The Unemployment Correlation Puzzle

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Abstract

Models with labor market frictions have been criticized because they generate too little volatility in employment subject to technology shocks. Shimer (2009) shows that real wage rigidity increases employment volatility significantly. We introduce two additional frictions in the model: sticky prices and real rigidities in the form of habit persistence and investment adjustment costs. The extended model can still generate volatility in employment but real wage rigidity does not affect the transmission mechanism anymore. Moreover, and perhaps more importantly, nominal and real rigidities induce a negative correlation between employment and output ("unemployment correlation puzzle") whereas in the data we observe a strong positive correlation. Therefore,

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the model driven *only* by technology shocks cannot explain business cycle dynamics.
1 Introduction

Although labor market frictions have been introduced in general equilibrium business cycle models several years ago (see Andolfatto (1996), Merz (1995), DenHaan, Ramey and Watson (2000)), the debate on the empirical performance of the model has developed only recently. Costain and Reiter (2008), Hall (2005) and Shimer (2005 and 2009) were the first to show that fluctuations in labor productivity lead to small fluctuations in employment in models based on the seminal contributions by Pissarides (1985 and 2000) and Mortensen and Pissarides (1994). A positive technology shock increases labor productivity but has little impact on employment because wages increase a lot on impact, thereby squeezing profits. Since profits increase little, firms create few new jobs. Therefore, real wages are very volatile whereas employment exhibits limited volatility ("unemployment volatility puzzle") in models with labor market frictions. According to worker flow data used by Shimer (2005) unemployment is eleven times more volatile than labor productivity, whereas in the model it is just three times more volatile.

This failure of the model with labor market frictions has induced a lot of research on this issue. Hagedorn and Manovskii (2008) propose a different calibration for the value of leisure and the workers bargaining power. Hall and Milgrom (2008) modify the bargaining set-up. Barnichon (2008) and Sveen and Weinke (2008) argue that fluctuations in labor productivity can be driven by demand shocks and not only by technology shocks. However, the most accepted solution to the "unemployment volatility puzzle" is the introduction of some form of real wage rigidity in the model (see Shimer (2005 and 2009) and Hall (2005) for an ad hoc approach and Gertler and Trigari (2009) for a more elegant time dependent stickiness a la Calvo). Since wages are rigid, profits can increase on the impact of the shock thereby stimulating job creation. This effect amplifies the employment response and therefore brings the model closer to the data. In a recent and influential manuscript Shimer (2009) confirms the result in a fully fledged DSGE model with capital accumulation and
labor market frictions.

In this paper we investigate more deeply the conclusion that real wage rigidity can solve the unemployment volatility puzzle. Wage rigidity is an essential ingredient in medium scale DSGE models used for business cycle analysis, like Smets and Wouters (2003 and 2007) and Christiano, Eichenbaum and Evans (2005). However, these models include many other features that have been proven useful to explain business cycle facts and, in particular, monetary shocks. This is the case in models where employment adjusts only along the intensive margins (Smets and Wouters (2003 and 2007) and Christiano, Eichenbaum and Evans (2005)) but also in models with two margins of labor adjustment. Trigari (2009a) shows that sticky prices and habit persistence are essential to obtain plausible dynamics for monetary shocks. Thus, we believe that once we deviate from the benchmark model (where the only friction is in the labor market) there is no reason to insert in the model only wage rigidity and disregard the other frictions that have become standard in the business cycle literature. In particular we think that it is important to introduce sticky prices and real rigidities (in the form of habit persistence in consumption and investment adjustment costs) together with real wage rigidity in the context of a business cycle model with labor market frictions. It seems important to us to check whether real wage rigidity can still solve the unemployment volatility puzzle in this more complex and realistic setting.

We conduct our analysis in the framework developed by Blanchard and Gali (2008) that we extend to include capital accumulation. The labor market friction is modeled as hiring costs. We choose this framework for its simplicity but also because the unemployment volatility puzzle is less severe than in other models with labor market frictions (like models with search and matching frictions). The presence of capital accumulation is essential to break the so called "neutrality result" taken as a benchmark by Blanchard and Gali (2008) and reinterpreted by Shimer (2009). These authors show that unemployment is invariant to technology shocks in models that satisfy balanced growth with no capital accumulation.
As a preliminary exercise we confirm and strengthen the result by Shimer (2009). In a version of our model with flexible prices and no real rigidities, real wage rigidities amplify powerfully the transmission mechanism for technology shocks.

However, once we bring the model one step closer to standard models that are used in business cycle analysis, by adding sticky prices and real rigidities, we show that real wage rigidity does not affect business cycle dynamics anymore. In fact, a limited amount of price stickiness, habit persistence and investment adjustment costs do not limit employment volatility. In our simulation, employment is seven times more volatility than labor productivity. However, this volatility is only marginally affected by the presence of real wage rigidity.

Furthermore, the main result of our paper is that in the extended model a technology shock implies a positive comovement between labor productivity and unemployment. An improvement in technology increases labor productivity but decreases employment! This is not a feature that we observe in business cycles. This shortcoming, that we call "unemployment correlation puzzle" is in our opinion much more serious than the "unemployment volatility puzzle" that characterizes the benchmark model (with flexible prices and no real rigidities). Therefore, if we believe that business cycles are driven only by neutral technology shocks, the model cannot explain business cycle fluctuations.

This pattern echoes very famous results in Gali (1999) and Francis and Ramey (2005) in models where the adjustment is made along the intensive margin (hours worked) and unemployment is absent. The same mechanisms apply here. Nominal rigidities (in the form of sticky prices) and real rigidities (in the form of habit persistence and investment adjustment costs) slow down the response of aggregate demand to the shock and thus firms find it optimal to reduce the labor force. Moreover, real wage rigidity becomes almost irrelevant because sticky prices and real rigidities induce already a significant amount of endogenous real wage rigidity. Importantly, in our model monetary policy responds endogenously to the shock through a Taylor
Sticky prices and habits are both necessary to induce a large negative response of employment in our model. However, just one friction (habits or sticky prices) is sufficient to induce a contraction in employment for a given realistic value of investment adjustment costs.

Other papers in the literature are related to our contribution. Haefke, Sonntag and van Rens (2008) and Pissarides (2009) argue that real wage rigidity cannot solve the "unemployment volatility puzzle" because wages for new hires, the relevant wage series for search and matching models, are highly cyclical. Our paper also de-emphasizes the role played by real wage rigidity and shows other frictions can affect the employment response to technology shocks.

Sveen and Weinke (2008) show that, as long as prices are sticky, the unemployment volatility puzzle is still present in a model driven only by technology shocks. However, as pointed out by Shimer (2008), the absence of capital accumulation implies that their model is close to a set-up where the "neutrality result" holds. Our paper confirms the result by Sveen and Weinke (2008) in a model where large deviations from the "neutrality result" are induced by the explicit modeling of capital accumulation and by the assumption on the labor market proposed by Blanchard and Gali (2008).

The rest of the paper is organized as follows. In section 2 we present the model. In section 3 we discuss our results. Finally, we conclude in section 4.

## 2 The model

Our model features labor market frictions à la Blanchard and Gali (2008). In addition we allow for endogenous capital accumulation subject to investment adjustment costs as in Christiano, Eichenbaum and Evans (2005).

\footnote{Therefore our model is not subject to the criticism raised by Dotsey (2002) towards the Gali (1999) model.}
2.1 Households

Each household is composed by a continuum of family members of measure 1. Each period some family members are unemployed while others work for firms. We assume income and consumption pooling at the household level following Merz (1995) and Andolfatto (1996).

Households maximize the following intertemporal utility function

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left( \ln \left( C_t - eC_{t-1} \right) - \frac{N_t^{1+\eta}}{1 + \eta} \right),$$

(1)

where $\beta$ is the subjective discount factor, $N_t$ denotes the fraction of household members that are employed in period $t$ by household member $h$, $\eta$ is the inverse of the Frisch elasticity of labor supply, $e$ represents the degree of internal habit persistence and $C_t$ is a Dixit–Stiglitz consumption aggregate given by

$$C_t = \left( \int_0^1 C_t \left(i\right)^{1- \frac{1}{\epsilon}} \, di \right)^{\frac{1}{1-\epsilon}},$$

(2)

where $\epsilon$ is the elasticity of substitution between different varieties of goods $C_t \left(i\right)$.

Let $P_t \left(i\right)$ is the price of good $i$. The associated price index is then defined as

$$P_t = \left( \int_0^1 P_t \left(i\right)^{1-\epsilon} \, di \right)^{\frac{1}{1-\epsilon}}.$$  

(3)

The latter has the property that the minimum expenditure required to purchase a bundle of goods resulting in $C_t$ units of the composite good is given by $P_tC_t$.

The maximization is subject to a sequence of budget constraints which take the following form

$$P_t \left(C_t + \Psi_t^{-1} I_t\right) + D_t \leq D_{t-1}R_{t-1} + P_tW_tN_t + BU_t + T_t + P_tR^K_tK_t.$$  

(4)

where $I_t$ is the amount of the aggregate good acquired by the household for in-
vestment purposes and we have assumed that the elasticity of substitution is the same as for the consumption aggregate. Variables $R_t$ and $W_t$ are the gross nominal interest rate on bond holdings and the real wage, respectively, while $U_t \equiv 1 - N_t$ is period unemployment. Lump-sum transfers is denoted $T_t$, which includes dividends resulting from ownership of firms as well as lump-sum taxes, and $B$ is unemployment benefits, $\Psi_t$ represents a shock to the marginal efficiency of investment (investment specific technology shock). Last, we let households own the capital stock, $K_t$, and rent it out to firm at the real rental rate $R^K_t$. The capital rental market is assumed to be of perfect competition.

The capital accumulation equation be given by

$$K_{t+1} = (1 - \delta)K_t + \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right)I_t,$$  

(5)

where $\delta$ represents the depreciation rate and $S\left(\frac{I_t}{I_{t-1}}\right)$ is a function describing investment adjustment costs as in Christiano, Eichenbaum and Evans (2005). We assume that in steady state $S = 0$, $S' = 0$ and $S'' > 0$.

The consumer Euler equation implied by this structure takes the standard form

$$1 = R_t E_t \Lambda_{t,t+1}.$$  

(6)

where $\Lambda_{t,t+1} \equiv \beta \left\{ \left(\frac{\Omega_{t+1}}{\Omega_t}\right) \left(\frac{P_t}{P_{t+1}}\right) \right\}$ is the nominal stochastic discount factor and $\Omega_t$ represents the marginal utility of consumption

$$\Omega_t = \frac{1}{C_t - eC_{t-1}} - \beta e E_t \frac{1}{C_{t+1} - eC_t}.$$  

(7)

The first-order conditions with respect to investment can be written as follows

$$1 = Q_t \Psi_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\left(\frac{I_t}{I_{t-1}}\right)\right] + \beta E_t \left[\Lambda_{t,t+1} Q_{t+1} \Psi_{t+1} S'\left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2\right].$$
\[ Q_t = \beta E_t \left[ \Lambda_{t+1} R_{t+1} + Q_{t+1} (1 - \delta) \right] \]

where Tobin’s \( Q_t \) is equal to the ratio of the Lagrange multipliers attached to the capital accumulation equation and the budget constraint.

### 2.2 Firms

There is a continuum of monopolistically competitive firms indexed on the unit interval and each firm is assumed to produce a differentiated good, \( Y_t (i) \). Technology is Cobb-Douglas,

\[ Y_t (i) = K_t (i)^{1 - \alpha} (Z_t N_t (i))^\alpha, \]  

where \( Z_t \) indicates an exogenous labor-augmenting technology shock, and \( K_t (i) \) denotes the period \( t \) capital stock hired by firm \( i \). Last, \( N_t (i) \) denotes the number of employed workers in firm \( i \).

We follow Blanchard and Galí (2008) in assuming restrictions on firms’ hiring decisions. The law of motion of employment is given by

\[ N_t (i) = (1 - s) N_{t-1} (i) + L_t (i), \]  

where parameter \( s \) denotes the rate of separation and \( L_t (i) \) is the newly hired workers in firm \( i \). Moreover, it is implicit in this formulation that workers enters into productive activity immediately when they get hired, as in Blanchard and Galí (2008).

In order to hire firms face hiring costs. They are assumed to take the form per hire

\[ G_t = \Upsilon \left( \frac{L_t}{U_t^\vartheta} \right)^\vartheta. \]  

The hiring cost depends on aggregate labor market tightness, as parameterized by parameters \( \Upsilon \) and \( \vartheta \). Labor market tightness is measured by the fraction of aggregate
new hires to the amount of search unemployment, $U^S_t \equiv 1 - (1 - s) N_{t-1}$, i.e. the fraction of the labor force that is searching for a job at the beginning of period $t$.

Cost minimization on the part of households implies that demand for each individual good $i$ in period $t$ is given by

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t,$$  \hspace{1cm} (11)

Finally, we assume staggered price setting à la Calvo (1983), i.e. each period a measure $(1 - \theta)$ of randomly selected firms get to re-optimize their price while the remaining firms keep their prices constant. Given those assumptions each firm $i$ solves the following problem:

$$\max \sum_{k=0}^{\infty} \mathbb{E}_t \left\{ \Lambda^R_{t,t+1} \left[ Y_{t+k}(i) \frac{P_{t+k}(i)}{P_{t+k}} - W_{t+k}(i) N_{t+k}(i) - R^K_{t+k} K_{t+k}(i) - G_{t+k} L_{t+k}(i) \right] \right\}$$

s.t.

$$Y_{t+k}(i) = \left( \frac{P_{t+k}(i)}{P_{t+k}} \right)^{-\epsilon} Y_{t+k},$$

$$Y_{t+k}(i) = K_{t+k}(i)^{\alpha} (Z_{t+k} N_{t+k}(i))^{1-\alpha},$$

$$N_{t+k}(i) = (1 - s) N_{t+k-1}(i) + L_{t+k}(i),$$

$$P_{t+k+1}(i) = \begin{cases} 
  P^*_t(i) & \text{with prob. } (1 - \theta) \\
  P_{t+k}(i) & \text{with prob. } \theta 
\end{cases}.$$

The first-order condition for price-setting is given by

$$\sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left\{ \Lambda^R_{t,t+1} \frac{Y_{t+k}(i)}{P_{t+k}} [P^*_t(i) - \mu P_{t+k} MC_{t+k}(i)] \right\} = 0,$$  \hspace{1cm} (12)

where $\Lambda^R_{t,t+1} = \Lambda_{t,t+1} \frac{P_{t+1}}{P_t}$, $P^*_t(i)$ is the optimally chosen price, $\mu \equiv \frac{\epsilon}{\epsilon + 1}$ is the fric-

\footnote{The hiring cost fluctuates with the two shocks to insure that permanent shocks have no permanent effect on employment and hours.}
tionless markup, and \( MC_t (i) \) denotes firm \( i \)'s real marginal cost in period \( t \). From cost minimization we obtain

\[
MC_t (i) = \frac{R^K_t}{\alpha Y^t (i)/K^t (i)}.
\]

Note, however, that heterogeneity in prices and thereby in output does not translate into heterogeneity in the real marginal cost since firms have constant returns to scale in employment and capital. We therefore have \( MC_t (i) = MC_t \ \forall i \).

Equation (12) reflects the forward-looking nature of price-setting: firms take into account not only current but also future expected marginal costs. The remaining first-order condition reads

\[
W_t + G_t = (1 - \alpha) \frac{MC_t Y_t (i)}{N_t (i)} + (1 - s) E_t \{ \Lambda^R_{t+1} G_{t+1} \}
\]

where the left hand side gives the cost associated with hiring one additional worker. That cost includes both a wage payment and hiring costs. The right hand side gives the benefit from hiring one additional worker, i.e., the marginal savings in the cost of using capital associated with having an additional worker in place, as well as expected reductions in future hiring costs.

### 2.3 Wage negotiation

Households and firms engage in a Nash bargaining to negotiate the wage \( W^b_t \). The household’s value of a match with firm \( i \) is given by

\[
\tilde{W}_t (i) = W^b_t (i) - \chi \Omega^{-1} N^w_t + E_t \left\{ \Lambda^R_{t+1} \left[ (1 - s) \tilde{W}_{t+1} (i) \\
+ s \left( F_{t+1} \tilde{W}_{t+1} + (1 - F_{t+1}) \tilde{U}_{t+1} \right) \right] \right\}.
\]

\(^3\)Sveen and Weinke (2007) analyze the case where firms in addition to hiring costs face firm-specific labor-adjustment costs.
where \( \bar{W}_t \equiv \int_0^1 \bar{W}_t(i) \frac{L_t(i)}{L_t} di \) denotes the average value of a match and \( F_t \equiv \frac{L_t}{U_t} \) is the job-finding probability. The value of a match with firm \( i \) consists of three elements. First, the real wage income. Second, the associated disutility of supplying labor (expressed in units of consumption). Third, the expected discounted value of continuing the match in the next period, or of searching for a job.

The value of being unemployed after hiring has taken place is given by

\[
\bar{U}_t = B + E_t \left\{ \Lambda_{t,t+1}^R \left[ F_{t+1} \bar{W}_{t+1} + (1 - F_{t+1}) \bar{U}_{t+1} \right] \right\},
\]

(16)

which equals the unemployment benefit and the expected discounted value of looking for a job in the next period.

We follow Blanchard and Galí (2008) in assuming that newly hired workers become productive instantaneously. This implies that the value of a match for firm \( i \) corresponds to the cost of hiring a worker

\[
\bar{J}_t = G(F_t),
\]

(17)

which is independent of the firm. The value of an open vacancy for firm \( i \) is zero, given our assumptions.

The wage is chosen in such a way that the Nash product is maximized, which implies the following first order condition

\[
(1 - \phi) \bar{J}_t = \phi \left( \bar{W}_t(i) - \bar{U}_t \right),
\]

(18)

where \( (1 - \phi) \) denotes the weight of workers in the bargain. Next, we substitute for \( \bar{J}_t, \bar{U}_t \) and \( \bar{W}_t(i) \) in the last equation. Noting that \( \bar{W}_t(i) \) is equal across firms allows us to find the wage resulting from the bargain in the following way

\[
W_t^b(i) = \chi \Omega_t^{-1} N_t^n + \frac{1 - \phi}{\phi} \left[ G_t - (1 - \delta) E_t \left( \Lambda_{t,t+1}^R (1 - F_{t+1}) G_{t+1} \right) \right],
\]

(19)
As in Hall (2005) we model real wage rigidity by the following ad hoc partial adjustment mechanism:\textsuperscript{4}

\[ W_t = \gamma W_{t-1} + (1 - \gamma) W^b_t \]

### 2.4 Monetary policy

For simplicity, we assume a simple monetary policy rule according to which the central bank reacts to inflation ($\Pi_t$)

\[
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho_R} \left[ \left( \frac{\Pi_t}{\Pi} \right)^{\phi} \right]^{1-\rho_R}
\]

where $\rho_R$ denotes the degree of interest rate smoothing.

### 2.5 Market clearing and exogenous shocks

Market clearing for each variety $i$ requires at each point in time that

\[ Y_t(i) = Y_t^d(i). \tag{20} \]

The aggregate resource constraint is given by

\[ C_t + \Psi_t^{-1} I_t + \gamma \left( \frac{L_t}{U_t} \right)^\theta L_t = Y_t \]

Finally, clearing of the bond market requires that $B_t = 0$, which holds for all $t$.

The exogenous shocks are described by autoregressive processes:

\[
\log \Psi_t = \rho_{\Psi} \log \Psi_{t-1} + \varepsilon_{\Psi,t}
\]

\textsuperscript{4}We will soon introduce the more elegant staggered wage setting proposed in Gertler and Trigari (2009) in a real model and adapted to a monetary model by Gertler, Sala and Trigari (2008)).
\[
\log Z_t = \rho \log Z_{t-1} + \varepsilon_{Z,t}
\]

2.6 Calibration

Let us now discuss the values which we assign to the model parameters in most of the quantitative analysis that we are going to conduct. The period length is one quarter. We let $\beta$ be 0.99, which implies an annual steady state real interest rate of about 4 per cent.

We follow Golosov and Lucas (2007) and set the elasticity of substitution between goods, $\epsilon$, to 7. This implies a steady-state mark-up of about 20 per cent. Our baseline value for the Calvo parameter, $\theta$, is 0.6, which is a conservative choice with respect to the recent empirical finding of Nakamura and Steinsson (2006) that firms change their prices on average every third quarter (equivalent to $\theta$ equal to 0.67).

As far as monetary policy is concerned we set $\tau_x$ to 1.5 as originally suggested by Taylor (1993) and the parameter measuring interest rate smoothing, $\rho_r$, is set to 0 at the outset but we set it at 0.8 in the sensitivity analysis (to be added). These parameter values are reasonable given the empirical results in, e.g., Clarida et al. (2000).

We set $\eta$ equal to 1 as in Blanchard and Gali (2008) and many papers in the real business cycle literature. We follow Shimer (2005) in setting steady state period unemployment to 0.057 and the quarterly job-finding rate to 0.71.\footnote{We compute the quarterly rate as $0.34 \times \sum_{j=1}^{3} (1 - 0.34)^{j-1}$, where 0.34 is the corresponding monthly rate reported by Shimer.} Given our model this implies a separation rate of about 0.15\footnote{The values used in the literature range from 0.07 (Merz 1995) to 0.15 (Andolfatto 1996).} and steady-state search unemployment of about 0.20. Following Hall (2005) the unemployment benefit, $B$, is set to 40% of steady state labor income. In order to calibrate the elasticity in the hiring cost function, $\vartheta$, we follow Blanchard and Gali (2008) and use a simple relationship between the hiring cost model and the Mortensen and Pissarides (1994) model.
In the latter, the matching function is given by $L = \omega V^\gamma U^{1-\gamma}$, where $V$ denotes vacancies, $\gamma$ is the elasticity of the matching function and $\omega$ is a constant. In that framework the cost of hiring an additional worker is proportional to $V/L = \omega^{\frac{1}{\gamma}} F^{\frac{1-\gamma}{\gamma}}$. Estimates for $\gamma$ are typically around 0.5 and we correspondingly set $\vartheta = \frac{1-\gamma}{\gamma} = 1$.

We choose $\phi$ (the bargaining power parameter) equal to 0.5 as in Trigari (2009a and 2009b). Given the elasticity of the matching function, the first-order condition for employment and the wage equation, both evaluated in steady state, imply two conditions to pin down the steady state wage income $WH$ and parameter $\chi$. Last, we use $\chi$ to pin down hours in steady state to $1/3$ of available time.

We set the depreciation rate ($\delta$) at 0.025, the capital share ($\alpha$) at 0.33, the second derivative of the investment adjustment cost function evaluated in steady state ($\lambda_1$) at 0.8, the value estimated by Smets and Wouters (2007). The degree of habit persistence ($\epsilon$) is 0.8 and the degree of real wage rigidity ($\gamma$) is 0.7.

### 3 Results

In this section we present results based on various versions of the model described in section 2. First, we show that the "unemployment volatility puzzle" is not so severe in a version of the model where the only friction is given by hiring costs. Second, we confirm, as in Shimer (2009), that adding real wage rigidity in the model increases unemployment volatility. Third, we show that adding sticky prices and real rigidities the presence of real wage rigidities does not affect the equilibrium dynamics anymore because the other nominal and real frictions generate endogenous real wage rigidity. Finally, and perhaps most importantly, the "unemployment correlation puzzle" arises in the model with nominal and real rigidities.
3.1 Labor market frictions and the unemployment volatility puzzle

Costain and Reiter (2008), Shimer (2005 and 2009) and Hall (2005) have shown that models with labor market frictions have troubles at generating unemployment volatility in response to productivity shocks. Blanchard and Gali (2008) have shown that unemployment is even invariant to productivity shocks as long as there is no capital in the model and preferences are consistent with balanced growth (log-utility and both unemployment benefits and hiring costs proportional to productivity). This invariance result (or "neutrality result") has been confirmed by Shimer (2009 chapter 2) in a model with search and matching frictions. Our first objective is to measure the deviation from the "neutrality result" in our set-up where we explicitly model capital accumulation and where hiring costs and unemployment benefits are acyclical (since we do not deal with permanent shocks).

In figure 1 we present impulse response functions for a version of our model where we shut down all nominal and real rigidities (flexible wages, flexible prices, no habits and no investment adjustment costs). Therefore, the only friction left is on the hiring process. The model is then comparable to the one in Shimer (2009 chapter 3) or to the standard RBC model with labor market frictions (Andolfatto (1996) and Merz (1995)). From figure 1 and table 1 we see that our model exhibits too little volatility in employment (and thus also in unemployment) when compared to US data. However, we remark a considerable deviation from the neutrality result and from the standard RBC model with search and matching frictions. This relatively large response of employment, although still too small with respect to the data, is due to the assumptions on the hiring process. In particular, two features originally introduced by Blanchard and Gali (2008), favor a large employment volatility. The first is the assumption of instantaneous hiring. From (9) we see that new hires be-

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7 Computed statistics on US data and on the RBC model with labor market frictions are taken from Baleer (2008). The RBC model is a version of Merz (1995) where employment is a predetermined variable and the labor market friction is modeled as a search and matching process.
come immediately productive whereas the standard assumption in the literature is that new hires become productive with a one period delay (and therefore employment is a predetermined variable). The second feature is the assumption that the market for hires is always open. This implies that the value of a match for the firm is simply given by the hiring cost (17). In models with search and matching frictions firms have to post vacancies and this is costly. Moreover, it takes time to fill in vacancies. Here the hiring process is much smoother from the firm’s perspective: in case of separation the firm simply pays the hiring cost and hires immediately another worker. The combination of these two features allows employment to fluctuate more.  

The implied large deviation from the neutrality result provides an additional reason, other than simplicity, to use the Blanchard-Gali (2008) set-up to model labor market frictions.  

\[ \text{Table 1} \]

<table>
<thead>
<tr>
<th></th>
<th>st dev unemployment</th>
<th>st dev lab prod($\frac{\alpha}{\beta}$)</th>
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</thead>
<tbody>
<tr>
<td>US Data</td>
<td>11.41</td>
<td></td>
</tr>
<tr>
<td>RBC model based on Merz (1995)</td>
<td>3.12</td>
<td></td>
</tr>
<tr>
<td>Model with hiring costs and flexible wages</td>
<td>6.04</td>
<td></td>
</tr>
<tr>
<td>Model with hiring costs and $\gamma = 0.7$</td>
<td>9.75</td>
<td></td>
</tr>
<tr>
<td>Model with hiring costs and $\gamma = 0.8$</td>
<td>12.08</td>
<td></td>
</tr>
</tbody>
</table>

Thus, the first result of the paper is that the unemployment volatility puzzle is less severe in our model than in the standard RBC set-up.

### 3.2 Adding real wage rigidities

Shimer (2009 chapter 4) argues that real wage rigidities can limit the unemployment volatility puzzle. The idea is that in the model with flexible wages, the wage rate

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8Moreover, Furlanetto and Sveen (2009) show that these assumptions guarantee a larger response of the employment margin in models with two margins of labor adjustment (hours and employment).

9Since there is a large deviation from the neutrality result, the Shimer (2007) critique to the Sveen and Weinke (2007) paper does not apply here.
responds too much thereby squeezing profits and limiting job creation. Therefore, wage rigidity can limit by construction the increase in the real wage and facilitate the expansion in employment. This reasoning is confirmed in our model. Dotted lines in figure 1 and statistics in table 1 show that the employment response is larger on impact and is more volatile. The ratio of the unemployment standard deviation over the labor productivity standard deviation is now 9.75, still slightly lower than in the data (11.41), but much larger than in models with flexible wages.

Notice that our model features the same maximum employment response as in Shimer (2009, figure 4.1, once adjusted for the size of the shock). However, we achieve the same effect for a lower degree of real wage rigidity ($\gamma = 0.7$ in our model, $\gamma = 0.97$ in Shimer (2009)). This is due to the fact that the transmission mechanism for productivity shocks is more expansionary in our model for the reasons explained in the previous subsection. Therefore, so far we have confirmed and strengthened the fact that real wage rigidities can amplify the transmission mechanism for productivity shocks.

Interestingly, if we raise the parameter measuring real wage rigidity ($\gamma = 0.8$) the unemployment volatility puzzle is solved (last line of table 1). Thus, the second result of the paper is that if we are willing to accept a high degree of real wage rigidity (but still significantly lower than in Shimer (2009 chapter 4)), it is possible to replicate the unemployment volatility that we see in the data.

### 3.3 Adding sticky prices and real rigidities

In this subsection we analyze the role of sticky prices and real rigidities (in the form of habit persistence and investment adjustment costs) for the transmission mechanism of productivity shocks in our model. We believe that once we deviate from the frictionless benchmark (where the only friction is in the hiring process) there is no reason to insert in the model only real wage rigidity and disregard the other frictions that have become standard in the business cycle literature. In this paper
we concentrate on sticky prices, habit persistence and investment adjustment costs but other mechanisms like inflation indexation, variable capital utilization, Kimball demand curves or firm-specific capital would not invalidate our result. Notice that all these frictions have proven very useful at explaining monetary shocks (Christiano, Eichenbaum and Evans (2005), Eichenbaum and Fisher (2008), Sveen and Weinke (2005) among many others), fiscal shocks (Furlanetto and Seneca (2009)) and investment-specific shocks (Justiniano, Primiceri and Tambalotti (2008)).

In table 2 and figure 2 we consider two versions of our model: in the first (bold lines) we have sticky prices and real rigidities with flexible wages whereas in the second version (dashed lines) we allow also for real wage rigidity.

<table>
<thead>
<tr>
<th></th>
<th>st dev unemployment (\times \text{st dev lab prod} (\frac{1}{T}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>11.41</td>
</tr>
<tr>
<td>RBC model based on Merz (1995)</td>
<td>3.12</td>
</tr>
<tr>
<td>Model with hiring costs, sticky prices, real rig. and flexible wages</td>
<td>7.25</td>
</tr>
<tr>
<td>Model with hiring costs, sticky prices, real rig. and (\gamma = 0.7)</td>
<td>6.62</td>
</tr>
</tbody>
</table>

From table 2 we see that sticky prices and real rigidities make the ratio of the unemployment standard deviation over the labor productivity standard deviation even larger than in the model with flexible prices and no real rigidities (7.25 vs 6.04 in table 1). The reason is that unemployment is even more volatile than before. Therefore it seems that the unemployment volatility puzzle is less severe in a model with more frictions.

Furthermore, from table 2 we see also that in the model with more frictions real wage rigidity does not play any role. From statistics in table 2 and impulse responses in figure 2 we see that real wage rigidity does not affect anymore the transmission mechanism anymore. The reason is that sticky prices, real rigidities
and the assumptions on the hiring process affect importantly the wage equation 19 generating a significant amount of endogenous real wage rigidity.

Therefore, the third result of the paper is that real wage rigidity can amplify unemployment volatility only as long as prices are flexible and real rigidities are not in the model.

Finally, and perhaps more importantly, the main result of the paper comes from impulse responses in figure 2. The introduction of sticky prices and real rigidities affects crucially the transmission mechanism for technology shocks. We see that the unemployment response is large and positive. The model implies a positive correlation between output and unemployment.

This pattern echoes very famous results in Gali (1999) and Francis and Ramey (2005) in models where the adjustment is made along the intensive margin (hours worked) and unemployment is absent. In these papers a positive technology shock implies a contraction in hours worked on the impact of the shock. The same mechanisms apply here. Nominal rigidities (in the form of sticky prices) and real rigidities (in the form of habit persistence and investment adjustment costs) slow down the response of aggregate demand to the shock and thus firms find it optimal to reduce the labor force. As in Gali (1999), our result relies in part on a suboptimal monetary policy. However, note that, unlike Gali (1999), in our model monetary policy responds endogenously to the shock through an aggressive anti-inflationary Taylor rule. The employment response would be even more negative for a positive degree of interest rate smoothing.

Importantly, both sticky prices and habits are necessary to induce a large negative response. However, just one friction (habits or sticky prices) is sufficient to induce a contraction in employment for a given degree of investment adjustment costs (see figure 3).
4 Discussion and conclusion

We suggest two possible interpretations for our results. If we believe that technology shocks are the main driver of aggregate fluctuations, as most of the literature studying the model with labor market frictions, our paper shows that real wage rigidity is a solution to the unemployment volatility puzzle only as long as prices are flexible and real rigidities are absent. If we are willing to accept a moderate amount of price stickiness and reasonable real rigidities, real wage rigidity becomes irrelevant and the model delivers the "unemployment correlation puzzle" instead of the "unemployment volatility puzzle". In fact, in our model with sticky prices and real rigidities, employment is very volatile, more than in the model with no sticky prices and no real rigidities. Thus volatility is not an issue in the model. However, correlation is much more problematic. Our model implies a negative correlation between employment and output (and consumption). This is not a feature that we observe in business cycles. The "unemployment correlation puzzle" is in our opinion a much more serious shortcoming than the "unemployment volatility puzzle" that characterize the model with flexible prices and no real rigidities. Therefore, if we believe that business cycles are driven by neutral technology shocks, the model cannot explain business cycle fluctuations.

The second interpretation is less pessimistic. It can very well be that business cycles are the result of the interaction of many shocks with different characteristics. In that case, comparing unconditional data (that in principle can be driven by several shocks) with data generated by a one-shock model is not a sensible exercise. Then there are two solutions: either compare unconditional data to a model driven by several shocks (as in the literature on estimated DSGE models) or compare the model driven only by technology shocks with the empirical evidence conditional on technology shocks. The first approach is taken by Mandelman and Zanetti (2008) and Gertler, Sala and Trigari (2008). Both papers, although using very different models, estimate a negative response of employment on the impact of a technology shock.
The second approach has been taken in four recent papers looking at the impact of neutral technology shocks on unemployment using VARs (Canova, Michelacci and Lopez-Salido (2008a and 2008b), Balleer (2008) and Barnichon (2008)). All the four papers find that a positive neutral technology shock generates a positive and significant response of unemployment. According to these empirical results, our model would not generate any counterfactual dynamics. The "unemployment volatility puzzle" and the "unemployment correlation puzzle" would not be puzzles anymore but just the outcome of a comparison between unconditional and conditional evidence. A possible solution to reproduce the unconditional evidence would then be to introduce more shocks in the model. Sveen and Weinke (2008) show some promising results in that direction.\footnote{We are now working on investment specific shocks (as shown in the model in section 2).}
Figure 1: baseline model (no real rigidities and flexible prices)
Figure 2: model with sticky prices and real rigidities
Figure 3: model with sticky prices or real rigidities (flexible wages in both cases)
References


Haefke, C., M. Sonntag and T. van Rens, 2008. Wage rigidity and job creation, mimeo CREI.


Appendix: the linearized model

In what follows we consider a log-linear approximation to the equilibrium dynamics around a zero inflation steady state. Unless stated otherwise lower case letters denote the log-deviation of the original variable from its steady state value. The consumption Euler equation reads

\[ \omega_t = E_t \omega_{t+1} - (r_t - E_t \pi_{t+1} - \rho), \] (21)

where parameter \( \rho \) denotes the household’s time preference rate. Up to the first order aggregate production is given by

\[ y_t = \alpha k_t + (1 - \alpha) (z_t + n_t). \] (22)

The linearized first order conditions with respect to investment and capital read as follows

\[ i_t = \left( \frac{1}{1 + \beta} \right) (\beta E_t i_{t+1} + i_{t-1} + \lambda_1 (q_t + \psi_t)) \]

\[ q_t = - (r_t - E_t \pi_{t+1}) + (1 - \beta (1 - \delta)) r_{t+1}^k + \beta (1 - \delta) q_{t+1} \]

The capital accumulation equation is given by

\[ k_t = (1 - \delta) k_{t-1} + \delta i_t \]

Cost minimization by firms implies

\[ r_t^k = mc_t + y_t - k_t \]

Aggregating the linearized law of motion of firm-level employment results in

\[ n_t = (1 - s) n_{t-1} + sl_t. \] (23)
Linearized unemployment reads

\[ u_t = -(1 - s) \frac{N}{U} n_{t-1}, \]  

(24)

where we have used the notation that a variable without a time subscript denotes the steady state value of that variable. Period unemployment is given by

\[ u_t^M = -\frac{N}{UM} n_t. \]

(25)

Aggregating and linearizing the first order condition for firm-level employment implies

\[ w_t = \left( \frac{Y}{W} \right)^\theta (-\varphi f_t - \beta (1 - \delta) (-\varphi f_{t+1} + r_t - E_t \pi_{t+1})) + (1 - \alpha) \frac{MC*Y}{WN} (mc_t + y_t - n_t) \]

(26)

The following relationships holds true

\[ f_t = l_t - u_t. \]

(27)

The real wage is given by

\[ w_t^b = \frac{\chi N^\phi}{W\Omega} (\eta m_t - \omega_t) + \left( \frac{\phi Y}{W} \right)^\theta \varphi f_t - \left( \frac{\phi \beta (1 - \delta) (1 - F Y (\frac{L}{\Omega^\phi}))^\theta}{W} \left( -\omega_t + \omega_{t+1} \right) \right) - \left( \frac{(\varphi - (\varphi + 1) F) \phi \beta (1 - \delta) Y (\frac{L}{\Omega^\phi})^\theta}{W} \right) \]

(28)

The standard New Keynesian Phillips curve for inflation is derived

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa mc_t, \]

(30)

where parameter \( \kappa = \frac{(1-\beta\theta)(1-\theta)}{\theta} \).
The monetary policy rule is given by

$$r_t = \rho_r r_{t-1} + (1 - \rho_r) \left[ \rho + \tau_{zt} \pi_t \right]$$  \hspace{1cm} (31)

Finally, let us state the exogenous driving forces

$$z_t = \rho_z z_{t-1} + e_{zt},$$  \hspace{1cm} (32)

$$\psi_t = \rho_\psi \psi_{t-1} + e_{\psi t},$$  \hspace{1cm} (33)