paper is to show that our utility kernel resolves this puzzle, because it reduces the reliance on long-run risk and instead makes households display strong preferences for early resolution of uncertainty. On the other hand, with Epstein-Zin-Weil preferences and a power utility kernel, such a modification in the timing attitude would simply lead to counterfactually high RRA given its relatively high value of $u'(\cdot)/u(\cdot)$.

We also study the asset pricing implications of our new utility kernel in a dynamic stochastic general equilibrium (DSGE) model, where consumption and dividends are determined endogenously. Our GMM estimates reveal that the proposed utility kernel in this setting resolves the puzzlingly high RRA required in many DSGE models to explain asset prices, as it enables an otherwise standard New Keynesian model to match the equity premium, and the bond premium (i.e. the level and variability of the 10-year nominal term premium) with a low RRA of 5. In contrast, most existing New Keynesian models are only able to match these asset pricing moments by postulating counterfactually high RRA (see Rudebusch and Swanson (2012), Andreasen (2012), Swanson (2015), among others). The mechanism explaining this substantial improvement of the New Keynesian model is similar to the one offered in the long-run risk model, namely that our new utility kernel allows strong preferences for early resolution of uncertainty to coincide with low RRA.

We finally use the estimated extension of the long-run risk model and the New Keynesian model in two counterfactual experiments to explore the asset pricing implications of the timing attitude and long-run risk, respectively. The effects of the timing attitude is explored by putting the Epstein-Zin-Weil parameter to zero in both models, implying that RRA now is tightly linked to the IES. We find that this modification generates a very small reduction in RRA for the two models, but both models are now unable to explain asset prices. Our second counterfactual re-introduces strong preferences for early resolution of uncertainty but omits long-run risk. Here, we also find that both models cannot match asset prices, although the IES, the RRA, and the timing attitude are identical to their estimated values in each of the two benchmark models. These experiments therefore suggest that the mechanism, which enables Epstein-Zin-Weil preferences with our utility kernel to explain asset prices, is *not* to separate the IES from RRA, but instead to introduce strong preferences for early resolution to amplify effects of long-run risk.

The remainder of this paper is organized as follows. Section 2 introduces our new utility kernel within the long-run risk model. Section 3 estimates this extension of the long-run risk model and study its empirical performance. Section 4 considers a New Keynesian model with the proposed utility kernel and explore its empirical performance. Concluding comments are provided in Section 5.[2]

2 A Long-Run Risk Model

This section extends the long-run risk model with a new utility kernel. The representative household is introduced in Section 2.1, and the exogenous processes for consumption and dividends are specified in Section 2.2. We present the new utility kernel in Section 2.3 and derive the implied IES and RRA in Section 2.4. We finally study the asset pricing properties of our new utility kernel in Section 2.5 using an analytical second-order perturbation solution.

2.1 The Representative Household

Consider a representative household with recursive preferences as in Epstein and Zin (1989) and Weil (1990). Using the convenient formulation proposed in Rudebusch and Swanson

[2] All technical derivations and proofs are deferred to an online appendix available from the homepage of the authors or on request.

(2012), the value function V_t is given by

$$V_t = \mathcal{U}_t + \beta \mathbb{E}_t [V_{t+1}^{1-\alpha}]^{\frac{1}{1-\alpha}} \tag{1}$$

for $\mathcal{U}_t > 0$, where $\mathbb{E}_t$ is the conditional expectation given information in period t.[3] The subjective discount factor is given by $\beta \in (0,1)$, and $\mathcal{U}_t \equiv \mathcal{U}(C_{h,t}, C_t)$ denotes the utility kernel as a function of home consumption $C_{h,t}$ and market consumption C_t. For higher values of the Epstein-Zin-Weil parameter $\alpha \in \mathbb{R} \setminus \{1\}$, these preferences generate higher levels of risk aversion if $\mathcal{U}_t$ always is positive, and vice versa for $\mathcal{U}_t < 0$. This allows (1) to separate the IES and RRA, which otherwise have a perfectly inverse relationship when $\alpha = 0$ and (1) simplifies to standard expected utility.

Another important property of (1) is to embed the household with preferences for resolution of uncertainty. This behavioral property is determined by the aggregation function in (1), i.e. by $f\left(\mathcal{U}_t, \mathbb{E}_t\left[V_{t+1}^{1-\alpha}\right]\right) \equiv \mathcal{U}_t + \beta\left(\mathbb{E}_t\left[(V_{t+1})^{1-\alpha}\right]\right)^{\frac{1}{1-\alpha}}$, where the household displays preferences for early (late) resolution of uncertainty if $f(\cdot,\cdot)$ is convex (concave) in its second argument (see Weil (1990)). The formulation in (1) therefore implies preferences for early (late) resolution of uncertainty if $\alpha > 0$ ($\alpha < 0$).[4] Given that α controls the degree of curvature in $f(\cdot,\cdot)$ with respect to $\mathbb{E}_t\left[V_{t+1}^{1-\alpha}\right]$, it seems natural to consider α as an crude measure for the strength of the household's timing attitude. That is, we will say that a household with a large (small) value of α displays strong (weak) preferences for early resolution of uncertainty when $\mathcal{U}_t > 0$, and correspondingly for late resolution of risk.[5]

The representative household has access to a complete market for state contingent claims $A_{t+1}(s)$, paying 1 unit of market consumption in period $t+1$ if state s is materialized. Resources are spent on C_t and state contingent claims, and we therefore have the following budget restriction

$$C_t + \mathbb{E}_t\left[M_{t,t+1} A_{t+1}\right] = A_t, \tag{2}$$

where $M_{t,t+1}$ denotes the real stochastic discount factor. Note that this budget restriction is independent of $C_{h,t}$, because this good is produced and consumed within the household.

[3] The specification in (1) is equivalent to the one in Epstein and Zin (1989), i.e. $\hat{V}_t^{\rho} = \hat{\mathcal{U}}_t^{\rho} + \beta \mathbb{E}_t\left[\hat{V}_{t+1}^{\hat{\alpha}}\right]^{\rho/\hat{\alpha}}$, if we let $V_t = \hat{V}_t^{\rho}$, $\mathcal{U} = \hat{\mathcal{U}}^{\rho}$, and $\alpha = 1 - \hat{\alpha}/\rho$. When $\mathcal{U}_t < 0$, we define $V_t = \mathcal{U}_t - \beta \mathbb{E}_t[(-V_{t+1})^{1-\alpha}]^{\frac{1}{1-\alpha}}$.

[4] The opposite sign restrictions apply when $\mathcal{U}_t < 0$.

[5] To our knowledge, there does not exist a measure quantifying the household's timing attitude for a general multi-good utility kernel with Epstein-Zin-Weil preferences as considered in (1). The measure proposed by Epstein et al. (2014) may be implemented when the utility kernel is defined across a single good by the power or the logarithm function but its extension to a general multi-good utility kernel is currently not available.

2.2 Consumption and Dividends

The process for market consumption is specified to be compatible with production economies displaying balanced growth.[6] Hence, we let

$$C_t \equiv Z_t \times \tilde{C}_t, \tag{3}$$

where $Z_t > 0$ is the balanced growth path of technology, or simply the productivity level, whereas $\tilde{C}_t$ refers to cyclical variation in consumption around Z_t. The level of Z_t coincides with long-lasting supply shocks in production economies and is typically specified with deterministic and stochastic trends. Uncertainty about the future productivity level in the economy is therefore captured by Z_t. Following Bansal et al. (2010), the second component $\tilde{C}_t$ in (3) introduces cyclical consumption risk, which in production economies originates from demand-related shocks, monetary policy shocks, or short-lived supply shocks (see Smets and Wouters (2007), Justiniano and Primiceri (2008), among others).

Inspired by the work of Bansal and Yaron (2004), we assume that the productivity level evolves as

$$\begin{array}{rcl} \log Z_{t+1} & = & \log Z_t + \log \mu_z + x_t + \sigma_z \sigma_t \varepsilon_{z,t+1} \\ x_{t+1} & = & \rho_x x_t + \sigma_x \sigma_t \varepsilon_{x,t+1} \end{array}, \tag{4}$$

with the conditional volatility σ_t for fluctuating economic uncertainty given by

$$\sigma_{t+1}^2 = 1 - \rho_\sigma + \rho_\sigma \sigma_t^2 + \sigma_\sigma \varepsilon_{\sigma,t+1}. \tag{5}$$

Here, $\varepsilon_{i,t+1} \sim \mathcal{NID}(0,1)$ for $i \in (z, x, \sigma)$ with $|\rho_x| < 1$ and $|\rho_\sigma| < 1$.[7] Hence, productivity has a deterministic trend when $\log \mu_z \neq 0$ and a stochastic trend for $\sigma_z > 0$ or $\sigma_x > 0$. Our specification in (3) implies the same trends in market consumption, where x_t introduces persistent changes in the growth rate of Z_t and captures long-run productivity risk. The innovation $\varepsilon_{z,t}$ does not generate any persistence in the growth rate of Z_t and is therefore referred to as short-run productivity risk.[8] Variation in consumption around Z_t is specified as in Bansal et al. (2010) by letting

$$\log \tilde{C}_{t+1} = \rho_{\tilde{c}} \log \tilde{C}_t + \sigma_{\tilde{c}} \sigma_t \varepsilon_{\tilde{c},t+1}, \tag{6}$$

[6] See King et al. (1988) and King and Rebelo (1999) for a detailed exposition.

[7] Although (5) does not enforce $\sigma_t^2 \geq 0$, we nevertheless maintain this specification for comparison with Bansal and Yaron (2004) and Bansal et al. (2010). Accounting for the non-negativity constraint on σ_t^2 may be done using a log-normal process for σ_t, as in Schorfheide et al. (2014), or the two specifications mentioned in Andreasen (2010). Asset prices may also for these alternative specifications be computed by the perturbation method as applied below.

[8] Hence, we follow the terminology from the long-run risk model (see for instance Bansal et al. (2010)), although variation in $\varepsilon_{z,t}$ has a permanent effect on the *level* of Z_t.

where $\varepsilon_{\tilde{c},t} \sim \mathcal{NID}(0,1)$ and $|\rho_{\tilde{c}}| < 1$.

To see how (3) to (6) relate to the existing literature, note that

$$\Delta c_{t+1} = \log \mu_z + x_t + \Delta \tilde{c}_{t+1} + \sigma_z \sigma_t \varepsilon_{z,t+1}, \tag{7}$$

where $\Delta c_{t+1} \equiv \log(C_{t+1}/C_t)$, $\tilde{c}_t \equiv \log \tilde{C}_t$, and $\Delta \tilde{c}_{t+1} \equiv \rho_{\tilde{c}} \Delta \tilde{c}_t + \sigma_{\tilde{c}} (\sigma_t \varepsilon_{\tilde{c},t+1} - \sigma_{t-1} \varepsilon_{\tilde{c},t})$. Our specification is thus similar to the one in Bansal et al. (2010) without jumps, which for $\sigma_{\tilde{c}} = 0$ reduces to the consumption process in Bansal and Yaron (2004) without cyclical risk.

Finally, as in Bansal and Yaron (2004), we let dividends D_t be positively correlated with productivity in the following way

$$\Delta d_{t+1} = \log \mu_d + \phi x_t + \sigma_d \sigma_t \varepsilon_{d,t+1}, \tag{8}$$

where $d_{t+1} \equiv \log D_{t+1}$, $\varepsilon_{d,t} \sim \mathcal{NID}(0,1)$, and ϕ denotes firm leverage (see Abel (1999)). For completeness, all innovations are assumed to be mutually uncorrelated at all leads and lags.

2.3 The Utility Kernel

Before presenting our new utility kernel, let us briefly motivate its construction. It is well-known that only the power- and the log-utility kernels are feasible in the baseline consumption-based asset pricing model when market consumption is trending. By accounting for ratio-habits in market consumption, we avoid this constraint and obtain greater flexibility in modeling $u(\cdot)$ and its derivatives compared to the standard power- and log-specifications. The presence of ratio-habits eliminate effects of long-run productivity risk through market consumption in the utility kernel, which is unfortunate for our study of the timing attitude, and we therefore also include home consumption as a way to re-introduce long-run productivity risk into the utility kernel.

More formally, the adopted specification for the utility kernel is given by

$$\mathcal{U}(C_{h,t}, C_t) = C_{h,t}^{\chi} u\left(\frac{C_t}{H_t}\right). \tag{9}$$

where we follow Benhabib et al. (1991), Greenwood and Hercowitz (1991), among others and allow for non-separability between home consumption $C_{h,t}$ and market consumption C_t. As in Abel (1990) and Fuhrer (2000), the household displays external habits H_t in C_t, which we include because they expand the set of functional forms for $u(\cdot)$ beyond the standard power- and log-specification, provided C_t/H_t is non-trending. To ensure that the marginal utility of home consumption is positive, we require that $\chi \in (0,1)$ and $u(\cdot) > 0$, in addition to the usual conditions that $u'(\cdot) > 0$ and $u''(\cdot) < 0$. It is also easy to show that strict concavity of $\mathcal{U}(C_{h,t}, C_t)$ is satisfied provided

$$(\chi - 1)\, u\left(\frac{C_t}{H_t}\right) u''\left(\frac{C_t}{H_t}\right) > \chi u'\left(\frac{C_t}{H_t}\right)^2, \tag{10}$$

meaning that the marginal utility of $C_{h,t}$ and C_t can not be too high relative to the product $u(\cdot)\, u''(\cdot)$. As we will demonstrate below, this condition is not particularly restrictive and clearly satisfied in all of the cases considered in the present paper.

As traditionally assumed in models with home goods, we assume that the production function of these goods is proportional to the productivity level, i.e. $Y_{h,t} = Z_t$, where $Y_{h,t}$ denotes the number of home goods produced.[9] The considered specification of consumption habits is given by $H_t = Z_t$, which in a simple way to ensure that habits are slow-moving and related to the trend in market consumption, as typically assumed in the literature (see for instance Campbell and Cochrane (1999)). It is obvious that more sophisticated habit specifications are compatible with (9), but we prefer our formulation in the interest of simplicity.[10]

Given the budget restriction in (2) and the assumption of strictly positive marginal utility of home consumption, it is clearly optimal for the household to consume all home goods, i.e. $C_{h,t} = Z_t$. Exploiting this optimality condition and the adopted habit specification, (9) simplifies to

$$\mathcal{U}(C_t) = Z_t^{\chi} u\left(\frac{C_t}{Z_t}\right). \tag{11}$$

Hence, an increase in the economy's productivity level Z_t may not only increase $\mathcal{U}$ through higher market consumption, but also through higher home consumption. Thus, home production leads to a direct effect of productivity shocks in the utility function, and this may increase the household's exposure to productivity risk compared to the standard long-run risk model without home consumption and habits. The effect of productivity risk is clearly increasing in the marginal utility of home consumption as determined by χ, whereas cyclical consumption risk $\tilde{C}_t = C_t/Z_t$ operates through $u(\cdot)$. Note also that the standard power-specification $\mathcal{U}(C_t) = C_t^{\chi}$ is nested by (11) when $u(C_t/Z_t) = (C_t/Z_t)^{\chi}$ and the utility from habit-adjusted market consumption coincides with the utility from home consumption.

[9] It is straightforward to extend our model to the more general production function ΩZ_t^{ω} for home goods with $\omega \in [0, 1]$, where $\Omega > 0$ may represent a fixed fraction of leisure or capital used in home production.

[10] Inspired by Fuhrer (2000), one could for instance let $H_t = (1 - \rho_H) \sum_{i=0}^{\infty} \rho_H^i C_{t-1-i}$, with the recursive formulation $H_t = (1 - \rho_H)\, C_{t-1} + \rho_H H_{t-1}$, but this extension comes at the expense of introducing the additional utility parameter ρ_H.

2.4 Behavioral Implications

The degree of intertemporal substitution in (11) at the steady state is given by

$$\mathrm{IES} = -\frac{u'(1)}{u''(1)}, \tag{12}$$

as the level of habit-adjusted consumption C_t/Z_t is equal to one at this point, according to (6). Using the expression in Swanson (2013), the relative risk aversion for (1) and (11) is

$$\mathrm{RRA} = \frac{1}{\mathrm{IES}} + \alpha \frac{u'(1)}{u(1)} \tag{13}$$

at the steady state. Hence, risk aversion is determined by the IES, the timing attitude as measured by α, and the ratio $u'(\cdot)/u(\cdot)$. On the other hand, χ does not affect RRA because the household in equilibrium has no margin to vary $C_{h,t}$ to better absorb shocks.[11] We also note that $u'(1)/u(1)$ plays a key role for RRA, as it determines how the household's timing attitude translates into higher or lower risk aversion. In other words, for a given IES and a given timing attitude, $u'(1)/u(1)$ determines RRA. To understand the intuition behind this effect, consider the case where $u'(1)/u(1)$ is low, and variation in $u(\cdot)$ therefore is relatively small compared to its steady state level. This implies that the value function only displays small variation, and we therefore have that the timing attitude α only has a small effect on RRA. This property of $u'(1)/u(1)$ is often overlooked in the existing literature, because much focus has been devoted to the power utility kernel $\mathcal{U}(C_t) = C_t^{\chi}$, where χ determines both the IES and $u'(1)/u(1)$. Our subsequent analysis will carefully outline the importance of $u'(1)/u(1)$ and show how a flexible specification of this ratio may be useful for explaining asset prices from consumption behavior.

Finally, with marginal utility of market consumption given by $\mathcal{U}'(C_t) = Z_t^{\chi-1}u'(C_t/Z_t)$, the stochastic discount factor becomes

$$M_{t,t+1} = \beta \left(\frac{\mathbb{E}_t \left[V_{t+1}^{1-\alpha} \right]^{\frac{1}{1-\alpha}}}{V_{t+1}} \right)^{\alpha} \frac{u'(C_{t+1}/Z_{t+1})}{u'(C_t/Z_t)} \frac{Z_{t+1}^{\chi-1}}{Z_t^{\chi-1}}. \tag{14}$$

Hence, the standard ratio of marginal utilities $u'(C_{t+1}/Z_{t+1})/u'(C_t/Z_t)$ is extended with a correction for home goods $Z_{t+1}^{\chi-1}/Z_t^{\chi-1}$ and the Epstein-Zin-Weil term, where the time attitude α determines how surprises in the value function (i.e. $\mathbb{E}_t \left[V_{t+1}^{1-\alpha} \right]^{\frac{1}{1-\alpha}}/V_{t+1}$) affect the stochastic discount factor and hence asset prices. The expression in (14) also reveals that

[11] Unreported results show that χ, and hence marginal utility of home consumption, has a small impact on RRA outside of steady state, for which (13) serves as a good approximation.

home consumption through χ has a strong effect on asset prices in our model, although χ does not affect the IES or RRA. The main asset pricing implications of home consumption go through three channels: i) the value function, ii) the ratio of marginal utilities, and iii) the conditional covariance between $M_{t,t+1}$ and dividends as both Δc_{t+1} and Δd_{t+1} co-move with the productivity level, i.e. Z_t.

2.5 Understanding Asset Prices

To explain how the IES, the RRA, and the timing attitude affect asset prices in our new utility kernel, we follow Bansal and Yaron (2004) and consider a simplified version of the long-run risk model without stochastic volatility, i.e. $\sigma_\sigma = 0$. Given the unspecified form of $u(\cdot)$, the stochastic discount factor is not necessarily log-linear and asset prices can not be computed using the log-normal method as in Bansal and Yaron (2004) or the approach taken in Hansen et al. (2008). Instead, we use the perturbation method of Judd and Guu (1997) to derive a novel analytical second-order approximation to the long-run risk model with an unspecified functional form for $u(\cdot)$.[12]

To obtain our analytical approximation, we first solve for the log-transformed value function $v_t \equiv \log V_t$ and the *twisted* log-transformed value function $ev_t \equiv \log \mathbb{E}_t \left[e^{(1-\alpha)\left(v_{t+1}+\chi \log \mu_{z,t+1}\right)} \right]$ where $\mu_{z,t} \equiv Z_t / Z_{t-1}$. These approximations are then used to solve for the net risk-free rate $r_t^f \equiv \log R_t^f$, the price-dividend ratio $pd_t \equiv \log \left(P_t / D_t\right)$, and finally the net expected equity return $r_t^{m,e} \equiv \mathbb{E}_t \left[r_{t+1}^m \right]$, where $r_t^m \equiv \log R_t^m$. In the interest of space, we only provide the solution for the risk-free rate and the expected equity return.

Proposition 1 *The second-order approximation to the risk-free rate r_t^f and the expected equity return $r_t^{m,e}$ at the steady state are given by*

$$
\begin{aligned}
r_t^f &= r_{ss} + r_{\tilde{c}} \tilde{c}_t + r_x x_t + \frac{1}{2} r_{\tilde{c}\tilde{c}} \tilde{c}_t^2 + \frac{1}{2} r_{\sigma\sigma}^f \\
r_t^{m,e} &= r_{ss} + r_{\tilde{c}} \tilde{c}_t + r_x x_t + \frac{1}{2} r_{\tilde{c}\tilde{c}} \tilde{c}_t^2 + \frac{1}{2} r_{\sigma\sigma}^{m,e}
\end{aligned}
$$

[12] The analytical perturbation approximation may also be applied to the version of the long-run risk model with stochastic volatility, but its additional state variables complicate the model solution and make the effects of our new utility kernel less transparent.

where

$$
\begin{aligned}
r_{ss} &= -\log\beta-(\chi-1)\log\mu_z \\
r_{\tilde{c}} &= (1-\rho_{\tilde{c}})\frac{u''(1)}{u'(1)} \\
r_x &= 1-\chi \\
r_{\tilde{c}\tilde{c}} &= \left(1-\rho_{\tilde{c}}^2\right)\left(\frac{u'''(1)}{u'(1)}+\frac{u''(1)}{u'(1)}-\left(\frac{u''(1)}{u'(1)}\right)^2\right) \\
r_{\sigma\sigma}^f &= -\alpha v_x^2\sigma_x^2-[1+(1-\alpha)\chi(\chi-2)]\sigma_z^2-\sigma_{\tilde{c}}^2\left(\frac{u'''(1)}{u'(1)}+\frac{u''(1)}{u'(1)}(1-2\alpha v_{\tilde{c}})+\alpha v_{\tilde{c}}^2\right) \\
r_{\sigma\sigma}^{m,e} &= -(1-\kappa_1)pd_{\sigma\sigma}+\kappa_1\left(pd_{\tilde{c}\tilde{c}}+pd_{\tilde{c}}^2\right)\sigma_{\tilde{c}}^2+\kappa_1\left(pd_{xx}+pd_x^2\right)\sigma_x^2+\sigma_d^2
\end{aligned}
$$

and $\kappa_1=\frac{\beta\mu_z^{(\chi-1)}\mu_d}{1+\beta\mu_z^{(\chi-1)}\mu_d}$. *The derivatives of* v_t *and* pd_t *are provided in Appendix A.*

Proposition 1 shows that the expressions for r_t^f and $r_t^{m,e}$ only differ in their uncertainty corrections $r_{\sigma\sigma}^f$ and $r_{\sigma\sigma}^{m,e}$, which is explained by the fact that all the remaining terms represent a perfect foresight approximation. The steady state value of r_t^f and $r_t^{m,e}$ is directly affected by the impact of long-run productivity risk in the utility kernel as controlled by χ, where high values of χ ensure low returns in the steady state. To see how these results relate to Bansal and Yaron (2004), recall that the standard power utility specification is nested by our model when

$$
u\left(\frac{C_t}{Z_t}\right)=\frac{1}{1-1/\psi}\left(\frac{C_t}{Z_t}\right)^{1-1/\psi} \quad \text{and} \quad \chi=1-1/\psi. \tag{15}
$$

Given this particular specification, we obtain the well-known result $r_{ss}=-\log\beta+\log\mu_z/\psi$, where a high ψ =IES ensures low steady state returns, for instance $\psi=1.5$ as in Bansal and Yaron (2004). Returning to our general utility kernel in (11), the first-order effects from variation in $\tilde{c}_t$ may be re-expressed as $-(1-\rho_{\tilde{c}})/\text{IES}$ using (12), showing that $r_{\tilde{c}}<0$ due to the negative auto-correlation in $\Delta\tilde{c}_t$. A similar negative effect on the risk-free rate from cyclical consumption risk is reported in Bansal et al. (2010). We also find the traditional positive effect of long-run risk, i.e. $r_x>0$, because $\chi<1$. For the standard power kernel in (15), note also that r_x^f simplifies to $1/\psi>0$ as in Bansal and Yaron (2004). The uncertainty correction in the risk-free rate depends on i) the size of the shocks, ii) the curvature of $u(\cdot)$, iii) the timing attitude α, and iv) how cyclical consumption risk affects the value function through $v_{\tilde{c}}$. The same holds for $r_{\sigma\sigma}^{m,e}$ following an inspection of the approximated expression for pd_t provided in Proposition A.2.

To further explore the implications of our new utility kernel and how it relates to the equity premium and the risk-free rate puzzle, Proposition 2 provides the unconditional mean of the risk-free rate and the ex ante equity premium.

Proposition 2 *The unconditional mean of the risk-free rate* $\mathbb{E}\left[r_t^f\right]$ *and the ex ante equity premium* $\mathbb{E}\left[r_{t+1}^m - r_t^f\right]$ *in a second-order approximation at the steady state are given by*

$$\begin{aligned}\mathbb{E}\left[r_t^f\right] &= r_{ss}^f - \frac{\sigma_x^2}{2}\frac{\alpha\kappa_0^2\chi^2}{\left(1-\kappa_0\rho_x\right)^2} - \frac{\sigma_z^2}{2}\left[1+(1-\alpha)\,\chi\,(\chi-2)\right] \\ &\quad -\frac{\sigma_{\tilde{c}}^2}{2}\left[\frac{u''(1)^2}{u'(1)^2} - 2\alpha\frac{u''(1)}{u'(1)}\frac{u'(1)}{u(1)}\frac{1-\kappa_0}{1-\kappa_0\rho_{\tilde{c}}} + \alpha\frac{u'(1)^2}{u(1)^2}\frac{\left(1-\kappa_0\right)^2}{\left(1-\kappa_0\rho_{\tilde{c}}\right)^2}\right]\end{aligned}$$

and

$$\begin{aligned}\mathbb{E}\left[r_{t+1}^m - r_t^f\right] &= \sigma_x^2\frac{\alpha\kappa_0\kappa_1}{\left(1-\kappa_0\rho_x\right)\left(1-\kappa_1\rho_x\right)}\chi\left(\phi+\chi-1\right) \\ &\quad +\sigma_{\tilde{c}}^2\left[\kappa_1\frac{1-\rho_{\tilde{c}}}{1-\kappa_1\rho_{\tilde{c}}}\frac{u''(1)^2}{u'(1)^2} - \alpha\kappa_1\frac{\left(1-\rho_{\tilde{c}}\right)\left(1-\kappa_0\right)}{\left(1-\kappa_0\rho_{\tilde{c}}\right)\left(1-\kappa_1\rho_{\tilde{c}}\right)}\frac{u''(1)}{u'(1)}\frac{u'(1)}{u(1)}\right].\end{aligned}$$

The auxiliary parameters are $\kappa_0 = \beta\mu_z{}^{\chi}$ *and* $\kappa_1 = \frac{\beta\mu_z^{(\chi-1)}\mu_d}{1+\beta\mu_z^{(\chi-1)}\mu_d}$.

Proposition 2 shows that the mean risk-free rate is given by its steady level r_{ss}^f minus uncertainty corrections for each of the shocks affecting consumption. The first uncertainty correction $-\frac{1}{2}\sigma_x^2\alpha\kappa_0^2\chi^2/\left(1-\kappa_0\rho_x\right)^2$ relates to long-run productivity risk and has a negative impact on $\mathbb{E}\left[r_t^f\right]$ if $\alpha > 0$ and the household has preferences for early resolution of risk. Importantly, this correction becomes more negative for larger values of χ and α. The second correction $-\frac{\sigma_z^2}{2}\left[1+(1-\alpha)\,\chi\,(\chi-2)\right]$ relates to short-run productivity risk and is also negative, given $\chi \in (0,1)$ and $\alpha > 1$. The uncertainty correction attached to cyclical consumption risk has three terms (in the bracket). The first term is given by $-1/2\sigma_{\tilde{c}}^2\left(u''(1)/u'(1)\right)^2 = -1/2\sigma_{\tilde{c}}^2/\text{IES}^2$ and decreases for lower values of the IES. The second term $\sigma_{\tilde{c}}^2\alpha\frac{u''(1)}{u'(1)}\frac{u'(1)}{u(1)}\frac{1-\kappa_0}{1-\kappa_0\rho_{\tilde{c}}}$ is also negative, given $\alpha > 0$ and the restrictions on $u(\cdot)$ and its derivatives. The final term $-\frac{\sigma_{\tilde{c}}^2}{2}\alpha\frac{u'(1)^2}{u(1)^2}\frac{(1-\kappa_0)^2}{(1-\kappa_0\rho_{\tilde{c}})^2}$ is also negative.

The equity premium depends positively on x_t if $\phi + \chi > 1$ and $\alpha > 0$. We also note that the effect is increasing in i) the persistency of x_t as determined by ρ_x, ii) the timing attitude α, iii) firm leverage ϕ, and iv) the marginal utility of home consumption as controlled by χ. The impact of cyclical consumption risk on the equity premium is determined by two terms, which both are positive given $\alpha > 0$ and the restrictions on $u(\cdot)$ and its derivatives. The magnitude of these terms dependent on the curvature of $u(\cdot)$, the timing attitude α, and the persistency of the cyclical risk $\rho_{\tilde{c}}$.

To summarize our insights from these analytical expressions, recall that existing models tend to generate too low equity premia and too high risk-free rates. Given identical returns for equity and the risk-free rate under perfect foresight, we thus require a positive uncertainty correction in $\mathbb{E}\left[r_{t+1}^{m} - r_{t}^{f}\right]$ and a negative uncertainty correction in $\mathbb{E}\left[r_{t}^{f}\right]$ to simultaneously resolve the equity premium and the risk-free rate puzzle. The utility kernel we propose in (9) does exactly so for high values of α and χ in combination with low values of the IES. We also note that household's timing attitude α has a separate effect on asset prices beyond the IES and RRA, which is consistent with the experimental evidence in Chew and Ho (1994) and van Winden et al. (2011). Another interesting observation from our analytical approximation is that the functional form of $u(\cdot)$ matters beyond what is implied by RRA and IES.

3 Estimation Results: The Long-Run Risk Model

This section studies the ability of this modified long-run risk model to explain key features of the post-war U.S. stock market. We first describe the model solution and estimation methodology in Section 3.1. The estimation results for the standard long-run risk model are provided in Section 3.2 as a natural benchmark. Section 3.3 to 3.5 consider different functional forms for $u(\cdot)$ in our utility kernel, whereas Section 3.6 studies the performance of the long-run risk model on moments not included in the estimation. We finally consider three counterfactual experiments in Section 3.7 to understand the relative importance of the IES, the RRA, and the timing attitude when explaining asset prices.

3.1 Model Solution and Estimation Methodology

We solve for asset prices using a perturbation approximation as in Section 2.5, but use a third-order expansion to allow for time-variation in risk premia. This approximation is computed using the algorithm of Binning (2013). The estimation is carried out on quarterly data, as this data frequency strikes a good balance between getting a reasonably long sample and providing reliable measures of consumption and dividend growth.[13] Consistent with the common calibration procedure for the long-run risk model, we let one period in the model correspond to one month and time-aggregate the teoretical moments to the frequency in the data.

Our quarterly data set is from 1947Q1 to 2014Q4, where we use the same five variables as in Bansal and Yaron (2004): i) the log-transformed price dividend ratio pd_t, ii) the real

[13] Although the long-run risk model often is calibrated to moments in annual data, as in Bansal and Yaron (2004), its performance is remarkably robust and carries over to quarterly data (see Bansal et al. (2012*b*) and Beeler and Campbell (2012)).

risk-free rate r_t^f, iii) the market return r_t^m, iv) consumption growth Δc_t, and v) dividend growth Δd_t.[14] All variables are stored in $\mathbf{data}_t$ with dimension 5×1. We explore whether our model can match the means, the variances, the contemporaneous covariances, and the persistence in these five variables. Hence, we let

$$\mathbf{q}_t \equiv \begin{bmatrix} \mathbf{data}_t \\ vec\left(\mathbf{data}_t \mathbf{data}_t'\right) \\ diag\left(\mathbf{data}_t \mathbf{data}_{t-1}'\right) \end{bmatrix},$$

where $diag\,(\cdot)$ denotes the diagonal elements of a matrix. Letting $\boldsymbol{\theta}$ contain the model parameters, the GMM estimator of Hansen (1982) is then given by the value of $\boldsymbol{\theta}$ that minimizes the objective function

$$Q = \left(\frac{1}{T}\sum_{t=1}^{T} \mathbf{q}_t - \mathbb{E}\left[\mathbf{q}_t\left(\boldsymbol{\theta}\right)\right]\right)' \mathbf{W}_T \left(\frac{1}{T}\sum_{t=1}^{T} \mathbf{q}_t - \mathbb{E}\left[\mathbf{q}_t\left(\boldsymbol{\theta}\right)\right]\right),$$

where $\frac{1}{T}\sum_{t=1}^{T} \mathbf{q}_t$ denotes the empirical moments and $\mathbf{W}_T$ is the weighting matrix. The model-implied moments $\mathbb{E}\left[\mathbf{q}_t\left(\boldsymbol{\theta}\right)\right]$ are computed in closed form as in Andreasen et al. (2013) using a pruning scheme when constructing the approximated model solution.[15] We adopt the conventional 2-step implementation of GMM and use a diagonal weighting matrix based on the variance of the sample moments in a preliminary first step, before obtaining our final estimate $\hat{\boldsymbol{\theta}}$ using the optimal weighting matrix.[16]

3.2 The Benchmark Model

As a natural benchmark, we first consider the standard long-run risk model, which corresponds to a version of our model without home production and habits. That is, we let

$$\mathcal{U}(C_t) = \frac{1}{1 - 1/\psi} C_t^{1-1/\psi}. \tag{16}$$

For comparability with nearly all calibrations of the long-run risk model, we let RRA $= 10$ and IES $= 1.5$ as in Bansal and Yaron (2004).[17] Table 1 shows that x_t generates a small but very persistent component in consumption growth with $\hat{\sigma}_x = 0.0004$ and $\hat{\rho}_x = 0.9802$. As in

[14] Details on the data sources and data construction are provided in the online appendix.

[15] Omitting the pruning scheme for the approximated model solution and computing unconditional moments by the Monte Carlo method gives nearly identical results.

[16] The weigthing matrices are in both steps computed by the estimator of Newey and West (1987) using 15 lags, but our results are robust to using either 10 or 20 lags.

[17] The desired level of risk aversion is obtained by setting α appropriately using (13). Estimating the RRA and the IES give nearly identical results to those provided in Tables 1 and 2.

the calibration of Bansal et al. (2012a), we also find the conditional volatility to be highly persistent ($\hat{\rho}_{\sigma} = 0.9942$), as it amplifies the volatility channel in the model. Variation in cyclical consumption risk is much more mean-reverting ($\tilde{\rho}_{\tilde{c}} = 0.17$), but still important given the relatively large value of $\hat{\sigma}_{\tilde{c}} = 0.0033$. We also note that the constraint on the effective discount factor $\beta^* \equiv \beta\mu_z^{1-1/\psi} < 1$ for (1) with (16) is binding, suggesting that the standard utility kernel at least along this dimension is constrained in its ability to fit the data.

< Table 1 about here >

Table 2 verifies the common finding in the literature that the standard long-run risk model is able to explain several asset pricing moments. In particular, the model provides a very satisfying fit to the mean and standard deviation of the price-dividend ratio, the risk-free rate, and the market return. The only possible exception is the mean market return which is somewhat lower than in the data (5.44% vs. 6.92%), but still well within the 95% confidence interval. Omitting cyclical consumption risk by letting $\sigma_{\tilde{c}} = 0$ as in Bansal and Yaron (2004), does not affect the mean of asset prices but lowers the standard deviation of the risk-free rate from 2.21% to 1.66%.[18] Table 2 also shows that our estimated version of the long-run risk model underestimates the persistence in the risk-free rate (0.57 vs. 0.87) and overestimates the standard deviation in consumption growth (2.90% vs. 2.04%) as well as its persistence (0.57 vs. 0.31).

< Table 2 about here >

The last part of Table 2 reports the contemporaneous correlations, where we observe that consumption growth in the standard long-run risk model is too highly correlated with the price-dividend ratio (0.20 vs. 0.03), which is similar to the finding reported in Beeler and Campbell (2012). We further note that consumption growth is also too strongly correlated with the risk-free rate (0.36 vs. 0.16). Conventional two-sided t-tests based on the reported standard errors for the sample moments in Table 2 further show that the difference in $corr\left(pd_t, \Delta c_t\right)$ has a P-value of 6.44% and is therefore statistical significant at the 10% level. The P-values for the differences in $corr\left(r_t^f, \Delta c_t\right)$ is 4.88% and hence statistical significant at the 5% level.[19]

[18] Similar effects of cyclical consumption risk are reported in Bansal et al. (2012a).

[19] Using the log-normal method and the calibration in Bansal and Yaron (2004), the long-run risk model implies $corr\left(pd_t, \Delta c_t\right) = 0.547$ and $corr\left(r_t^f, \Delta c_t\right) = 0.581$, whereas the corresponding empirical moments on annual data are 0.061 and 0.356, respectively. The slightly modified calibration in Bansal et al. (2012a) with less long-run risk gives $corr\left(pd_t, \Delta c_t\right) = 0.368$ and $corr\left(r_t^f, \Delta c_t\right) = 0.473$. Thus, the elevated correlations for $corr\left(pd_t, \Delta c_t\right)$ and $corr\left(r_t^f, \Delta c_t\right)$ also appear in calibrated versions of the long-run risk model using annual data.

To understand why consumption growth is too highly correlated with pd_t and r_t^f, recall that the standard long-run risk model relies on the power utility kernel with an IES = 1.5 and RRA = 10. Equation (13) then implies a relative low value of the Epstein-Zin-Weil parameter ($\alpha = 28$) and hence a somewhat modest effect from early resolution of uncertainty. To explain the level of the risk-free rate and the market return, the model therefore requires high persistence in x_t to amplify the long-run risk channel (see Section 2.5). But, this high level of persistence in x_t generates too much variability and persistence in consumption growth according to Table 2. Using the analytical approximation in Section 2.5, we also observe that

$$cov(\Delta c_t, pd_t) = \frac{\phi + \chi - 1}{1 - \kappa_1 \rho_x} \rho_x \frac{\sigma_x^2}{1 - \rho_x^2} + \frac{(1 - \rho_{\tilde{c}})^2}{1 - \kappa_1 \rho_{\tilde{c}}} \frac{1}{\text{IES}} \frac{\sigma_{\tilde{c}}^2}{1 - \rho_{\tilde{c}}^2} \tag{17}$$

$$cov(\Delta c_t, r_t^f) = (1 - \chi)\, \rho_x \frac{\sigma_x^2}{1 - \rho_x^2} - (1 - \rho_{\tilde{c}})^2 \frac{1}{\text{IES}} \frac{\sigma_{\tilde{c}}^2}{1 - \rho_{\tilde{c}}^2} \tag{18}$$

which both are increasing in ρ_x. Hence, an undesirable effect of the high persistence in x_t is to amplify the co-movement of consumption growth with respect to pd_t and r_t^f. To illustrate the sizable effect of the long-run risk channel, consider lowering ρ_x from its estimated value of 0.98 to 0.95, with all remaining parameters unchanged compared to Table 1. This modification ensures that consumption growth now closely matches its standard deviation (2.27 vs. 2.04) and persistence (0.30 vs. 0.31), in addition to being less correlated with pd_t and r_t^f, as shown in (17) and (18). But the model is now unable to explain the equity premium puzzle, as the average market return falls from 5.44% to just 2.19%.

3.3 A New Utility Kernel: a Power Specification

We next explore whether our more flexible utility kernel with home production and habits enables the long-run risk model to resolve this puzzle for consumption growth without distorting the fit of the remaining moments. Given the popularity of the power function, the most obvious specification is probably to let

$$\mathcal{U}(C_t) = \frac{Z_t^{\chi}}{1 - 1/\psi} \left(\frac{C_t}{Z_t} \right)^{1-1/\psi}, \tag{19}$$

where χ and ψ are free parameters.[20] The GMM estimates in Table 1 reveal that $\hat{\chi} = 0.51$, the IES is $\hat{\psi} = 1.02$, and $\widehat{\text{RRA}} = 13.52$. A Wald test clearly rejects the restriction $1 - 1/\psi = \chi$

[20] For instance, the recent paper by Creal and Wu (2016) considers Epstein-Zin-Weil preferences with ratio-habits when the utility kernel has a power specification. We also note that the utility kernel in (19) may be re-expressed as $\mathcal{U}(C_t) = \frac{Z_t^{\chi - 1 + 1/\psi}}{1 - 1/\psi} C_t^{1-1/\psi}$, showing that (19) is identical to a model without habits but with a modified curvature parameter for Z_t.

from the benchmark model in (16) at all conventional significance levels, as $1-1/\hat{\psi}=0.02$ and thus substantially lower than $\hat{\chi}$. The condition for concavity in (10) reduces to $\chi\psi<1$ for (19), which clearly holds given these estimates. We also note that $\hat{\psi}$ is just above its lower bound of one, which is the borderline for $\mathcal{U}(C_t)$ to be positive, as required for home consumption to have positive marginal utility.

Table 2 further shows that this version of the long-run risk model displays less persistence in x_t with $\hat{\rho}_x=0.645$, and the model therefore improves the fit of the auto-correlations and some contemporaneous correlations compared to the benchmark model. In particular, $corr\left(pd_t,\Delta c_t\right)$ and $corr\left(r_t^f,\Delta c_t\right)$ are now substantially lower than in the benchmark model and no longer significantly different from the empirical moments. Unfortunately, this improvement comes at the cost of a slightly worse fit to all standard deviations, except for consumption growth.

To evaluate the overall goodness of the fit for the long-run risk model with (19), Table 2 also reports the value of the objective function Q^{step2} in step 2 of our GMM estimation and the related P-value for the well-known J-test for model misspecification (see Hansen (1982)). The benchmark model and our extension(s) are not rejected by the data, but we note that the J-test has low power given our fairly short sample ($T=271$). The values of Q^{step2} are unfortunately not comparable across models, because they are computed for model-specific weighting matrices. To facilitate model comparison, we therefore introduce the following measure for goodness of fit

$$Q^{scaled}=\sum_{i=1}^{n}\left(\frac{m_i^{data}-m_i^{model}}{1+m_i^{data}}\right)^2, \tag{20}$$

where m_i^{data} and m_i^{model} refer to the scaled moments in the data and the model, respectively, as reported in Table 2.[21] Although the moments in (20) are weighted differently than in the estimation, Q^{scaled} may nevertheless serve as a natural summary statistics for model comparison from an economic perspective. We find that the benchmark model has a value of $Q^{scaled}=1.14$ and therefore marginally dominates an extension based on (19) with $Q^{scaled}=1.17$. Thus, the better fit of several auto- and contemporaneous correlations in this extension of the long-run risk model does not compensate for its weaker performance in matching standard deviations when using the weights in (20).

[21] The difference $m_i^{data}-m_i^{model}$ in (20) is standardized by $1+m_i^{data}$, as oppose to just m_i^{data}, to ensure that moments close to zero do not get very large weights.

3.4 A New Utility Kernel: a Semi-Parametric Specification

An important insight from our analytical expressions in Section 2.5 is that the functional form of $u(\cdot)$ has a substantial impact on asset prices. Hence, the inability of (19) to improve the overall fit of the benchmark model may be due to the adopted power-specification of $u(\cdot)$. Given that our setup does not restrict the functional form of $u(\cdot)$, we can easily explore this possibility by considering a semi-parametric version of our utility kernel, where $u\left(\frac{C_t}{Z_t}\right)$ is approximated using a second-order Taylor expansion at the steady state, where C_t/Z_t is equal to one. Hence, we momentarily let

$$\mathcal{U}(C_t) = Z_t^{\chi}\left(u(1) + u'(1)\left(\frac{C_t}{Z_t} - 1\right) + \frac{1}{2}u''(1)\left(\frac{C_t}{Z_t} - 1\right)^2\right). \tag{21}$$

The value of $u(1)$ is not empirically identified and therefore normalized to one, leaving only $u'(1)$ and $u''(1)$ as free parameters.[22] Although such a semi-parametric formulation is rarely considered, it may nevertheless provide useful insight into the properties of the long-run risk model, because it introduces three separate parameters to determine the three behavioral characteristics of Epstein-Zin-Weil preferences. That is, i) $u'(1)$ determines RRA, ii) $u''(1)$ controls the IES, and iii) α reflects the timing attitude. This formulation of Epstein-Zin-Weil preferences therefore separates the IES and RRA *without* imposing the well-known constraint on the timing attitude. Nevertheless, we should emphasize that this semi-parametric extension of the long-run risk model is not our preferred specification, but that it only serves to guide our choice of a more appropriate functional form for $u(\cdot)$ than the conventional power specification.

Table 1 shows that the preferred utility kernel is characterized by $\hat{\chi} = 0.46$, $\hat{u}'(1) = 5.190\times10^{-4}$, and $\hat{u}''(1) = -0.0042$, which imply $\hat{u}'(1)/u(1) = 5.190\times10^{-4}$ and $|\hat{u}''(1)/\hat{u}'(1)| = 8.0259$. Thus, these estimates differ substantially from the kernel in (19) by having a much larger value of $|\hat{u}''(1)/\hat{u}'(1)|$, which corresponds to a fairly low IES of 0.12.[23] With RRA estimated to 9.67, we find $\hat{\alpha} = 3,171$ and hence very strong preferences for early resolution of uncertainty, whereas the long-run risk channel is substantially reduced with $\hat{\rho}_x = 0.15$.

Table 2 further shows that (21) greatly improves the fit of the long-run risk model, which now matches most of the considered moments. For the contemporaneous correlations with consumption growth, we note that $corr\,(pd_t, \Delta c_t)$ and $corr\left(r_t^f, \Delta c_t\right)$ are matched by the model. As a result, this version of the long-run risk model has a very low value of

[22] Unreported results reveal that third- and fourth-order derivatives of $u(\cdot)$ are also not identified in our case, although these terms in principle may affect the third-order perturbation approximation.

[23] We also note that the concavity condition in (10) is clearly satisfied given these estimates, as $(\hat{\chi} - 1)\,\hat{u}''(1)$ is much larger than $\hat{\chi}\hat{u}'(1)^2$.

$Q^{scaled} = 0.42$, implying that it provides a substantially better overall fit to the data than the benchmark model with $Q^{scaled} = 1.14$.

3.5 A New Utility Kernel: an Exponential Power Specification

Our semi-parametric estimates show that home production, habits, *and* a sufficiently flexible utility kernel improve the performance of the long-run risk model. A natural question is what functional form of $u(\cdot)$ is capable of reproducing these semi-parametric estimates? Our proposed specification is given by

$$\mathcal{U}(C_t) = \frac{Z_t^{\chi}}{\chi}\left(\frac{1-e^{-\tau(C_t/Z_t)}}{\tau}\right)^{\chi}, \tag{22}$$

which we refer to as the exponential power utility kernel. The utility from home consumption remains controlled by χ, whereas the effect from habit-adjusted market consumption is determined by an exponential function indexed by $\tau > 0$. The utility from this latter part is raised to the power of χ to ensure that (22) reduces to the standard power utility kernel in (16) when $\tau \to 0$.[24] To illustrate some of the properties of the exponential power utility kernel and establish some intuition about τ, consider Figure 1 which plots the function $\mathcal{U}(C_t)$ in (22) for different values of τ. These plots reveal that τ controls the concavity of $u(\cdot)$ and hence the speed by which marginal utility decreases for higher consumption.

< Figure 1 about here >

To further study the properties of (22), consider its two steady state ratios

$$u'(1)/u(1) = \frac{\chi\tau}{e^{\tau}-1} \quad \text{and} \quad u''(1)/u'(1) = -\tau\left(1+\frac{1-\chi}{e^{\tau}-1}\right). \tag{23}$$

Hence, a higher value of τ lowers $u'(1)/u(1)$ and increases $|u''(1)/u'(1)|$. Given that the IES$= -u'(1)/u''(1)$, the latter corresponds to a lower IES for a higher value of τ. In contrast, the power specification in (19) implies that $u'(1)/u(1) = 1 - 1/\psi$ and $u''(1)/u'(1) = -\frac{1}{\psi}$, and this functional form is therefore unable to jointly generate a low value of $u'(1)/u(1)$ and a high value of $u''(1)/u'(1)$, as required to match the data according to our semi-parametric estimation results. The preference parameter τ also effects relative risk aversion, which is given by

$$\text{RRA} = \frac{1}{\text{IES}} + \alpha\frac{\chi\tau}{e^{\tau}-1}. \tag{24}$$

[24] Another alternative would be to let $\mathcal{U}(C_t) = \frac{Z_t^{\chi}}{1-1/\psi}\left(\frac{1-e^{-\tau(C_t/Z_t)}}{\tau}\right)^{1-1/\psi}$, but unreported results suggest that χ, ψ, and τ are not jointly identified.

The first term in (24) coincides with the measure of RRA obtained with standard expected utility, i.e. $\alpha = 0$, and increases in τ. The second term in (24) is due to the presence of Epstein-Zin-Weil preferences and decreases in τ. To understand this effect, recall that the second term of RRA equals $\alpha u'(1)/u(1)$, meaning that higher values of τ generate a larger reduction in $u'(1)$ compared to $u(1)$. This effect is also evident from Figure 1 as $u(\cdot)$ becomes very flat for a wide range of consumption levels when τ is large (i.e. very low marginal utility). This in turn makes the value function less responsive to changes in future consumption when τ increases, and we therefore see a reduction in the required compensation for holding risky assets, i.e. a lower RRA.[25]

The final column of Table 1 shows that $\hat{\tau} = 8.95$ with a standard error of 1.03. Hence, $\hat{\tau}$ is far from zero, meaning that (22) improves upon the standard power utility kernel in (16). The concavity condition in (10) simplifies to $\frac{e^{\tau(C_t/Z_t)}}{1+e^{\tau(C_t/Z_t)}} > \chi$ for (22) and is therefore clearly satisfied given that $\hat{\chi} = 0.51$. In line with previous calibrations of the long-run risk model, we also find a low degree or risk aversion with $\widehat{\text{RRA}} = 9.78$. The two steady state ratios in (23) are $u'(1)/u(1) = 0.0006$ and $|u''(1)/u'(1)| = 8.95$, and hence very close to the semi-parametric estimates in Section 3.4. This implies that the IES based on (22) is 0.11, which is consistent with the regression evidence in Hall (1988).

The last column of Table 2 shows that this extension of the long-run risk model provides a very satisfying fit to all means, standard deviations, and auto-correlations. The model is also very successful in matching contemporaneous correlations, where we in particular note that $corr(pd_t, \Delta c_t)$ and $corr\left(r_t^f, \Delta c_t\right)$ are no longer significantly different from the corresponding sample moments.[26] This result is mainly explained by the fact that $\hat{\rho}_x = 0.38$ is this version of the long-run risk model and hence substantially lower than in the benchmark model. Table 2 further shows that this improvement is achieved without distorting the fit of the remaining moments, as this fully parametric extension of the long-run risk model with just one additional parameter also gives a good overall fit with $Q^{scaled} = 0.50$, and hence clearly improves upon the benchmark model with $Q^{scaled} = 1.14$.

3.6 Additional Model Implications

In addition to the moments used in the estimation, the long-run risk model is also frequently evaluated based on its ability to reproduce several stylized relationships for the U.S. stock

[25]For completness, the absolute risk aversion implied by (22) is $\tau\left(1 + \frac{1-\chi}{e^{\tau}-1}\right)\left(e^{r_{ss}^f - \mu_z} - 1\right) + \alpha\frac{\chi\tau}{e^{\tau}-1}\left(e^{r_{ss}^f - \mu_z} - 1\right)$ at the steady state, and hence affected by other structural parameters than τ, although the exponential function enters in (22).

[26]The P-values for the related two-sided t-tests are 96.6% for $corr(pd_t, \Delta c_t)$ and 29.9% for $corr\left(r_t^f, \Delta c_t\right)$.

market. Appendix B shows that the exponential power utility kernel in (22) preserves the ability of the long-run risk model to predict excess market returns from the price-dividend ratio, and that the model does well in matching the relationship between consumption growth and the price-dividend ratio at various leads and lags. We also show in Appendix B, that (22) enables the long-run risk model to match the negative relationship between economic uncertainty and the price-dividend ratio, despite having an IES well below one.

3.7 The Key Mechanisms

Having documented the strong empirical performance of the exponential power utility kernel, we next consider three experiments to illustrate some of the key mechanisms in the model. The first experiment we consider is to gradually increase τ from $\tau \to 0$ to its estimated value. Table 3 shows that a higher value of τ generates a fast reduction in the IES and a substantial increase in the required value of α to ensure a constant RRA. This in turn has desirable effects on asset prices as a higher value of τ i) reduces $\mathbb{E}[pd_t]$ and $\mathbb{E}[r_t^f]$, ii) increases $\mathbb{E}[r_t^m]$, and iii) generates more variability in pd_t, r_t^f, and r_t^m.

To understand the effects of τ on $\mathbb{E}[r_t^f]$ and $\mathbb{E}[r_t^m]$ in greater detail, Table 3 also decomposes their values based on Proposition 2.[27] Here, we emphasize to effects. First, lowering the IES through higher values of τ does not affect $r_{ss} = -\log\beta - (\chi - 1)\log\mu_z$, which in contrast increases rapidly in the standard utility kernel when reducing the IES through large negative values of χ. Second, a high value of τ lowers $u'(\cdot)/u(\cdot)$ and allows for strong preferences for early resolution of uncertainty through a high α without making the household very risk averse. To understand the effect of increasing α for a *given* level of RRA, recall that the household is indifferent to resolution of uncertainty when $\alpha = 0$, and all uncertainty corrections are therefore either very small or absent. This case is well-represented by the first column in Table 3 where $\tau \to 0$. Now suppose we increase α to make the household prefer early resolution of uncertainty, but without affecting the RRA. This modification makes the certain cash flow from the one-period risk-free bond more attractive, and a lower risk-free rate is therefore required. This effect explains why we see larger negative corrections for long-run and short-run productivity risk as well as cyclical consumption risk in $\mathbb{E}[r_t^f]$ when increasing α. On the other hand, uncertain future dividends from equity become less attractive for higher values of α due to the presence of long-run productivity risk. A household with strong preferences for early resolution of uncertainty therefore requires a larger compensation

[27] This decomposition exploits the fact that the unconditional mean of any variable in the pruned state-space system with Gaussian shocks is identical at second- and third order (see Andreasen et al. (2013)). The contribution from stochastic volatility is then given by the difference between the unconditional mean at third order and the mean implied by Proposition 2, which omits stochastic volatility.

for long-run productivity risk (and its stochastic volatility) to hold equity compared to the case of $\alpha = 0$. Table 3 shows that these effects on $\mathbb{E}[r_t^m]$ for higher values of α dominate the larger negative risk corrections from short-run and cyclical risk, which are similar to those for the risk-free rate and hence constitute a pure discounting effect.

< Table 3 about here >

The second experiment we consider is to omit long-run productivity risk in the model by letting $\sigma_x = 0$. The fourth column in Table 3 shows that this modification has profound implications, as the model now generates a too high level for the the price-dividend rate (7.81 vs. 3.50) and the risk-free rate (1.91 vs. 0.83), whereas the average market return is too low (2.08 vs. 6.92). Note also that the model without long-run productivity risk generates insufficient variability in pd_t, r^f, and r_t^m. Thus, the exponential power utility kernel and the strong timing attitude do not alleviate the dependence on long-run productivity risk in the model, although habit-adjusted consumption $\tilde{C}_t = C_t/Z_t$ is estimated to be highly persistent with $\rho_{\tilde{C}} = 0.9962$ (see Table 1).

Our final experiment is motivated by the main research question in the present paper, that is whether the key effect of Epstein-Zin-Weil preferences is to separate the IES from RRA or to impose a timing attitude on households? We first note that the RRA implied by our exponential power utility kernel is only slightly larger than the inverse of the IES, and hence nearly satisfies the key relationship from expected utility. This implies that the Epstein-Zin-Weil parameter α almost has no effect on RRA and primarily serves to specify the household's timing attitude in our estimated extension of the long-run risk model. To explore the quantitative effect of the timing attitude on asset prices, we let $\alpha = 0$ to consider the case without Epstein-Zin-Weil preferences, where the household is indifferent between early and late resolution of uncertainty. The fifth column of Table 3 shows that this modification only lowers the RRA from 9.78 to 8.95, but it nevertheless has a profound impact on the model, which largely displays the same properties as when omitting long-run productivity risk. That is, the model is simply unable to match both the level and the variability of asset prices without Epstein-Zin-Weil preferences, and hence strong preferences for early resolution of uncertainty. The final column in Table 3 shows that this finding is not explained by the small reduction in RRA from 9.78 to 8.95, as a version of our model with RRA$= 9.78$ but a slightly lower IES $= 0.102$ gives largely the same asset pricing implications as seen with RRA $= 8.95$ in the fifth column. This suggests that the main effect of Epstein-Zin-Weil preferences with our exponential power utility kernel is *not* to separate the IES from RRA but instead to introduce strong preferences for early resolution of uncertainty.

4 A New Keynesian Model

To provide further support for the considered Epstein-Zin-Weil preferences with the exponential power utility kernel, we next show that they also help to explain asset prices in an otherwise standard New Keynesian model. The processes for consumption and dividends are here derived within the model and are not assumed to be exogenously given as in the long-run risk model. In addition to explaining the equity premium, we also show that these preferences enable the New Keynesian model to match the level and the variability of the nominal term premium and hence resolve the bond premium puzzle (see Rudebusch and Swanson (2008)). Our main finding is that we can explain these puzzles with a low RRA of 5, whereas most existing New Keynesian models require extreme levels of RRA to match these moments.

We proceed by presenting our New Keynesian model in Section 4.1, the adopted estimation routine in Section 4.2, and the estimation results in Section 4.3. We finally study the key mechanisms in our modified New Keynesian model in Section 4.4.

4.1 Model Description

The considered model is specified along the lines of Rudebusch and Swanson (2012) and Swanson (2015) for comparability with much of the existing macro-finance literature building on the standard New Keynesian models (see Hordahl et al. (2008), Andreasen (2012), among others).

4.1.1 Household

The representative household is similar to the one considered in Section 2, except for a variable labor supply L_t. To match the persistence in consumption growth, we follow much of the New Keynesian tradition and extend our consumption habits with bC_{t-1}. These modifications are included in the exponential power utility kernel by letting

$$\mathcal{U}\left(C_{h,t}, C_t, L_t\right) = C_{h,t}^{\chi}\left[\frac{1}{\chi}\left(\frac{1-e^{-\tau\left(\frac{C_t-bC_{t-1}}{Z_t}\right)}}{\tau}\right)^{\chi} + \varphi_0 \frac{\left(1-L_t\right)^{1-\frac{1}{\varphi}}}{1-\frac{1}{\varphi}}\right], \tag{25}$$

where $\chi \in (0,1)$, $\tau > 0$, $\varphi_0 > 0$, and $\varphi \in \mathbb{R} \setminus \{1\}$.[28] As in Section 2, we consider the case where the production function of home goods is proportional to the technology level, i.e. $Y_{h,t} = Z_t$, with $Y_{h,t}$ denoting the number of home goods produced.

[28]The conditions for strict concavity of $\mathcal{U}\left(C_{h,t}, C_t, L_t\right)$ are discussed in Appendix C.

The real budget constraint for the household is given by $\mathbb{E}_t\left[M_{t,t+1}\frac{X_{t+1}}{\pi_{t+1}}\right]+C_t=\frac{X_t}{\pi_t}+W_tL_t+D_t$, where X_t is nominal state-contingent claims, π_t denotes gross inflation, W_t is the real wage, and D_t is real dividend payments from firms. Given this budget restriction and strictly positive marginal utility of home consumption, it is clearly optimal for the household to consume all home goods, i.e. $C_{h,t}=Z_t$. This reduces the intertemporal optimization problem for the household to C_t, L_t, and the portfolio of state-contingent claims, meaning that the degree of RRA can be obtained using the general formulas in Swanson (2013). Accordingly, the Epstein-Zin-Weil preferences in (1) with the utility kernel in (25) implies

$$\text{RRA}=\frac{\tilde{C}_{ss}}{\frac{\exp\left\{\tau\left(1-b/\mu_{Z,ss}\right)\tilde{C}_{ss}\right\}-1}{\tau\left(\exp\left\{\tau\left(1-b/\mu_{Z,ss}\right)\tilde{C}_{ss}\right\}-\chi\right)}+\varphi\tilde{W}_{ss}\left(1-L_{ss}\right)}+\alpha\frac{\tilde{C}_{ss}\chi}{\frac{\exp\left\{\tau\left(1-b/\mu_{Z,ss}\right)\tilde{C}_{ss}\right\}-1}{\tau}+\frac{\chi\tilde{W}_{ss}\left(1-L_{ss}\right)}{1-\frac{1}{\varphi}}},\tag{26}$$

where $\tilde{C}_{ss}$ and $\tilde{W}_{ss}$ refer to the steady state of consumption and the real wage in the normalized economy without trending variables. The deterministic trend in consumption is given by $\mu_{Z,ss}$ which coincides with the deterministic trend in productivity, specified below in (28). Simple inspection of (26) reveals that the first term is undetermined in τ, whereas the second term decreases in τ for the same reason as outlined in Section 3.5. The IES at the steady state is given by

$$\text{IES}=\frac{1}{\tau\tilde{C}_{ss}}\frac{1-\exp\left\{\tau\left(1-b/\mu_{Z,ss}\right)\tilde{C}_{ss}\right\}}{\chi-\exp\left\{\tau\left(1-b/\mu_{Z,ss}\right)\tilde{C}_{ss}\right\}},\tag{27}$$

which converges to the familiar expression $\left(1-\frac{b}{\mu_{Z,ss}}\right)/\left(1-\chi\right)$ when $\tau\rightarrow 0$.

4.1.2 Firms

Final output Y_t is produced by a perfectly competitive representative firm, which combines a continuum of differentiated intermediate goods $Y_t\left(i\right)$ using the production function $Y_t=\left(\int_0^1 Y_t\left(i\right)^{\frac{\eta-1}{\eta}}di\right)^{\frac{\eta}{\eta-1}}$ where $\eta>1$. This implies that the demand for the ith good is $Y_t\left(i\right)=\left(\frac{P_t(i)}{P_t}\right)^{-\eta}Y_t$, where $P_t\equiv\left(\int_0^1 P_t\left(i\right)^{1-\eta}di\right)^{\frac{1}{1-\eta}}$ denotes the aggregate price level and $P_t\left(i\right)$ the price of the ith good.

The differentiated goods are produced by intermediate firms using the production function $Y_t\left(i\right)=Z_tA_tK_{ss}^{\theta}L_t\left(i\right)^{1-\theta}$, where K_{ss} and $L_t\left(i\right)$ denote capital and labor services at the ith firm, respectively. Productivity shocks are allowed to have the traditional stationary component A_t, but also a non-stationary component Z_t to generate long-run risk in the model. For the stationary shocks, we let $\log A_{t+1}=\rho_A\log A_t+\sigma_A\varepsilon_{A,t+1}$, where $\left|\rho_A\right|<1$, $\sigma_A>0$, and $\varepsilon_{A,t+1}\sim\mathcal{NID}\left(0,1\right)$. Similarly for the non-stationary shocks, we introduce

$\mu_{Z,t+1} = Z_{t+1}/Z$ and let

$$\log\left(\frac{\mu_{Z,t+1}}{\mu_{Z,ss}}\right) = \rho_Z \log\left(\frac{\mu_{Z,t}}{\mu_{Z,ss}}\right) + \sigma_Z \varepsilon_{Z,t+1}, \tag{28}$$

where $|\rho_Z| < 1$, $\sigma_Z > 0$, and $\varepsilon_{Z,t+1} \sim \mathcal{NID}(0,1)$.[29]

Intermediate firms can freely adjust their labor demand at the given market wage W_t and are therefore able to meet demand in every period. Price stickiness is introduced via Calvo contracts, where a fraction ζ of randomly selected firms can not set the optimal nominal price $P_t(i)$ of the good they produce and instead let $P_t(i) = \pi_{ss} P_{t-1}(i)$.

4.1.3 The Central Bank and Aggregation

The central bank sets the one-period nominal interest rate i_t as

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left(i_{ss} + \beta_\pi \log\left(\frac{\pi_t}{\pi_{ss}}\right) + \beta_y \log\left(\frac{Y_t}{Z_t Y_{ss}}\right)\right),$$

based on a desire to close the inflation and the output gap with $\beta_n = [0,5]$ for $n = \{\pi, y\}$, subject to smoothing changes in the policy rate with $\rho_i \in [0,1]$. Note that the inflation gap accounts for steady-state inflation π_{ss}, and that the output gap is expressed in deviation from the balanced growth path as in Justiniano and Primiceri (2008), Rudebusch and Swanson (2012), among others.

Summing across the heterogeneous firms implies $Y_t S_{t+1} = Z_t A_t K_{ss}^{\theta} L_t^{1-\theta}$, where $L_t \equiv \int_0^1 L_t(i)\,di$ is aggregated labor demand and S_{t+1} is the price dispersion index. As in Rudebusch and Swanson (2012), $\delta K_{ss} Z_t$ units of output are used to maintain a constant capital stock, meaning that the aggregate resource constraint is given by $Y_t = C_t + \delta K_{ss} Z_t$.

4.1.4 Equity and Bond Prices

Equity is defined as a claim on aggregate dividends from firms, i.e. $D_t = Y_t - W_t L_t$, and its real price is therefore $1 = \mathbb{E}_t\left[M_{t,t+1} R_{t+1}^m\right]$ where $R_{t+1}^m = \left(D_{t+1} + P_{t+1}^m\right)/P_t^m$.

The price in period t of a default-free zero-coupon bond $B_t^{(n)}$ maturing in n periods with a face value of one dollar is $B_t^{(n)} = \mathbb{E}_t\left[\frac{M_{t,t+1}}{\pi_{t+1}} B_{t+1}^{(n-1)}\right]$ for $n = 1, ..., N$ with $B_t^{(0)} = 1$. Its yield-to-maturity with continuously compounding is then $i_t^{(n)} = -\frac{1}{n}\log B_t^{(n)}$. Following Rudebusch

[29] The specification of long-run productivity risk adopted in the endowment model, i.e. (4), could also be used in the New Keynesian model, but we prefer the more parsimonious specification in (28) for comparability with the existing DSGE literature (see for instance Justiniano and Primiceri (2008), Altig et al. (2011), Swanson (2015), among others). This difference explains the slightly different notation used in (28) for $\mu_{Z,t}$, $\mu_{Z,ss}$, σ_Z, and $\varepsilon_{Z,t+1}$ compared to the corresponding parameters in (4).

and Swanson (2012), we define term premia as $\Psi_t^{(n)} = i_t^{(n)} - \widetilde{i}_t^{(n)}$, where $\widetilde{i}_t^{(n)}$ is the yield-to-maturity on a zero-coupon bond $\widetilde{B}_t^{(n)}$ under risk-neutral valuation, i.e. $\widetilde{B}_t^{(n)} = e^{-i_t}\mathbb{E}_t\left[\widetilde{B}_{t+1}^{(n-1)}\right]$ with $\widetilde{B}_t^{(0)} = 1$.

4.2 Model Solution and Estimation Methodology

As in Section 3, we approximate the model solution by a third-order perturbation approximation and estimate the model by GMM using unconditional first and second moments computed as in Andreasen et al. (2013). The selected series describing the macro economy and the bond market are given by Δc_t, π_t, i_t, $i_t^{(40)}$, $\Psi_t^{(40)}$, and $\log L_t$, where one time period in the model corresponds to one quarter. The 10-year nominal interest rate and its term premium (obtained from Adrian et al. (2013)) are available from 1961Q3, leaving us with quarterly data from 1961Q3 to 2014Q4. We include all means, variances, and first-order auto-covariances of these six variables for the estimation, in addition to nine contemporaneous covariances related to the correlations reported at the end of Table 5. To examine whether our New Keynesian model is able to match the equity premium, we also include the mean of the net market return $r_t^m = \log R_t^m$ in the set of moments.[30] Finally, the GMM estimation is implemented using the conventional two-step procedure outlined in Section 3.1.

We estimate all structural parameters in the model except for a few badly identified parameters. That is, we let $\delta = 0.025$ and $\theta = 1/3$ as typically considered for the U.S. economy. We also let $\eta = 6$ to get an average markup of 20% and impose $\varphi = 1/4$ to match a Frisch labor supply elasticity in the neighborhood of 0.5. Finally, we set the ratio of capital to output in the steady state equal to 2.5 as in Rudebusch and Swanson (2012).

4.3 Estimation Results

We proceed by first considering Epstein-Zin-Weil preferences with a standard power utility kernel in Section 4.3.1, i.e. $\mathcal{U}(C_t, L_t) = \frac{1}{\chi}(C_t - bC_{t-1})^{\chi} + Z_t^{\chi}\frac{\varphi_0}{1-1/\varphi}(1-L_t)^{1-\frac{1}{\varphi}}$, before exploring the performance of the exponential power utility kernel in Section 4.3.2.

4.3.1 A Standard Power Utility Kernel

Given that RRA is hard to estimate accurately in the New Keynesian model, the analysis is conducted by conditioning the estimation on different values of RRA. Following Kaltenbrunner and Lochstoer (2010), we first let RRA = 5, which is within the middle range of reasonable values for RRA suggested by Mehra and Prescott (1985). The estimated coefficients are

[30] Details on the data sources and data construction are provided in the online appendix.

summarized in Table 4 and are all fairly standard, except for a high steady state inflation ($\hat{\pi}_{ss} = 1.14$) and high curvature in consumption utility ($\hat{\chi} = -13.4$), which gives an IES of 0.034. Table 5 shows that the model does well in matching all means (including the 10-year term premium and market return), but that this comes at the cost of too much variability in consumption growth (3.31% vs. 1.80%) and labor supply (2.93% vs. 1.61%). These results just iterate the finding in Rudebusch and Swanson (2008) that the standard New Keynesian model with low RRA can not match key asset pricing moments without distorting the macro economy.

< Table 4 about here >

We next follow Swanson (2015) and increase RRA to 60, although such extreme level of risk aversion is hard to justify based on micro-evidence. Table 5 shows that the New Keynesian model now reproduces all means without generating too much variability in the macro economy, except for a slightly elevated standard deviation in the log-transformed labor supply (2.44% vs. 1.61%). High risk aversion also helps in matching most auto- and contemporaneous correlations, and the model therefore has a much better overall fit with $Q^{scaled} = 0.268$ compared to $Q^{scaled} = 1.012$ with RRA$= 5$.

< Table 5 about here >

4.3.2 The Exponential Power Utility Kernel

We finally estimate our proposed utility kernel in (25) when conditioning on a realistic level of risk aversion with RRA $= 5$. Table 4 shows that $\hat{\tau} = 16.9$ and hence far from zero, meaning that the exponential power utility kernel also helps the New Keynesian model to explain postwar U.S. data. Correcting for habits and $\tilde{C}_{ss} = 0.8$, the scale-adjusted estimate of τ is $\hat{\tau}\left(1 - \hat{b}\right)\tilde{C}_{ss} = 6.5$ and hence somewhat similar to $\hat{\tau} = 8.95$ in the long-run risk model - at least when accounting for estimation uncertainty. Using (27), we thus estimate a relatively low IES of 0.07.

Table 5 shows that the New Keynesian model now matches all means and standard deviations, except for the labor supply that displays the same degree of variability as in the standard New Keynesian model with RRA $= 60$. Subject to this qualification, the New Keynesian model now explains the equity premium with a low RRA $= 5$. We also match the mean and the variability of the 10-year nominal term premia, implying that our New Keynesian model also explains the bond premium puzzle with low RRA. The auto- and contemporaneous correlations are also well matched, and our extension of the New Keynesian model therefore has a slightly better overall fit with $Q^{scaled} = 0.255$ compared to $Q^{scaled} = 0.268$ for the standard New Keynesian model with RRA $= 60$.

4.4 The Key Mechanisms

We next run three experiments to explore some of the key mechanisms in the New Keynesian model with the exponential power utility kernel in (25). The first experiment considered in Table 6 illustrates the implications of gradually increasing τ. As for the long-run risk model in Section 3.7, we emphasize two effects. First, a higher value of τ reduces the IES without affecting returns in the steady state. Second, increasing τ lowers $u'(\cdot)/u(\cdot)$ and allows for strong preferences for early resolution of uncertainty through a high α without making the household very risk averse. The large value of α then amplifies the existing risk corrections and enables the model to explain asset prices with low RRA.

Our second experiment abstracts from long-run productivity risk by letting $\sigma_Z = 0$. The fourth column in Table 6 shows that this modification has very large effects in the model, which now is unable to explain both the level and the variability of π_t, i_t, $i_t^{(40)}$, and $\Psi_t^{(40)}$. Thus, long-run productivity risk is also an essential feature of the New Keynesian model.

Our final experiment is motivated by the main research question in the present paper, that is whether the key effect of Epstein-Zin-Weil preferences is to separate the IES from RRA or to impose a timing attitude on households? The fifth column in Table 6 therefore considers the case where $\alpha = 0$ and the household is indifferent between early and late resolution of uncertain. Although this modification only has a small effect on RRA (reducing it from 5 to 2.28) it nevertheless has a profound impact on the model, which largely displays the same properties as when omitting long-run productivity risk. In other words, the New Keynesian model is simply unable to explain asset prices without Epstein-Zin-Weil preferences, and hence strong preferences for early resolution of uncertainty. The final column in Table 6 shows that this result is not explained by the fall in RRA, as a version of the New Keynesian model with $\alpha = 0$ and RRA $= 5$ through stronger habits ($b = 0.86$) is also unable to explain asset prices.

Thus, we confirm the result from the long-run risk model, namely that the main effect of Epstein-Zin-Weil preferences with our exponential power utility kernel is *not* to separate the IES from RRA but instead to introduce strong preferences for early resolution of uncertainty. This finding also helps to clarify why consumption habits may struggle to match asset prices in DSGE models, although they allow for additional flexibility in setting the IES and RRA (see Rudebusch and Swanson (2008)). The reason being that consumption habits do not introduce preferences for early resolution of uncertainty, which we have shown is essential to explain asset prices within our DSGE model.

< Table 6 about here >

5 Conclusion

This paper highlights the importance of the timing attitude for explaining asset prices based on consumption behavior. To do so, we introduce a new utility kernel to obtain greater flexibility in setting the IES, the RRA, *and* the timing attitude compared to the standard power-specification adopted in Epstein and Zin (1989) and Weil (1990). The proposed exponential power utility kernel has one additional parameter τ that increases the speed by which marginal utility in the power kernel decreases for higher consumption. That is when $\tau \rightarrow 0$, we recover the standard power utility kernel, and hence the traditional implementation of Epstein-Zin-Weil preferences. The main benefit of introducing this additional parameter is to obtain greater flexibility in setting $u'(\cdot)/u(\cdot)$ and $u''(\cdot)/u'(\cdot)$ compared to the power utility kernel, where a single coefficient determines both ratios. The desired specification is obtained with a relatively high value of τ, as it generates a low value of $u'(\cdot)/u(\cdot)$ and hence enables strong preferences for early resolution of uncertainty to coincide with low RRA. We then show that this basic mechanism is able to explain asset pricing puzzles in both endowment and production economies. In particularly, we resolve a puzzle in the long-run risk model that consumption growth is too highly correlated with the price-dividend ratio and the risk-free rate, and we resolve the puzzlingly high RRA in the New Keynesian model to match asset prices. Our analysis also reveals that the main effect of Epstein-Zin-Weil preferences with our utility kernel is *not* to separate the IES from RRA but instead to introduce strong preferences for early resolution of uncertainty. This conclusion is thus opposite to the traditional motivation for considering Epstein-Zin-Weil preferences, which we show relies on using the standard power utility kernel.

A A Perturbation Approximation under Homoscedasticity

Proposition A.1 *The second-order approximated log-transformed value function v_t and the log-transform twisted value function ev_t at the steady state are given by*

$$v_t = v_{ss} + v_{\tilde{c}}\tilde{c}_t + v_x x_t + \frac{1}{2}v_{\tilde{c}\tilde{c}}\tilde{c}_t^2 + v_{\tilde{c}x}\tilde{c}_t x_t + \frac{1}{2}v_{xx}x_t^2 + \frac{1}{2}v_{\sigma\sigma}, \tag{29}$$

$$ev_t = ev_{ss} + ev_{\tilde{c}}\tilde{c}_t + ev_x x_t + \frac{1}{2}ev_{\tilde{c}\tilde{c}}\tilde{c}_t^2 + ev_{\tilde{c}x}\tilde{c}_t x_t + \frac{1}{2}ev_{xx}x_t^2 + \frac{1}{2}ev_{\sigma\sigma} \tag{30}$$

where

$$\begin{aligned}
v_{ss} &= \log\left(\frac{1_{\{u(1)>0\}}u(1) - 1_{\{u(1)<0\}}u(1)}{1-\kappa_0}\right) \\
v_{\tilde{c}} &= \frac{u'(1)}{u(1)}\frac{1-\kappa_0}{1-\rho_{\tilde{c}}\kappa_0} \\
v_x &= \frac{\kappa_0}{1-\rho_x\kappa_0}\chi \\
v_{\tilde{c}\tilde{c}} &= \frac{1-\kappa_0}{1-\rho_{\tilde{c}}^2\kappa_0}\left[\frac{u''(1)}{u(1)} + \frac{u'(1)}{u(1)}\right] - \left[\frac{u'(1)}{u(1)}\frac{1-\kappa_0}{1-\rho_{\tilde{c}}\kappa_0}\right]^2 \\
v_{\tilde{c}x} &= \frac{u'(1)}{u(1)}\frac{1-\kappa_0}{1-\rho_{\tilde{c}}\kappa_0}\chi\left[\frac{\rho_{\tilde{c}}\kappa_0}{1-\rho_{\tilde{c}}\rho_x\kappa_0} - \frac{\kappa_0}{1-\rho_x\kappa_0}\right] \\
v_{xx} &= \frac{\kappa_0}{1-\rho_x^2\kappa_0}\frac{1-\kappa_0}{(1-\rho_x\kappa_0)^2}\chi^2 \\
v_{\sigma\sigma} &= \frac{\kappa_0}{1-\kappa_0}\left[v_{\tilde{c}\tilde{c}}\sigma_{\tilde{c}}^2 + (1-\alpha)v_{\tilde{c}}^2\sigma_{\tilde{c}}^2 + v_{xx}\sigma_x^2 + (1-\alpha)v_x^2\sigma_x^2 + (1-\alpha)\chi^2\sigma_z^2\right]
\end{aligned}$$

and

$$\begin{aligned}
ev_{ss} &= (1-\alpha)\left(\log\left(\frac{1_{\{u(1)>0\}}u(1) - 1_{\{u(1)<0\}}u(1)}{1-\kappa_0}\right) + \chi\log\mu_z\right) \\
ev_{\tilde{c}} &= (1-\alpha)\rho_{\tilde{c}}\frac{u'(1)}{u(1)}\frac{1-\kappa_0}{1-\rho_{\tilde{c}}\kappa_0} \\
ev_x &= \frac{(1-\alpha)\chi}{1-\rho_x\kappa_0} \\
ev_{\tilde{c}\tilde{c}} &= (1-\alpha)\rho_{\tilde{c}}^2\frac{1-\kappa_0}{1-\rho_{\tilde{c}}^2\kappa_0}\left[\frac{u''(1)}{u(1)} + \frac{u'(1)}{u(1)}\right] - (1-\alpha)\rho_{\tilde{c}}^2\left[\frac{u'(1)}{u(1)}\frac{1-\kappa_0}{1-\rho_{\tilde{c}}\kappa_0}\right]^2 \\
ev_{x\tilde{c}} &= (1-\alpha)\rho_x\rho_{\tilde{c}}v_{x\tilde{c}} \\
ev_{xx} &= (1-\alpha)\rho_x^2\frac{\kappa_0}{1-\rho_x^2\kappa_0}\frac{1-\kappa_0}{(1-\rho_x\kappa_0)^2}\chi^2 \\
ev_{\sigma\sigma} &= \frac{1-\alpha}{1-\kappa_0}\left[v_{\tilde{c}\tilde{c}}\sigma_{\tilde{c}}^2 + (1-\alpha)v_{\tilde{c}}^2\sigma_{\tilde{c}}^2 + v_{xx}\sigma_x^2 + (1-\alpha)v_x^2\sigma_x^2 + (1-\alpha)\chi^2\sigma_z^2\right]
\end{aligned}$$

and $\kappa_0 = \beta\mu_z^{\chi}$.

Proposition A.2 *The second-order approximated log-transformed price-dividend ratio pd_t at the steady state is given by*

$$pd_t = pd_{ss} + pd_{\tilde{c}}\tilde{c}_t + pd_x x_t + \frac{1}{2}pd_{\tilde{c}\tilde{c}}\tilde{c}_t^2 + pd_{\tilde{c}x}\tilde{c}_t x_t + \frac{1}{2}pd_{xx}x_t^2 + \frac{1}{2}pd_{\sigma\sigma},$$

where

$$\begin{aligned}
pd_{ss} &= \log\frac{\kappa_1}{1-\kappa_1} \\
pd_{\tilde{c}} &= -\frac{1-\rho_{\tilde{c}}}{1-\kappa_1\rho_{\tilde{c}}}\frac{u''(1)}{u'(1)} \\
pd_x &= \frac{\phi+\chi-1}{1-\kappa_1\rho_x} \\
pd_{\tilde{c}\tilde{c}} &= -pd_{\tilde{c}}^2 - 2\frac{u''(1)}{u'(1)}pd_{\tilde{c}} - \frac{1-\rho_{\tilde{c}}^2}{1-\kappa_1\rho_{\tilde{c}}^2}\left(\frac{u'''(1)}{u'(1)} + \frac{u''(1)}{u'(1)}\right) \\
pd_{\tilde{c}x} &= -pd_{\tilde{c}}pd_x + \frac{\kappa_1\rho_{\tilde{c}}pd_{\tilde{c}}(\chi-1+\phi)}{1-\kappa_1\rho_{\tilde{c}}\rho_x} - \frac{(1-\rho_{\tilde{c}})\frac{u''(1)}{u'(1)}(\chi-1+\phi+\rho_x\kappa_1 pd_x)}{1-\kappa_1\rho_{\tilde{c}}\rho_x} \\
pd_{xx} &= -pd_x^2 + \frac{(\chi-1+\phi)^2}{1-\rho_x^2\kappa_1} + 2\kappa_1\rho_x\frac{\chi-1+\phi}{1-\rho_x^2\kappa_1}pd_x \\
pd_{\sigma\sigma} &= \frac{\sigma_d^2}{1-\kappa_1} + \frac{\sigma_z^2}{1-\kappa_1}\left[\alpha + (1-\alpha)(1-\chi)^2\right] \\
&\quad + \frac{\sigma_{\tilde{c}}^2}{1-\kappa_1}\Big[\alpha v_{\tilde{c}}^2 - 2\alpha\kappa_1 pd_{\tilde{c}}v_{\tilde{c}} + \kappa_1 pd_{\tilde{c}\tilde{c}} \\
&\qquad + \kappa_1 pd_{\tilde{c}}^2 - 2\alpha\frac{u''(1)}{u'(1)}v_{\tilde{c}} + 2\kappa_1 pd_{\tilde{c}}\frac{u''(1)}{u'(1)} + \frac{u'''(1)}{u'(1)} + \frac{u''(1)}{u'(1)}\Big] \\
&\quad + \frac{\sigma_x^2}{1-\kappa_1}\left[\alpha v_x^2 - 2\alpha\kappa_1 pd_x v_x + \kappa_1 pd_{xx} + \kappa_1 pd_x^2\right]
\end{aligned}$$

and $\kappa_1 = \frac{\beta\mu_z^{(\chi-1)}\mu_d}{1+\beta\mu_z^{(\chi-1)}\mu_d}$. *The expressions for $v_{\tilde{c}}$ and v_x are provided in Proposition A.1.*

B The Long-Run Risk Model: Additional Model Implications

The first set of moments we consider relate to the ability of the price-dividend ratio to predict the excess market return, consumption growth, and dividend growth. The first column of Figure 2 shows that our extension of the long-run risk model with an exponential power utility kernel preserves the good performance of the benchmark model and almost perfectly reproduces the ability of a high price-dividend ratio to forecast low excess market returns at all considered horizons. As found in Beeler and Campbell (2012), the price-dividend ratio does not forecast either consumption or dividend growth in the data, which is also matched when using the exponential power utility kernel in the long-run risk model.

As a supplement to these univariate predictability tests, Bansal et al. (2012*a*) suggest expanding the information set in these forecast regressions by consumption growth and the risk-free rate. The R-squared for these multivariate regressions are provided in the second column of Figure 2, showing that the long-run risk model with the exponential power utility kernel also in this case reproduces the desired degree of predictability in excess market returns. For consumption and dividend growth, our estimated version of the standard long-run risk model generally produces too much predictability. A similar finding is reported in Beeler and Campbell (2012) for two calibrated versions of this model. On the other hand, our extension of the long-run risk model with an exponential power utility kernel generates too little predictability in consumption and dividend growth.

< Figure 2 about here >

Following Beeler and Campbell (2012), we also study the ability of the price-dividend ratio to explain past and future consumption growth. Figure 3 shows that our estimated version of the standard long-run risk model implies that past and future consumption growth is too highly correlated with the price-dividend ratio compared to empirical evidence. A similar finding is reported in Beeler and Campbell (2012) for two calibrated versions of the benchmark model. On the other hand, our extension of the long-run risk model with an exponential power utility kernel implies that past and future consumption growth are completely uncorrelated with the price-dividend ratio as seen in the data.

The last two charts in Figure 3 explore the relationship between consumption volatility and the price-dividend ratio. As in Bansal and Yaron (2004), we measure the conditional volatility σ_t by the absolute value of the residual from an AR-model for consumption growth. In line with empirical evidence, our extension of the long-run risk model has the properties that i) a high price-dividend ratio predicts future low volatility and ii) high uncertainty forecasts a low price-dividend ratio (see also Bansal et al. (2005)). Hence, our extension of the long-run risk model with an exponential power utility kernel matches the negative relationship between volatility and the price-dividend ratio with an IES well below one. This is in contrast to the standard long-run risk model, which only reproduces this negative relationship with an IES larger than one, as emphasized in Bansal and Yaron (2004).

< Figure 3 about here >

C New Keynesian Model: Concavity Condition

The utility kernel in (25) is strictly concavity if $\mathcal{U}_{CC} < 0, \mathcal{U}_{CC}\mathcal{U}_{LL} > 0$, and

$$\mathcal{U}_{CC}\mathcal{U}_{LL}\mathcal{U}_{C_hC_h} - \mathcal{U}_{LL}\mathcal{U}_{CC_h}^2 - \mathcal{U}_{C_hL}^2\mathcal{U}_{CC} < 0. \tag{31}$$

We know that $\mathcal{U}_{CC} < 0$, whenever $\tau > 0$ and $\chi < 1$. This implies that $\mathcal{U}_{CC}\mathcal{U}_{LL} > 0$, provided $\varphi_0 > 0$ and $\varphi > 0$. Simple algebra implies that (31) reduces to

$$-\mathcal{A}\chi\left(\chi-1\right)\left[\frac{1}{\chi}\left(\frac{1-e^{-\tau\left(\frac{C_t-bC_{t-1}}{Z_t}\right)}}{\tau}\right)^{\chi}+\varphi_0\frac{\left(1-L_t\right)^{1-\frac{1}{\varphi}}}{1-\frac{1}{\varphi}}\right]$$

$$+\chi^2\left(\frac{1-e^{-\tau\left(\frac{C_t-bC_{t-1}}{Z_t}\right)}}{\tau}\right)^{\chi}\frac{\tau}{e^{\tau\left(\frac{C_t-bC_{t-1}}{Z_t}\right)}-1}+\chi^2\left(1-\varphi\right)\mathcal{A}\varphi_0\frac{\left(1-L_t\right)^{1-\frac{1}{\varphi}}}{1-\frac{1}{\varphi}}<0,$$

where $\mathcal{A}=\left[\left(\chi-1\right)\left(\frac{\tau}{e^{\tau\left(\frac{C_t-bC_{t-1}}{Z_t}\right)}-1}\right)-\tau\right]$. This condition is satisfied in the steady state for the estimates reported in Table 4.

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Table 1: The Long-Run Risk Model: The Structural Parameters

Estimation results using data from 1947Q1 to 2014Q4. The reported estimates are from the second step in GMM with the weigthing matrix estimated by 15 lags in the Newey-West estimator. The model has a monthly time frequency with model-implied moments time-aggregated to a quarterly time frequency. In column (1), the values of RRA and ψ are calibrated and standard errors are therefore not available.

	Benchmark	Extended Model		
	(1) Power utility kernel	(2) Power utility kernel	(3) Semi-parametric utility kernel	(4) Exponential power utility kernel
$u'(1)$	–	–	5.19×10^{-4} (0.0002)	–
$u''(1)$	–	–	-0.0042 (0.0019)	–
χ	–	0.5106 (0.0066)	0.4637 (0.9579)	0.5144 (0.4998)
RRA	10	13.515 (2.4618)	9.6716 (3.1934)	9.7809 (0.9846)
ψ	1.5	1.0181 (0.0119)	–	–
τ	–	–	–	8.9505 (1.0252)
β	0.9995^a (*n.a.*)	0.9980 (0.0010)	0.9981 (0.0019)	0.9987 (0.0008)
$\rho_{\tilde{c}}$	0.1651 (0.0272)	0.0041 (6.1043)	0.9998 (0.0004)	0.9962 (0.0018)
ρ_x	0.9802 (0.0030)	0.6450 (0.2446)	0.1510 (1.1698)	0.3756 (0.6678)
ρ_σ	0.9942 (0.0008)	0.9933 (0.0020)	0.9889 (0.0094)	0.9941 (0.0016)
μ_z	1.0014^a	1.0018 (0.0028)	1.0019 (0.0001)	1.0018 (0.0001)
μ_d	1.0006 (0.0001)	1.0015 (0.0011)	1.0015 (0.0001)	1.0014 (0.0006)
ϕ	1.8858 (0.0414)	3.6231 (2.6532)	2.2725 (4.6025)	1.9909 (2.4266)
$\sigma_{\tilde{c}}$	0.0033 (0.0003)	0.0017 (0.0080)	0.0025 (0.0004)	0.0025 (0.0004)
σ_z	0.0021 (0.0004)	0.0024 (0.0006)	0.0008 (0.0018)	0.0008 (0.0017)
σ_d	0.0147 (0.0009)	0.0129 (0.0014)	0.0133 (0.0013)	0.0148 (0.0014)
σ_x	0.0004 (0.0001)	0.0009 (0.0005)	0.0013 (0.0023)	0.0012 (0.0015)
σ_σ	0.0876 (0.0134)	0.0775 (0.0267)	0.1199 (0.0984)	0.1042 (0.0353)
Memo				
IES	1.50	1.02	0.12	0.11
α	28.00	703.42	$3,171$	$1,390$

[a] The coefficient is at the boundary of its domain as $\beta\mu_z^{1-1/\psi} < 1$ and its standard error is therefore not available (n.a.).

Table 2: The Long-Run Risk Model: Fit of Moments

Except for the price-dividend ratio, all means and standard deviations are expressed in annualized percent. Moments are annualized through a multiplication of 400, except for the standard deviation of the market return which is multiplied by 200. All model-implied moments in columns (2) to (5) are from the unconditional distribution, whereas the empirical data moments in column (1) are the sample means. In column (1), figures in parentesis refer to the standard error of the empirical moment, computed based on the Newey-West estimate (with 15 lags) of the co-variance matrix for the considered set of moments.

		Benchmark	Extended Model		
	(1)	(2)	(3)	(4)	(5)
	Data	Power utility kernel	Power utility kernel	Semi-parametric utility kernel	Exponential power utility kernel
Means					
pd_t	3.495 (0.095)	3.514	3.505	3.519	3.499
r_t^f	0.831 (0.411)	0.997	0.975	1.393	1.100
r_t^m	6.919 (1.842)	5.440	8.737	8.520	6.001
Δc_t	1.905 (0.209)	1.735	2.189	2.229	2.108
Δd_t	2.391 (0.979)	0.692	1.775	1.836	1.722
Stds					
pd_t	0.421 (0.060)	0.392	0.361	0.430	0.422
r_t^f	2.224 (0.378)	2.206	1.782	1.914	1.986
r_t^m	16.445 (1.201)	16.521	15.351	15.658	16.559
Δc_t	2.035 (0.169)	2.904	1.859	1.791	1.875
Δd_t	9.391 (1.221)	8.710	7.516	7.173	8.040
Persistence					
$corr(pd_t, pd_{t-1})$	0.982 (0.150)	0.980	0.980	0.985	0.983
$corr\left(r_t^f, r_{t-1}^f\right)$	0.866 (0.085)	0.574	0.930	0.909	0.925
$corr\left(r_t^m, r_{t-1}^m\right)$	0.084 (0.058)	0.002	-0.006	0.005	-0.006
$corr\left(\Delta c_t, \Delta c_{t-1}\right)$	0.306 (0.130)	0.571	0.253	0.220	0.248
$corr\left(\Delta d_t, \Delta d_{t-1}\right)$	0.396 (0.092)	0.298	0.207	0.138	0.143

Table 2: Long-Run Risk Model: Fit of Moments (continued)

		Benchmark	Extended Model		
	(1)	(2)	(3)	(4)	(5)
	Data	Power utility kernel	Power utility kernel	Semi-parametric utility kernel	Exponential power utility kernel
Correlations					
$corr\left(pd_t, r_t^f\right)$	0.035 (0.250)	0.437	0.952	0.157	0.543
$corr\left(pd_t, r_t^m\right)$	0.058 (0.073)	0.112	0.078	0.058	0.073
$corr\left(pd_t, \Delta c_t\right)$	0.025 (0.093)	0.197	0.007	0.005	0.021
$corr\left(pd_t, \Delta d_t\right)$	-0.017 (0.132)	0.109	0.005	0.000	0.001
$corr\left(r_t^f, r_t^m\right)$	0.023 (0.062)	0.093	0.060	0.052	0.042
$corr\left(r_t^f, \Delta c_t\right)$	0.161 (0.102)	0.362	0.132	0.059	0.055
$corr\left(r_t^f, \Delta d_t\right)$	-0.168 (0.102)	0.303	0.104	0.035	0.036
$corr\left(r_t^m, \Delta c_t\right)$	0.233 (0.065)	0.063	0.032	0.047	0.140
$corr\left(r_t^m, \Delta d_t\right)$	0.104 (0.050)	0.275	0.262	0.265	0.282
$corr\left(\Delta c_t, \Delta d_t\right)$	0.062 (0.062)	0.374	0.274	0.158	0.156
Goodness of fit					
Q^{Step2}	-	0.062	0.050	0.039	0.037
J-test: P-value	-	0.112	0.195	0.311	0.449
Q^{scaled}	-	1.145	1.171	0.417	0.497

Table 3: The Long-Run Risk Model: Analyzing the Exponential Power Kernel

Moments are computed using a third-order perturbation approximation and represented as in Table 2. Unless stated otherwise, all parameters attain the estimated values from column (4) in Table 1. For decomposing $\mathbb{E}[r_t^f]$ and $\mathbb{E}[r_t^m]$, the contribution from the steady state, long-run risk, short-run risk, and cyclical risk are computed based on Proposition 2, while the contribution from stochastic volatility is given by the difference between the unconditional mean and the sum of these four terms.

	(1)	(2)	(3)	(4)	(5)	(6)
		$\chi = 0.51$		$\sigma_x = 0$	$\alpha = 0$	$\alpha = 0$
	$\tau \to 0$	τ=5	τ=$\hat{\tau}^{GMM}$	τ=$\hat{\tau}^{GMM}$	τ=$\hat{\tau}^{GMM}$	τ=9.7806
Means						
pd_t	7.287	6.589	3.499	7.812	7.369	7.376
r_t^f	2.611	2.276	1.100	1.909	2.320	2.260
r_t^m	2.540	3.180	6.001	2.084	2.486	2.484
Stds						
pd_t	0.017	0.131	0.422	0.252	0.229	0.250
r_t^f	0.563	0.945	1.986	1.405	1.381	1.497
r_t^m	5.395	6.838	16.559	9.332	8.750	9.263
Decomposing $\mathbb{E}[r_t^f]$						
r_{ss}^f	2.631	2.631	2.631	2.631	2.631	2.631
Long-run risk	-0.011	-0.159	-0.809	0	0	0
Short-run risk	-0.005	-0.080	-0.406	-0.406	0.000	0.000
Cyclical risk	-0.005	-0.117	-0.317	-0.317	-0.311	-0.371
Stochastic volatility	0.000	0.000	0.000	0.000	0.000	0.000
Decomposing $\mathbb{E}[r_t^m]$						
r_{ss}^m	2.631	2.631	2.631	2.631	2.631	2.631
Long-run risk	0.013	0.199	1.011	0	0	0
Short-run risk	-0.005	-0.080	-0.406	-0.406	-0.000	0.000
Cyclical risk	-0.005	-0.116	-0.314	-0.314	-0.309	-0.368
Stochastic volatility	-0.095	0.545	3.079	0.173	0.163	0.222
Memo						
RRA	9.78	9.78	9.78	9.78	8.95	9.78
IES	2.06	0.20	0.11	0.11	0.11	0.1022
α	18	273	1,390	1,390	0	0

Table 4: The New Keynesian Model: The Structural Parameters

Estimation results using data from 1961Q3 to 2014Q4. The reported estimates are from the second step of GMM with the weigthing matrix estimated by 15 lags in the Newey-West estimator. The estimate of β in column (1) is on the boundary and the standard error is therefore not available.

	Benchmark		Extended Model
	(1)	(2)	(3)
	RRA=5	RRA=60	RRA=5
β	0.9999	0.9955	0.9908
		(0.0032)	(0.0019)
b	0.5085	0.6970	0.5157
	(0.0093)	(0.0718)	(0.1499)
χ	-13.3678	-4.0710	0.8148
	(0.9909)	(1.3655)	(0.5563)
τ	$\rightarrow 0$	$\rightarrow 0$	16.9450
			(6.8596)
ζ	0.5195	0.7300	0.6677
	(0.0164)	(0.0143)	(0.0351)
β_π	1.4232	2.2226	1.1810
	(0.0176)	(0.2670)	(0.0788)
β_y	0.2175	0.7255	0.0190
	(0.0179)	(0.3458)	(0.0422)
μ_Z	1.0040	1.0055	1.0052
	(0.0001)	(0.0004)	(0.0004)
π_{ss}	1.1407	1.0300	1.0431
	(0.0113)	(0.0017)	(0.0065)
L_{ss}	0.3364	0.3355	0.3367
	(0.0005)	(0.0005)	(0.0009)
ρ_r	0.5890	0.8872	0.5975
	(0.0261)	(0.0252)	(0.0605)
ρ_A	0.9953	0.9878	0.9909
	(0.0004)	(0.0006)	(0.0012)
ρ_Z	0.1009	0.4818	0.6272
	(0.0086)	(0.0895)	(0.3229)
σ_A	0.0125	0.0051	0.0082
	(0.0003)	(0.0006)	(0.0010)
σ_Z	0.0130	0.0040	0.0029
	(0.0004)	(0.0002)	(0.0019)
Memo			
IES	0.034	0.061	0.074
$\mathcal{U}_{ss}$	$-274,455$	-572	0.121
α	-1.31	-32.54	171.62

Table 5: The New Keynesian Model: Fit of Moments

All variables are expressed in annualized terms in percentage, except for the mean of $\log(l_t)$. All model-implied moments in columns (2) to (5) are from the unconditional distribution, whereas the empirical data moments in column (1) are given by the sample means. In column (1), figures in parentesis refer to the standard error of the empirical moment, computed based on the Newey-West estimate (with 15 lags) of the co-variance matrix for the considered set of moments.

		Benchmark		Extended Model
	(1)	(2)	(3)	(4)
	Data	RRA=5	RRA=60	RRA=5
Means				
Δc_t	1.975 (0.253)	1.595	2.182	2.055
π_t	3.890 (0.512)	3.391	3.417	3.432
i_t	4.999 (0.682)	5.188	5.040	5.061
$i_t^{(40)}$	6.497 (0.617)	6.556	6.463	6.507
$\Psi_t^{(40)}$	1.663 (0.251)	1.741	1.583	1.678
$\log L_t$	−1.081 (0.003)	−1.081	−1.080	−1.081
r_t^m	5.527 (1.786)	4.760	5.390	5.419
Stds				
Δc_t	1.802 (0.137)	3.313	1.442	1.361
π_t	2.716 (0.342)	2.938	2.744	2.696
i_t	3.173 (0.478)	2.942	2.615	2.912
$i_t^{(40)}$	2.621 (0.441)	2.672	2.308	2.547
$\Psi_t^{(40)}$	1.165 (0.167)	1.084	1.092	1.109
$\log L_t$	1.619 (0.162)	2.926	2.437	2.476
Persistence				
$corr\,(\Delta c_t, \Delta c_{t-1})$	0.529 (0.082)	0.506	0.709	0.777
$corr\,(\pi_t, \pi_{t-1})$	0.953 (0.037)	0.777	0.858	0.894
$corr\,(i_t, i_{t-1})$	0.949 (0.044)	0.947	0.989	0.981
$corr\left(i_t^{(40)}, i_{t-1}^{(40)}\right)$	0.976 (0.066)	0.991	0.981	0.985
$corr\left(\Psi_t^{(40)}, \Psi_{t-1}^{(40)}\right)$	0.937 (0.063)	0.995	0.988	0.991
$corr\,(\log L_t, \log L_{t-1})$	0.932 (0.545)	0.753	0.942	0.969

Table 5: The New Keynesian Model: Fit of Moments (continued)

		Benchmark		Extended Model
	(1)	(2)	(3)	(4)
	Data	RRA=5	RRA=60	RRA=5
Correlations				
$corr\left(\Delta c_t, \pi_t\right)$	-0.184 (0.136)	0.275	-0.045	-0.112
$corr\left(\Delta c_t, i_t\right)$	0.021 (0.181)	0.143	-0.039	-0.037
$corr\left(\Delta c_t, \Psi_t^{(40)}\right)$	-0.036 (0.165)	-0.008	-0.022	-0.045
$corr\left(\pi_t, i_t\right)$	0.703 (0.059)	0.932	0.877	0.965
$corr\left(\pi_t, i_t^{(40)}\right)$	0.585 (0.146)	0.822	0.851	0.859
$corr\left(\pi_t, \Psi_t^{(40)}\right)$	0.236 (0.146)	0.442	0.419	0.377
$corr\left(i_t, i_t^{(40)}\right)$	0.900 (0.043)	0.853	0.869	0.878
$corr\left(i_t, \Psi_t^{(40)}\right)$	0.424 (0.222)	0.358	0.381	0.377
$corr\left(i_t^{(40)}, \Psi_t^{(40)}\right)$	0.757 (0.252)	0.698	0.766	0.721
Goodness of fit				
Q^{Step2}	-	0.061	0.053	0.058
J-test: P-value	-	0.605	0.672	0.494
Q^{scaled}	-	1.012	0.268	0.255

Table 6: The New Keynesian Model: Analyzing the Exponential Power Kernel
All moments are computed using a third-order perturbation and represented as in Table 5. Unless stated otherwise, all parameters attain the estimated values from column (3) in Table 4.

	(1)	(2)	(3)	(4)	(5)	(6)
	$\chi = 0.815$			$\sigma_Z = 0$	$\alpha = 0$	$\alpha = 0, b = 0.86$
	$\tau \to 0$	$\tau = 10$	$\tau = \hat{\tau}^{GMM}$	$\tau = \hat{\tau}^{GMM}$	$\tau = \hat{\tau}^{GMM}$	$\tau = \hat{\tau}^{GMM}$
Means						
Δc_t	2.055	2.055	2.055	2.055	2.055	2.055
π_t	16.916	14.910	3.432	16.068	16.433	17.007
i_t	20.979	18.614	5.061	19.988	20.420	21.058
$i_t^{(40)}$	20.946	18.834	6.507	20.117	20.463	20.928
$\Psi_t^{(40)}$	0.058	0.427	1.678	0.3588	0.275	0.238
$\log L_t$	-1.089	-1.083	-1.081	-1.080	-1.080	-1.082
r_t^m	4.073	4.217	5.419	4.055	4.072	4.087
Stds						
Δc_t	4.190	1.955	1.361	1.113	1.708	2.284
π_t	3.356	4.585	2.696	5.245	5.637	10.413
i_t	3.231	4.995	2.912	5.894	6.301	10.269
$i_t^{(40)}$	2.679	4.120	2.547	4.870	5.070	4.480
$\Psi_t^{(40)}$	0.015	0.204	1.109	0.193	0.146	0.121
$\log L_t$	1.793	4.902	2.476	7.122	7.320	5.476
Memo						
RRA	5	5	5	5	2.28	5
IES	2.629	0.124	0.074	0.074	0.074	0.072
α	0.50	19.85	171.62	171.62	0	0

Figure 1: The Exponential Power Utility Kernel

All plots are done for $\chi = 1/3$.

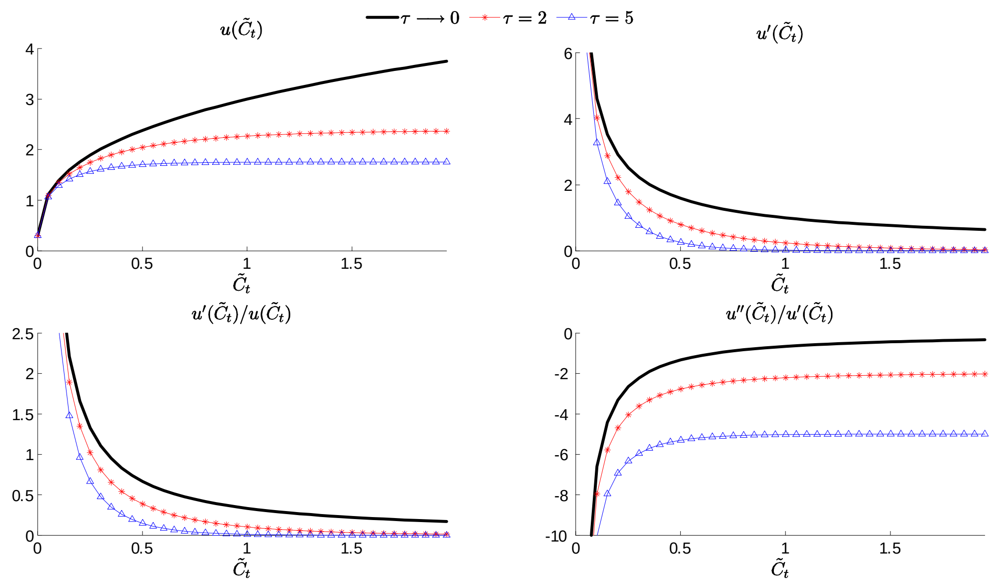

Figure 2: Predictive Regressions

All model-implied moments are computed given the estimated parameters in Table 1 using a simulated sample path of $1,000,000$ observations. The 95 percentage confidence bands are computed using the Newey-West estimator with $2 \times j$ lags for the univariate regressions, and for the multivariate regressions by the block bootstrap using a window of $2 \times j$ observations.

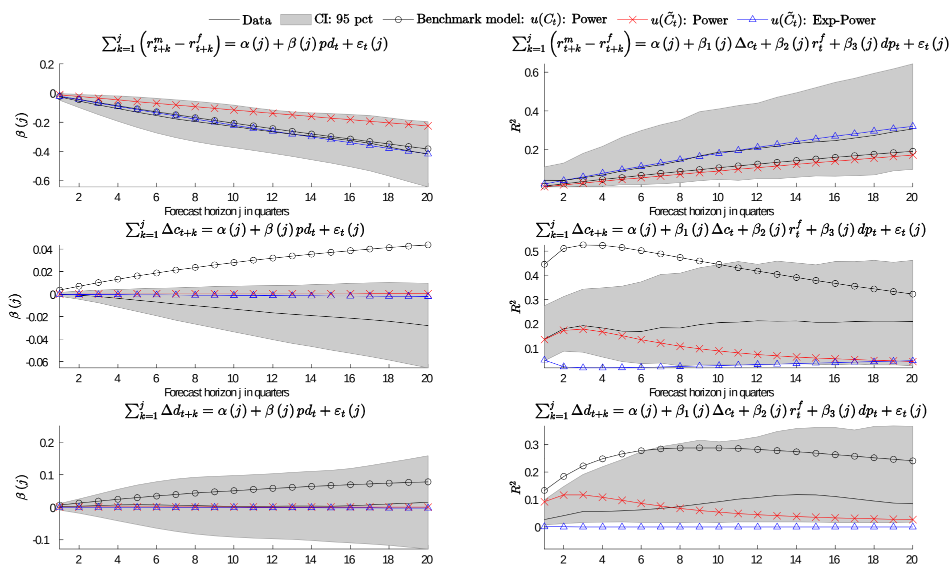

Figure 3: Properties of Consumption Growth and Volatility

All model-implied moments are computed given the estimated parameters in Table 1 using a simulated sample path of $1,000,000$ observations. The conditional volatility $\hat{\sigma}_t$ is estimated by $|\hat{u}_t|$ where $\hat{u}_t$ is the residual from the OLS regression $\Delta c_t = \alpha + \sum_{j=1}^{4} \beta(j) \Delta c_{t-j} + u_t$. The 95 percentage confidence bands are computed using the Newey-West estimator with $max(10, 2 \times j)$ lags for the two consumption growth regressions, whereas the lag length in the two volatility regressions are $2 \times j$.