Non-uniform staggered prices and output persistence

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Abstract

Staggered prices are a fundamental building block of New Keynesian dynamic stochastic general equilibrium models. Recent empirical evidence, however, suggest that deviations from uniform staggering are common. This paper analyses how such deviations affect the theoretical response to monetary policy shocks. I find that even large deviations from uniform staggering have small effects on the response in output. This suggests that a model of uniform staggering may serve well as an approximation to a more complicated model with some degree of synchronization in price setting.

1 Introduction

There is little disagreement in the literature that monetary policy shocks are far from neutral, but quite potent in producing long-lived effects on output, nor that this is a consequence of that prices and wages not being continuously reoptimized. There is also little disagreement that the boom in output prevails longer than the typical price or wage contract. The literature tries to explain this

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by staggered pricing models, in the spirit of Taylor (1980) and Calvo (1983), where only a fraction of price setters are allowed to reoptimize in every period. The key insight is that a price setter about to reoptimize realizes that some prices remain fixed and therefore sets a lower price than he would if all prices were changed at the same time.

The modern synthesis is the New-Keynesian DSGE model, which combines staggered price setting with the microfoundations of real business cycle models. This has become the workhorse model for monetary policy analysis. The pivotal assumption of staggered prices in this class of models has spurred an interest in investigating this issue empirically. It is commonplace to use the FK-index\(^1\) proposed by Fisher and Konieczny (2000), as a measure of the degree of synchronization in price setting. This index takes a value of 0 under uniform (perfect) staggering and it takes a value of 1 under perfect synchronization. A principal finding is that the degree of synchronization in price setting varies at different levels of aggregation in the demand system. In particular, staggering is most predominant when the FK-index is applied to data that has been aggregated to only contain a few broad product categories. In such cases the FK-index is commonly found be in the range 0.1 to 0.25, see e.g. Dhyne, Alvarez, Bihan, Veronese, Dias, Hoffmann, Jonker, Lunnemann, Rumler, and Vilmunen (2006) and Dhyne and Konieczny (2007). In contrast, considerable synchronization is typically found at low levels of aggregation. For instance, Cornille and Dossche (2006), using Belgian producer price data, find that for about 80 percent of the product categories at the NACE 4-digit level, the FK-index lies between 0.25 and 0.75.\(^2\)

\(^1\)Defined as
\[
\sqrt{\frac{1}{T} \sum_{t=1}^{T} (\alpha_t - \bar{\alpha})^2}
\]
where \(T\) is the sample size, \(\alpha_t\) the proportion of the firms changing their price between periods \(t - 1\) and \(t\), and \(\bar{\alpha}\) the sample mean of \(\alpha_t\).

\(^2\)Dhyne and Konieczny (2007) find a similar result for consumer prices. They also emphasize the importance of geographical aggregation, with synchronization being the highest at the city level. This view is confirmed by Veronese,
Both the Taylor and the Calvo model are built on the assumption that a constant fraction of all prices are reset every period, meaning that prices are uniformly staggered. But such a price setting behavior is, at the least, not consistent with the empirical evidence at low levels of aggregation. The crucial question is how sensitive staggered pricing models are to deviations from uniform staggering, and whether they still have the ability to generate sufficient output persistence, once the assumption of uniform staggering is relaxed. In the first part of this paper, I try to answer this question by solving a simple analytically tractable two-period Taylor pricing model. I study how deviations from uniform staggering affect the economy by varying the fraction of firms setting their prices in odd and even periods. The main finding is that the expected cumulative effect on output following a monetary policy shock is a non-linear function of the degree synchronization. Small deviations from uniform staggering have small effects on output much, but small deviations from perfect synchronization have large effects. This suggests that a model of uniform staggering may serve well as an approximation to a more complicated model with some degree of synchronization in price setting.

In the second part of the paper I extend the analysis to a full-fledged general equilibrium model. A model seeking to expand the conventional knowledge on how the timing of price changes affect the real economy should, in order to account for that the degree of synchronization varies at different levels of aggregation, plausibly contain more than one level in the demand system. I set up a model with many sectors, where each sector corresponds to an individual product category. Each sector further consists of a large number of firms, each having some monopoly power from the production of a different brand. Prices are uniformly staggered at the aggregate level, but there is some degree of synchronization within sectors.

Fabiani, and Sabbatini (2005), using Italian consumer price data.
While staggering at the aggregate level is enough to create some persistence, regardless of the degree of staggering at the sector level, the literature has recognized the need of real rigidities in order to create sufficient persistence in output. Real rigidities interact with staggering by introducing mechanisms that slow down price adjustment when demand is reallocated from adjusters to non-adjusters. Eichenbaum and Fisher (2007) emphasize that a model where the elasticity of demand varies along a firm’s demand curves, as in Kimball (1995), and firm-specific capital, where a firm’s marginal cost is an increasing function of the firm’s demand, is consistent with data.

For a plausible calibration I find that the increase in persistence, when real rigidities are introduced, is much larger when prices at the sector level are uniformly staggered, compared to perfectly synchronized. However, as in the simple two-period model, the mapping between the degree of synchronization and the expected cumulative effect on output is highly non-linear and deviations from uniform staggering have smaller effects on output, than deviations from perfect synchronization.

The remainder of this paper is organized as follows. In the next section I present and solve the simple two-period Taylor model analytically. Section 3 presents the full model and Section 4 the results from the numerical simulations. Section 5 concludes.

2 A model with an analytical solution

Assume an economic environment with a continuum of monopolistically competitive firms, indexed by $i \in [0,1]$. The firms are divided into two, \textit{ex ante} identical, cohorts. In each cohort, prices are set every other period and for two consecutive periods. The cohort consisting of price setters setting their prices in even periods encompasses a fraction $\alpha \in [0,1]$ of the firms and the cohort consisting of price setters setting their prices in odd periods a fraction $1 - \alpha$. In the absence of any
price setting frictions, the optimal time $t$ relative price is given by

$$\bar{p}_{it} - p_t = \phi y_t, \quad \phi > 0$$  \hspace{1cm} (1)

where $\bar{p}_{it}$ is the optimal nominal price, $p_t$ the price level and $y_t$ aggregate real activity. All lower case variables are expressed as log-deviations from their steady state values. If $\phi < 1$ prices are strategic complements and if $\phi > 1$ they are strategic substitutes. A firm allowed to reoptimize, sets its reset price $p^*_it$ as an average of the expected frictionless prices over the two periods the price remains fixed:

$$(p^*_it - p_t) + (p^*_it - E_t p_{t+1}) = \phi (y_t + E_t y_{t+1})$$  \hspace{1cm} (2)

The price level is obtained by integrating over all individual prices:

$$p_t = \int_0^1 p_{it} di.$$  \hspace{1cm} (3)

The model nests the Taylor (1980) model of uniformly staggered pricing as a special case when $\alpha = 1/2$. If instead $\alpha = 0 \ (= 1)$ all prices are set in odd (even) periods and price setting is perfectly synchronized. For intermediate values of $\alpha$, prices are neither uniformly staggered, nor perfectly synchronized. One can use the FK-index to map the value of $\alpha$ into a measure of the degree synchronization in price setting through the relation

$$FK = \sqrt{1 - 4\alpha (1 - \alpha)}.$$  \hspace{1cm} (4)
To close the model, aggregate demand is assumed to follow the quantity theory relation

\[ y_t = m_t - p_t, \]  

(5)

where money supply \( m_t \) is a random walk

\[ m_t = m_{t-1} + \varepsilon_t, \]  

(6)

and \( \varepsilon_t \) is a white noise process with finite variance.

Under these assumptions the model can be solved analytically. It turns out that output evolves according to the first order difference equation:

\[ y_t = \gamma_I y_{t-1} + \omega_I \varepsilon_t, \]  

(7)

where \( I \) is a variable taking a value of one in odd periods and a value of two in even periods, and \( \gamma_I \) and \( \omega_I \) are given by

\[
\gamma_1 = \frac{(1 - \alpha) \lambda_1 + \alpha}{\alpha \lambda_2 + (1 - \alpha) \lambda_2},
\]

(8)

\[
\omega_1 = (1 - \alpha) \lambda_1 + \alpha,
\]

(9)

and

\[
\gamma_2 = \frac{\alpha \lambda_2 + (1 - \alpha)}{(1 - \alpha) \lambda_1 + \alpha} \lambda_1,
\]

(10)

\[
\omega_2 = \alpha \lambda_2 + (1 - \alpha),
\]

(11)
where $\lambda_1$ and $\lambda_2$ are obtained by solving the system

\begin{align*}
\lambda_1 - A_1 (\lambda_1 \lambda_2 + 1) &= 0, \tag{12} \\
\lambda_2 - A_2 (\lambda_1 \lambda_2 + 1) &= 0, \tag{13}
\end{align*}

where

\begin{align*}
A_1 &= \frac{\alpha (1 - \phi)}{2(\phi + \alpha (1 - \phi))}, \tag{14} \\
A_2 &= \frac{(1 - \alpha)(1 - \phi)}{2(\phi + (1 - \alpha)(1 - \phi))}. \tag{15}
\end{align*}

The derivations are in Appendix A. In general, the response in output to a shock to money supply will be different depending on whether the shock hits in an even or an odd period. The model is especially easy to solve for a few special cases. First consider the case with perfect synchronization, which can be summarized in the following set of results:

**Proposition 1** If $\alpha = 0$, then the solution to the system in (8) - (11) is given by $\gamma_1 = 0$, $\gamma_2 = 0$, $\omega_1 = 0$, and $\omega_2 = 1$. If $\alpha = 1$, then the solution is given by $\gamma_1 = 0$, $\gamma_2 = 0$, $\omega_1 = 1$ and $\omega_2 = 0$.

**Corollary 2** If $\alpha = 0$ ($= 1$) a shock to money supply is neutral in odd (even) periods. If instead the shock occurs in an even (odd) period, all prices are fixed in advanced and the shock to money supply fully transmitted into output. Output is however not persistent and returns to steady state in the following period, when all price setters again are allowed to change their prices.

The inability of monetary policy to create persistent effects in output should be understood in the light of how prices are set in the economy. Combining (2) with (5) and (6), the optimal reset
price can be written as
\[ p_{it}^* = \phi m_t + \frac{(1 - \phi)}{2} (p_t + E_t p_{t+1}). \] (16)

Imposing \( p_{t+1} = p_t = p_{it}^* \) the optimal reset price set in this period exactly replicates the optimal frictionless price \( p_{it}^* = m_t \). Thus, the increase in money supply is completely absorbed into prices, whenever price setters are given the opportunity to reoptimize.

The second set of results pertains to the case when prices are neither strategic complements, nor strategic substitutes.

**Proposition 3** If \( \phi = 1 \), then, \( \forall \alpha \in [0, 1] \) the solution to the system in (8) - (11) is given by \( \gamma_1 = 0 \), \( \gamma_2 = 0 \), \( \omega_1 = \alpha \) and \( \omega_2 = 1 - \alpha \).

**Corollary 4** If \( \phi = 1 \), the effect on output, following an increase in money supply, in the period that the shock occurs is proportional to the fraction of non-adjusters in that period. Output is however not persistent and returns to steady state in the following period, when the first round of price adjustments is completed.

Again, the lack of persistence is a result of that price setters replicate the frictionless optimal price whenever they are allowed to reoptimize. The increase in money supply is completely absorbed into prices, once all price setters have been given the opportunity to reoptimize after the shock.

The third set of results pertains to the general case when prices are neither perfectly synchronized, nor necessarily uniformly staggered, and prices are either strategic substitutes or complements.

**Proposition 5** If \( \alpha \in (0, 1) \) and \( \phi \neq 1 \), then the stable solution (\( \gamma_1 \) and \( \gamma_2 \) inside the unit circle)
to the system in (12) and (13) is given by

\[ \lambda_1 = \frac{1 - \sqrt{1 - 4A_1A_2}}{2A_2} \]  

(17)

and

\[ \lambda_2 = \frac{1 - \sqrt{1 - 4A_1A_2}}{2A_1} \]  

(18)

In terms of output dynamics this implies that \( \omega_1 \) and \( \omega_2 \) are always positive, while \( \gamma_1 \) and \( \gamma_2 \) are positive (negative) if \( \phi < 1 \) (\( \phi > 1 \)). In the limiting case when \( \phi \to 0 \) the solution reduces to \( \gamma_1 = 1 \), \( \gamma_2 = 1 \), \( \omega_1 = 1 \) and \( \omega_2 = 1 \); in the limiting case when \( \phi \to \infty \) the solution reduces to \( \gamma_1 = -1 \), \( \gamma_2 = -1 \), \( \omega_1 = 0 \) and \( \omega_2 = 0 \).

Under the assumed restrictions on \( \alpha \) in the previous proposition and when prices are strategic complements, output is always more persistent, \( \gamma_1 \) and \( \gamma_2 \) are higher, compared to the case when prices are strategic substitutes. Moreover, when prices are strategic complements, output is always more persistent compared to when prices are perfectly synchronized. If instead prices are strategic substitutes, output is always less persistent compared to when prices are perfectly synchronized.

In the limiting case when \( \phi \to 0 \), an increase in the money supply has a permanent effect on output, whereas in the case \( \phi \to \infty \), an increase in the money supply is neutral and does not effect output. This, together with the previous propositions, implies that if \( \phi = 1 \) or any of the limiting cases described above, for any interior value of \( \alpha \), output dynamic does not depend on the degree of synchronization in price setting. These cases however are not very interesting from an economic perspective. Instead, one would plausibly assume that there is a finite degree of strategic complementarity in pricing, which yields the model the ability to produce temporary, but long-lived, effects on output, following a monetary injection. In this case price setters, by (16), care relatively
little about the money supply, but to greater extent about the current, and expectations about next period’s, price level. The insight that next period’s price setters follow the same rule, implies that if price setters today set a low price, meaning that the price level is low tomorrow, neither next period’s price setters find much incentive to increase their prices. As a result, the price level, and thereby output, adjust sluggishly.

How does the degree of synchronization in price setting affect the economy’s response to a shock to the money supply? In order to answer this question one needs a measure for comparing impulse responses for different values of $\alpha$. For this purpose it is convenient to work with the expected cumulative effect on output. As discussed in Carvalho (2006), this measure takes into account both the intensity and persistence of the response in output following a shock. In the Appendix I show that expected cumulative effect on output, $E_t \sum_{k=0}^{\infty} y_{t+k}$, following an unexpected shock of size $\varepsilon$ to money supply at time $t$, can be written as

$$C(\alpha, \phi) = \frac{[\omega_1 (1 + \gamma_2) + \omega_2 (1 + \gamma_1)] \varepsilon}{1 - \gamma_1 \gamma_2}.$$  

(19)

When the model is unable to produce persistence in output this measure takes on a particularly simple form.

**Proposition 6** If $\alpha = 0$, $\alpha = 1$ or $\phi = 1$ the expected cumulative effect on output is given by $\frac{\varepsilon}{2}$.

If prices are perfectly synchronized, a shock to money supply is, as described in Corollary 1, fully transmitted into output if the shock hits the economy in a period when prices are not adjusted. This causes an immediate increase in output of size $\varepsilon$; output however returns to steady state in the following period. If instead the shock hits the economy in a period when all prices are reoptimized there is no effect on output. Since $\varepsilon_t$ is a random walk and equally probable to hit the economy in
all periods, averaging over the response in output in odd and even periods yields the proposition above for \( \alpha = 0 \) and \( \alpha = 1 \). A similar argument can be applied to the case when \( \phi = 1 \).

**Proposition 7** If \( \alpha \in (0, 1) \) and \( \phi \neq 1 \) the expected cumulative effect on output is given by

\[
\left[ \frac{1}{1 - \phi} \Omega (\alpha, \phi) - 1 \right] \frac{\varepsilon}{2},
\]

where

\[
\Omega (\alpha, \phi) = \frac{1 + \sqrt{1 - 4A_1 A_2}}{\sqrt{1 - 4A_1 A_2}} \left[ 2 - (1 + \phi) \sqrt{1 - 4A_1 A_2} \right].
\]

The details of this derivation are in the Appendix. How the expected cumulative effect on output depends on \( \alpha \) crucially hinges on the properties of \( \Omega (\alpha, \phi) \), described in the following set of results.

**Proposition 8** If \( \alpha \in (0, 1) \) and \( \phi \neq 1 \):

\[
\arg \max_\alpha \Omega (\alpha, \phi) = \frac{1}{2},
\]

which follows by differentiating \( \Omega (\alpha, \phi) \) with respect to \( \alpha \), and setting the resulting derivative equal to zero.

The expected cumulative effect on output is thus the largest when uniform staggering, as in Taylor (1980), is assumed.

**Proposition 9** If \( \alpha \in (0, 1) \) and \( \phi \neq 1 \): \( \Omega (\alpha, \phi) \) is a strictly concave function with respect to \( \alpha \).

The proof is in the Appendix.
Corollary 10 For all permissible values of $\alpha$ and $\phi$:

$$C_{11}(\alpha, \phi) = \begin{cases} < 0 & \text{if } \phi < 1 \\ = 0 & \text{if } \phi = 1 \\ > 0 & \text{if } \phi > 1 \end{cases}$$

implying that $C$ is strictly concave (convex), with respect to $\alpha$, if prices are strategic complements (substitutes). If prices are neither strategic complements nor strategic substitutes, $C$ is both concave and convex (linear) with respect to $\alpha$.

It follows from Corollary 10 that $C(a, \phi)$, assuming $\phi \neq 1$, is the flattest around uniform staggering ($\alpha = \frac{1}{2}$) and the steepest when prices are perfectly synchronized. Thus, moving, say a distance $\tau$, from uniform staggering, so that $\alpha = \frac{1}{2} + \tau$, has a smaller effect on the expected cumulative effect on output, than moving the same distance from perfect synchronization. This indicates that assuming uniform staggering may be a reasonable approximation to the true price setting structure, even if there is some degree of synchronization among price setters. How accurate such an approximation is depends on both how far from uniform staggering the true price setting structure actually is and how steep $C(a, \phi)$ is around this point.

Figure 1 plots the mapping between $\alpha$ and $C(a, \phi)$ for different values of $\phi$; the expected cumulative effect on output has been normalized so that perfect synchronization corresponds to one. It turns out that quantitatively uniform staggering provides a close approximation to the true price setting structure for a wide range of values of $\alpha$. Chari, Kehoe, and McGrattan (2000) argue that in order to match the U.S. business cycle, when prices are uniformly staggered, one needs $\phi = 0.05$. Then, even for values of $\alpha$ as low as 0.2 the model is able to create a substantial expected cumulative effect on output. One can further assess this claim by mapping the associated
values on $\alpha$ into the corresponding FK-index. Figure 2 plots the relation between this index and the (normalized) expected cumulative effect on output. The right axis expresses the response as a fraction of that under uniform staggering. For instance, when the FK-index is 0.25, the expected cumulative effect on output is very close to uniform staggering, producing 97 percent of the staggered response. Increasing the amount of synchronization so that the FK-index is 0.5, the expected cumulative effect on output is still close to uniform staggering, 87 percent of the response remains. Even when the FK-index is 0.75, and synchronization dominates price setting, 67 percent of the response under uniform staggering remains. If prices are perfectly synchronized, on the other hand, the expected cumulative effect on output is only 22 percent of the that under uniform
Figure 2: The expected cumulative effect on output for different values of the FK-index when \( \phi = 0.05 \).

To understand the reason for this highly non-linear relation between \( \alpha \) and \( C(\alpha, \phi) \), consider how prices are set in the economy. The price setting condition in (16) says that price setters should not only care about the price level today, but also about the price level tomorrow. This, in turn, depends on the optimal price tomorrow, which depends on the price level in the subsequent period, three periods ahead from now, and so on. The price level is

\[
p_t = (1 - \alpha)p_t^* + \alpha p_{t-1}^*
\]  

(23)
in odd periods and

\[ p_t = \alpha p^*_{t} + (1 - \alpha) p^*_{t-1} \]  \hspace{1cm} (24)

in even periods. Combining with (16) one can write the optimal reset price as

\[ p^*_t = (1 - 2A_1) m_t + A_1 \left( p^*_{t-1} + E_t p^*_{t+1} \right) . \]  \hspace{1cm} (25)

Assume that \( t \) is odd, forwarding the price setting condition above, substituting back and rearranging, this yields

\[ p^*_t = \frac{[1 - 2A_1 + A_1 (1 - 2A_2)]}{1 - A_1 A_2} m_t + \frac{A_1}{1 - A_1 A_2} p^*_t - 1 + \frac{A_1 A_2}{1 - A_1 A_2} E_t p^*_t + 2. \]  \hspace{1cm} (26)

Using the definitions of \( A_1 \) and \( A_2 \) one can write

\[ \frac{A_1 A_2}{1 - A_1 A_2} = \frac{F(a, \phi)}{4\phi + 3F(a, \phi)} \]  \hspace{1cm} (27)

where \( F(a, \phi) = \alpha (1 - \alpha) (1 - \phi)^2 \). For a given \( \phi \), the coefficient on \( E_t p^*_t + 2 \) depends primarily on \( \alpha (1 - \alpha) \). As a consequence, the influence by future optimal prices on this period’s optimal price is the largest when \( \alpha = \frac{1}{2} \).

The original question why the expected cumulative effect on output is insensitive to deviations from uniform staggering can thus be traced back to the degree of "forwardlookingness" in price setting. As discussed previously, under perfect synchronization the optimal price setting condition reduces to a purely static problem, while there is a considerable degree of forwardlookingness under uniform staggering. The concavity of \( F(a, \phi) \) with respect to \( \alpha \), however, implies that small deviations from uniform staggering have small effects on the price setting behavior, so that today’s
optimal price still is heavily influenced by future optimal prices.

3 The full model

This section describes the behavior of households and firms in the full model.

3.1 Households

The economy consists of a large number of sectors. The representative household derives utility from
the consumption of composite goods, according to the following generalized (Kimball) aggregator:

\[ 1 = \int \frac{1}{0} F \left( \frac{C_{it}}{C_t} \right) d_i, \]  

(28)

where \( C_{it} \) denotes consumption of the composite good in sector \( i \in [0,1] \) and \( C_t \) is aggregate
demand. Each composite good consists of a large number of brands, produced in each sector, from
which the household derives utility according to the aggregator:

\[ 1 = \int \frac{1}{0} G \left( \frac{C_{ijt}}{C_t} \right) d_j, \]  

(29)

where \( C_{ijt} \) denotes consumption of brand \( j \in [0,1] \) in sector \( i \). The household allocates consumption
expenditure between the different brands in different sectors to

\[ \min_{C_{ijt}} \int \frac{1}{0} \int \frac{1}{0} P_{ijt} C_{ijt} d_j d_i, \]  

(30)

where \( P_{ijt} \) denotes the price of brand \( j \) in sector \( i \), subject to (28) and (29) for a given \( C_t \). The
decision can be seen as sequential: in the first step, the household allocates consumption across
sectors and in the second step it allocates across brands within each sector. Solving this problem backwards yields that demand for brand $j$ in sector $i$ is given by

$$C_{ijt} = G^{t-1} \left( \frac{P_{ijt}}{P_{it}} D_{it} \right) C_{it}, \quad (31)$$

where

$$P_{it} = \frac{1}{C_{it}} \int_0^1 P_{ijt} C_{ijt} dj \quad (32)$$

is the price level in sector $i$, and

$$D_{it} = \int_0^1 G' \left( \frac{C_{ijt}}{C_{it}} \right) \frac{C_{ijt}}{C_{it}} dj. \quad (33)$$

Taking (31) as given, it follows that demand for the sector $i$ composite good is given by

$$C_{it} = F^{t-1} \left( \frac{P_{it}}{P_t} D_t \right) C_t, \quad (34)$$

where

$$P_t = \frac{1}{C_t} \int_0^1 P_{it} C_{it} di, \quad (35)$$

is the aggregate price level, and

$$D_t = \int_0^1 F' \left( \frac{C_{it}}{C_t} \right) \frac{C_{it}}{C_t} di. \quad (36)$$

Demand for a specific brand thus depends on the brand’s relative price in the sector, but also on the sector’s relative price level and aggregate demand.
The representative household solves the intertemporal problem:

\[
\max \sum_{k=0}^{\infty} \beta^k \left\{ \frac{1}{1-\sigma_C} C_t^{1-\sigma_C} - \frac{1}{1+\sigma_N} N_{t+k}^{1+\sigma_N} \right\},
\]

where \( N_t = \int_0^1 \int_0^1 N_{ijt} dji \) is the number of working hours supplied, subject to the budget constraint

\[
B_t + P_tC_t = R_{t-1}B_{t-1} + W_t N_t + \Phi_t,
\]

where \( B_t \) denotes bond holdings, \( R_t \) is the gross interest rate, \( W_t \) the nominal wage, and \( \Phi_t \) dividends from firm ownership. The associated first order conditions for consumption and bond holdings reduce to the Euler equation

\[
C_t^{1-\sigma_C} = \beta E_t \frac{R_t}{\Pi_{t+1}} C_{t+1}^{1-\sigma_C},
\]

where \( \Pi_t = \frac{P_t}{P_{t-1}} \) is the time \( t \) inflation rate. In addition, the first order condition for labor supply is

\[
\frac{N_t^{\sigma_N}}{C_t^{1-\sigma_C}} = \frac{W_t}{P_t}.
\]

3.2 Firms

Brand \( j \) in sector \( i \) is produced by a monopolist with technology

\[
Y_{ijt} = N_{ijt}^\gamma,
\]
where \( N_{ijt} \) denotes the firm’s input of labor, rented at the perfectly competitive economy wide labor market and capital is assumed to be fixed at the firm level\(^3\). All firms are owned by the households, and profits are paid out in the form of dividends at the end of each period.

Each sector is divided into \( N \) ex ante identical cohorts of size \( \alpha_1 \alpha_2 \ldots \alpha_N \), where \( \alpha_i \in [0,1] \forall i \) and \( \sum_{i=1}^{N} \alpha_i = 1 \). The cohorts are arranged within each sector according to their sectorial indices, so that firms indexed \( j \in [0, \alpha_1] \) represent the first cohort in sector \( i \), firms indexed \( j \in [\alpha_i, \alpha_i + \alpha_{i+1}] \) the second, and so on; Figure 3 illustrates this graphically for the case when \( N = 4 \). Prices are fixed for \( N \) periods and set in a staggered fashion. In particular, the firms in the first cohort in each sector set their prices at time \( t = 0, N, 2N, \ldots \), the second cohort at \( t = 1, N + 1, 2N + 1, \ldots \), and so on. This assumed behavior on price setting has two important implications: \( i \)) prices are uniformly staggered at the aggregate level because a constant fraction of all prices are reset in each period. \( ii \)) given that not all cohorts are of equal size, there is some degree of synchronization at the level of the individual sector.

Given these restrictions, a firm’s objective is to maximize the profit stream:

\[
E_t^{i} \sum_{k=0}^{N-1} \beta^k \Delta_{i,t+k} \left[ P_{ijt+k} Y_{ijt+k} - W_{t+k} N_{ijt+k} \right],
\]

Figure 3: The size of cohorts within each sector when \( N=4 \).

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\(^3\)This corresponds to the limiting case when adjustment costs approach infinity in the model of Eichenbaum and Fisher (2007). As discussed in Woodford (2005), this simplifying assumption is of little consequence for price setting dynamics.
where $\Delta_{t,t+k}$ is the firm’s subjective discount factor, subject to the demand and technological constraints in (31) and (40). This implies the price setting rule

$$
E_i^t \sum_{k=0}^{N-1} \beta^k \Delta_{t,t+k} Y_{ijt+k} [(1 - \eta(X_{ijt+k})) P_{ijt+k} - \eta(X_{ijt+k}) S_{ijt+k}] = 0, \quad (42)
$$

where $S_{ijt} = \frac{1}{2} W_{it} X_{ijt}^{1-\gamma}$ is the marginal cost and $\eta(X_{ijt}) = \frac{-G''(X_{ijt})}{X_{ijt} G'(X_{ijt})}$ is the elasticity of demand, at $X_{ijt} = \frac{Y_{ijt}}{Y_{it}}$, between brands in the sector.

### 3.3 Price setting

To understand how the Kimball aggregator and firm-specific capital affect price setting, it is instructive to log-linearize the price setting condition above, which yields

$$
E_i^t \sum_{k=0}^{N-1} \beta^k (p_{ijt} - p_{it+k}) = E_i^t \sum_{k=0}^{N-1} \beta^k [\tilde{s}_{ijt+k} + \epsilon_\mu (y_{ijt+k} - y_{it+k})], \quad (43)
$$

where $\epsilon_\mu$ is the elasticity of the desired markup, with respect to relative demand, and $\tilde{s}_{it}$ is the log-linear approximation to $\frac{\tilde{s}_{it}}{P_{it}}$. In the Dixit-Stiglitz model the markup is constant and $\epsilon_\mu = 0$; when an aggregator on the form in (29) is used instead, $\epsilon_\mu$ can be larger than zero. Then, an increase in a firm’s relative demand increases the firm’s desired markup, by lowering the demand elasticity. Further, log-linearizing (31) yields

$$
y_{ijt} - y_{it} = -\eta (p_{ijt} - p_{it}) , \quad (44)
$$
where $\eta$ is the steady state demand elasticity between brands in the same sector. Substituting this into (43), and rearranging, one obtains

$$E_t \sum_{k=0}^{N-1} \beta^k (p_{ijt} - p_{it+k}) = AE_t \sum_{k=0}^{N-1} \beta^k s_{ijt+k},$$

where

$$A = \frac{1}{1 + \eta \epsilon}. \tag{46}$$

When $A < 1$, a firm contemplating to raise its price, following an increase in its marginal cost, plausibly realizes that this also lowers relative demand and thus reduces the desired markup. This alleviates the desired price increase and reduces the pass-through from marginal cost into prices.

When capital is firm-specific, a firm’s marginal cost is an increasing function of the firm’s own output. Log-linearizing one finds that

$$s_{ijt} = \omega_p y_{ijt} + w_t$$

$$= \omega_p (y_{ijt} - y_{it}) + \omega_p (y_{it} - y_t) + s_t, \tag{47}$$

where $\omega_p = \frac{1-\gamma}{\eta}$, $s_{it} = \tilde{s}_{it} + (p_{it} - p_t)$, and $s_t = [\sigma_c - 1 + (\sigma N + 1) (\omega_p + 1)] y_t$ is aggregate real marginal cost. One can apply a similar argument, as in the case with the Kimball aggregator, to see why this reduces the required price increase, following an increase in marginal cost. A firm contemplating to raise its price, plausibly realizes that this also lowers demand, thus reducing marginal cost, and again alleviating the required price increase.

Log-linearizing (34) yields

$$y_{it} - y_t = -\theta (p_{it} - p_t), \tag{48}$$
where $\theta$ is the steady state demand elasticity between composite goods in different sectors. Combining this with (47) and substituting into (43), one obtains

$$E_t \sum_{k=0}^{N-1} \beta^k (p_{ijt} - p_{it+k}) = ADE_t \sum_{k=0}^{N-1} \beta^k s_{t+k} - \frac{AD}{C} E_t \sum_{k=0}^{N-1} \beta^k (p_{it+k} - p_{t+k}),$$

(49)

where

$$C = \frac{1}{1 + \theta \omega_p},$$

(50)

and

$$D = \frac{1}{1 + \eta \omega_p A},$$

(51)

4 Aggregate dynamics

The model is closed by assuming that monetary policy is implemented by the interest rate rule:

$$r_t = (1 - \rho_r) (\phi_y \pi_t + \phi_y y_t) + \rho_r r_{t-1} + \nu_t,$$

(52)

where

$$\nu_t = \rho_r \nu_{t-1} + \epsilon_t^r,$$

(53)

and $\epsilon_t^r$ is a white noise process with finite variance.

In calibrating the model, one period is assumed to correspond to a quarter; Table 1 summarizes the calibration. I set $\eta$ to 11 and $\theta$ to 1/2 in order to match the high price elasticity between brands and the low price elasticity between product categories, as estimated by e.g. Hausman, Leonard, and Zona (1994) and, more recently, Levin and Yun (2008); this implies a 10 percent steady state
markup. The value on $\epsilon_\mu$ implies that $A = 0.2326$. The value on $\gamma$ implies that $\omega_\mu = 1/2$ and that $C = 4/5$ and $D = 0.4388$. Following Carrillo, Fève, and Matheron (2007) the degree of interest rate smoothing is assumed to be relatively low, while shocks to the interest rate rule are highly persistent. In addition, the calibration of $\sigma_N$ are $\sigma_C$ are consistent with log utility in consumption and leisure.

What remains is to set $\alpha_1$, $\alpha_2$, $\alpha_3$ and $\alpha_4$. Unlike the simple two-period model, the mapping between the (sector level) FK-index and the size of the cohorts is, in general, not unique. One way to overcome this problem is to assume that the size of the cohorts are generated according to the function:

$$\alpha_i = \frac{(1 - x)}{(1 - x^4)} x^i. \quad (54)$$

Substitution of this into the definition of the FK-index, yields that the mapping between the FK-index and $x$ is given by

$$x^4 + cx^3 + cx^2 + cx + 1 = 0, \quad (55)$$

where

$$c = \frac{2}{3} \left(\frac{3FK^2 + 1}{FK^2 - 1}\right). \quad (56)$$
This quasi-symmetric quartic equation has two real roots and both have the same implications for \( \alpha_1, \alpha_2, \alpha_3 \) and \( \alpha_4 \).

To explore how the degree of synchronization, at the sector level, affects aggregate output in this model, I again calculate the expected cumulative effect on output for various values on the FK-index; this is plotted in Figure 4. The expected cumulative effect on output has been normalized so that one corresponds to the case without real rigidities from the Kimball aggregator and firm-specific capital, i.e. when assuming the Dixit-Stiglitz aggregator \( (A = 1) \) and a rental market for capital \( (C = D = 1) \). The right axis expresses the response as a fraction of that under uniform staggering.

When prices are uniformly staggered at the sector level, \( \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 1/4 \), the expected cumulative effect on output is more than 3.2 times larger than in the economy without real rigidities. Price setting within sectors are staggered and demand is being reallocated between firms in the same sector. However, because the proportion of reoptimizers is the same in all sectors, all firms, regardless of which sector they belong to, set the same price, whenever they are able to reoptimize. As a consequence, \( p_{it} = p_t \) and there is no reallocation of demand between sectors. In this case, (49) is given by

\[
E_i \sum_{k=0}^{N-1} \beta^k (p_{ijt} - p_{t+k}) = ADE_i \sum_{k=0}^{N-1} \beta^k s_{t+k}.
\]

Instead assuming that prices are perfectly synchronized, \( \alpha_1 = 1 \) and \( \alpha_2 = \alpha_3 = \alpha_4 = 0 \), the expected cumulative effect on output is only 35 percent of the response under uniform staggering. All firms in a sector change their prices at the same time and there is no reallocation of demand between sectors.

---

4Using the change of variable \( z = x + x^{-1} \), (55) can be written as the quadratic equation \( z^2 + cz + (c - 2) = 0 \). It follows that \( x = \frac{z \pm \sqrt{z^2 - 4}}{2} \) where \( z = -c \pm \sqrt{c^2 - 4(c - 2)} \). For each root of \( z \), the first root of \( x \) is the multiplicative inverse of the second root.
Figure 4: The expected cumulative effect on output, following a shock to the interest rate rule.

between firms in the same sector. However, because price setting is coordinated within sectors, demand will be reallocated between sectors. Since $p_{ijt} = p_{it}$ in this case, the price setting condition in (49) reduces to

$$E_t \sum_{k=0}^{N-1} \beta^k (p_{ijt} - p_{t+k}) = CE_t \sum_{k=0}^{N-1} \beta^k s_{t+k}. \quad (58)$$

There is still less than complete pass-through from marginal cost into prices. However, because the demand elasticity between sectors, $\theta$, is very low, implying that $C$ is close to one, this effect is small and, as a consequence, the expected cumulative effect on output is not much higher than without real rigidities. For the given calibration, $AD$ is much smaller than $C$, so that the pass-through of marginal cost into prices is much lower when prices are uniformly staggered and the expected
cumulative effect on output is much larger.

Because there are more real rigidities at the sector level than at the aggregate level, the economy where output is being reallocated within sectors produces a more persistent response in output, following an interest rate shock, than a model where output is being reallocated between sectors. But we are primarily interested in the empirically more realistic case when prices at the sector level are neither uniformly staggered, nor perfectly synchronized. Starting from a situation of non-uniform staggering and increasing the amount of staggering, increases the reallocation of demand between firms in the same sector, but decreases the reallocation of demand between different sectors. The former, however, more than compensates the latter. As indicated by Figure 4, the relationship between the degree of synchronization in price setting and the expected cumulative effect on output is, as in the two-period model, non-linear, so that starting from a situation with perfect staggering and introducing a small amount of synchronization, has a smaller effect on the expected cumulative effect on output, than performing the opposite experiment. For instance, 96 percent of the uniformly staggered response remains when the FK-index is 0.25, 85 percent when the FK-index is 0.5, and 65 percent when the FK-index is 0.75. Hence, also in this model, uniform staggering provides a reasonably close approximation to the true price setting structure, for empirically plausible values of the FK-index.

5 Conclusion

Staggered price setting is crucial in creating persistent effects on output following monetary policy shock. In the literature it is routinely assumed that a constant fraction of firms reoptimize their prices each period. In the present paper I relax this assumption and allow for various degrees of synchronization in price setting. The main result is that deviations from uniform staggering
have modest effects on the expected cumulative effect on output. This suggests that the standard
uniformly staggered pricing model is a reasonably accurate approximation to a more complicated
model, where prices are neither uniformly staggered nor perfectly synchronized.

It also suggests that non-adjusters have a disproportionately large influence on the behavior
of the aggregate economy, so that, when prices are strategic complements, the failure of even a
small group of price-setters to reoptimize is sufficient to create persistent output effects. This result
places the paper on about an equal footing with e.g. Haltiwanger and Waldman (1989) and Fehr and
Tyran (2008), who find that when agents vary in their ability to form expectations, rational agents
mimic the behavior of those less sophisticated, the latter, as a result, having a disproportionately
large effect on the aggregate outcome. Similarly, Dixon and Kara (2007) and Carvalho (2006) find
that in economies with the same average duration of price or wage contracts, a monetary policy
shocks yields a larger and more persistent response, when some contracts of longer durations are
present.
References


DIXON, H., AND E. KARA (2007): “Persistence and Nominal Inertia in a Generalized Taylor Econ-


