

# An Incomplete Markets Explanation to the UIP puzzle \*

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November 30, 2010

## Abstract

A large literature has related the failure of interest rate parity in the foreign exchange market to the existence of a time-varying risk premium. Nevertheless, most modern open economy DSGE models imply a (near) perfect interest rate parity condition. This paper presents a stylized two-country incomplete-markets model in which countries have strong precautionary motives because they face international liquidity constraints, the presence of which successfully generates a time-varying risk premium: the country that has accumulated debt after experiencing relative worse times has stronger precautionary motives and its asset carries a risk premium.

*Keywords:* Uncovered Interest Rate Parity, Incomplete Markets  
Precautionary Savings, Time-Varying Risk Premium

*JEL-Codes:* F31, F41, G12, G15

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\*I thank Refet Gürkaynak and participants at the 9th MNB-CEPR Macroeconomic Policy Research Workshop for useful comments. The usual disclaimer applies.

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# 1 Introduction

Large deviations from uncovered interest parity (UIP) are a strong regularity in the data. This paper addresses this puzzling empirical fact on international relative prices in a simple two-country model with incomplete international financial markets. In particular, financial markets are incomplete in the sense that countries face borrowing constraints on an internationally traded bond, the presence of which generates a time-varying risk-premium in the foreign exchange market.

The paper connects to a long literature, both empirical and theoretical. Uncovered interest rate parity, which states that the (nominal) interest rate differential should be equated to the expected change in the exchange rate, is a central feature of virtually all general equilibrium open economy models. While it would appear logical that investors would demand higher interest rates on currencies that are expected to fall in value, empirical evidence for advanced economies suggests that currency prices for high interest rates tend, instead, to appreciate. This departure from uncovered interest rate parity, also known as the forward premium anomaly, has been extensively documented. Following a seminal paper of Fama (1984), one strand of the literature argues that the failure of this interest parity relationship can be attributed to a time-varying risk premium on foreign exchange<sup>1</sup>.

Empirically, the violation of the uncovered interest rate parity condition is usually tested by a simple regression of (actual) exchange rate variations on the nominal interest rate differential. Under the assumption of rational expectations and risk neutral agents, the forward exchange rate is an unbiased estimator of the future spot exchange rate and therefore, since covered interest rate parity holds, the UIP regression should theoretically deliver a regression coefficient of 1. This is severely violated in the data, where, for advanced economies, the UIP coefficient is generally found to be much lower than 1, typically even negative.

This paper poses the question under which conditions a risk-premium can be generated that disturbs the otherwise tight link between the interest rate differential and subsequent

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<sup>1</sup>Other explanations include the 'peso problem' (i.e. that agents need to learn about structural changes of the economy over time and that during this transitional learning period, market participants make systematic prediction errors) or 'noise trading' (i.e. that agents are actually irrational because they believe the value of an asset depends on information else than economic fundamentals).

exchange rates. In the two-country model of the present paper, a time-varying risk premium arises from the presence of frictions in international financial markets. Countries are assumed to be able to trade an international bond, but are subject to borrowing constraints, which state that a country cannot borrow more than 100% of its output. Both countries have preferences over consumption of domestic and foreign goods, and are specialized in the production of one type of good (which, for simplicity, is given as an endowment each period).

Typically, when there are no frictions in international borrowing and lending, a country that experiences a shock that lowers the value of its income out of which it needs to consume from, it would like to access international financial markets in order to achieve a smooth consumption path. On the other hand, when borrowing constraints are present, there is a possibility that these constraints become binding at any point in time. As a result that country might not be able to make use of the international financial market for its consumption smoothing purposes, and instead will want to save for bad times. Consequently, close to the constraints, when precautionary motives become large, consumption risk is increasingly less shared across countries. The strength of this precautionary motive on interest rates varies over the cycle. While the effect is strongest close to the constraint, it also has a non-negligible effect when bond holdings are far from (either country's) constraint, introducing a wedge between the interest rate differential and the expected change in exchange rates. The precautionary effect can thus be interpreted as a time-varying risk premium on foreign exchange. A regression of exchange rate variations on the interest rate differential from simulated model data shows that the proposed mechanism can successfully lower the implied UIP regression coefficient to 0.68.

Finally, it should be noted that the mechanism that generates the time-varying risk premium is not connected to nominal risks in the economy. In fact, the model presented in section 2 is an entirely real model. It can be shown, however, that the exchange rate premium can be decomposed into a term relating to real risk, a term relating to nominal (inflation) risks, and the interaction of the two. Hollifield and Yaron (2001) perform such a decomposition and argue that empirically the inflation risk and the interaction terms seem to be of minor importance, and that models of exchange rate risk premia should focus on real risk, as opposed to nominal risk. Iwata and Tanamee (2009) run both standard nominal UIP regressions and

equivalent 'real' UIP regressions of the real exchange rate change on the real interest rate differential. They find that the resulting UIP slope coefficient from the regression in real terms is no longer negative, but still very different from 1, confirming large deviations from interest rate parity.<sup>2</sup> Recent theoretical contributions that propose models with a real risk explanation are Verdelhan (2010) and Martin (2010).

The literature dedicated to the uncovered interest rate parity puzzle is enormous and cannot possibly be covered here. A number of surveys, by Froot and Thaler (1990), Lewis (1995), and Engel (1996) provide a good overview of the literature until then. Engel (1999) examines the properties of the foreign exchange risk premium in sticky price general equilibrium models, finding that while such models are capable of producing large enough risk premia, the implied premia are typically constant<sup>3</sup>. More recently, Alvarez, Atkeson and Kehoe (2009) stress the importance of time-varying risk premia resulting from endogenous market segmentation, Bacchetta and van Wincoop (2010) find costs of actively managing foreign exchange portfolios responsible for the failure of UIP, and Burnside, Eichenbaum and Rebelo (2009) suggest a microstructure approach in addressing the puzzle.

The rest of the paper is structured as follows. Section 2 lays out the model structure, section 3 discusses parameterization and briefly comments on the solution technique. Section 4 presents the results. The nonlinear model with borrowing constraints is contrasted to a number of interesting comparison cases: complete international financial markets, the standard incomplete-markets bond-economy without borrowing constraints, and the case of financial autarky. The comparison is presented in terms of a sample simulated time path, results from a UIP regression and business cycle stylized facts from a simulation, and some selected policy functions of interest. Finally, section 5 concludes.

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<sup>2</sup>In particular, based on quarterly data from a sample of 43 countries, the authors find slope coefficients of the standard nominal UIP regression to be -0.57 for developed and 0.32 for developing countries, while they find slope coefficients of 0.22 and 0.13, respectively, for the case of the real regressions.

<sup>3</sup>As long as the exogenous forcing process has constant variance.

## 2 The Model

### 2.1 Model Setup

The world economy consists of two countries, Home and Foreign, each of which specializes in production of one type of (traded) good. All idiosyncratic risk is assumed to be perfectly insured among residents of a country, i.e. within-country financial markets are complete. We can therefore think of a representative consumer in each country that maximizes the expected sum of future discounted utilities from consumption,  $C_t$ :

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t), \quad (1)$$

where  $\beta$  is the discount factor. The utility function  $u(C_t)$  is taken to be of the constant relative risk aversion form,  $u(C_t) = (1/(1-\sigma)) [C_t^{1-\sigma} - 1]$ , where  $\sigma$  is the coefficient of relative risk aversion. Aggregate consumption is a constant elasticity of substitution (CES) basket over domestic good and foreign good consumption:

$$C_t = \left[ \gamma^{\frac{1}{\omega}} C_{H,t}^{\frac{\omega-1}{\omega}} + (1-\gamma)^{\frac{1}{\omega}} C_{F,t}^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}, \quad (2)$$

where  $C_{H,t}$  and  $C_{F,t}$  are the home country's consumption of home and foreign goods. Parameter  $\gamma$  is the degree of home bias in consumption, parameter  $\omega$  is the intratemporal elasticity of substitution between domestic and foreign consumption goods, which in this model corresponds to the trade elasticity. The foreign representative agent faces an equivalent problem, where foreign variables are denoted with an asterisk. Agents of each country receive an exogenous country-specific (and therefore good-specific) endowment  $Y_t$  or  $Y_t^*$  respectively in every period  $t$ . I abstract from modeling a production side, and assume instead that outputs arrive exogenously each period, following a bivariate autoregressive process of order 1:

$$\begin{pmatrix} y_t \\ y_t^* \end{pmatrix} = \begin{pmatrix} \rho & \psi \\ \psi & \rho \end{pmatrix} \begin{pmatrix} y_{t-1} \\ y_{t-1}^* \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \varepsilon_t^* \end{pmatrix}, \quad (3)$$

where  $\rho$  and  $\psi$  are coefficients describing the autocorrelation and spillover properties of the process, and  $\varepsilon_t$  and  $\varepsilon_t^*$  are normally distributed mean-zero shocks with variance  $\sigma_\varepsilon$  and correlation  $\rho_\varepsilon$ . Throughout the remainder of the paper I denote with lowercase variables the

logarithm of original variables, that is,  $y_t = \log(Y_t)$ . Mean income,  $\bar{Y}$  and  $\bar{Y}^*$ , is normalized to one.

Asset markets are incomplete in the sense that countries are only allowed to trade in one-period risk-free bonds,  $B_{H,t}$  and  $B_{F,t}$ , which, respectively, promise one unit of domestic or foreign consumption good in the next period and trade at price  $\frac{1}{R_t}$  and  $\frac{1}{R_t^*}$ , where  $R_t$  and  $R_t^*$  are the gross domestic and foreign real interest rate. Furthermore, define the real exchange rate,  $Q_t$ , as the relative price of the foreign final consumption good to the domestic final consumption good, and  $p_{H,t}$  ( $p_{F,t}$ ) are the relative price of the domestic (foreign) consumption good to the price of final consumption. The terms of trade are defined as  $\tau_t = p_{F,t}/p_{H,t}$ . We can then write the home country's budget constraint as:

$$\frac{B_{H,t+1}}{R_t} + \frac{Q_t B_{F,t+1}}{R_t^*} = B_{H,t} + Q_t B_{F,t} + p_{H,t} Y_t - C_t. \quad (4)$$

Even though agents are assumed to be able to trade risk-free bonds in order to smooth their consumption, they cannot do so unrestrictedly. In particular, I assume that the home country's debt level cannot exceed some fraction  $K$  of the level of its steady state output (which is normalized to 1):<sup>4</sup>

$$B_{H,t+1} + Q_t B_{F,t+1} \geq -K \quad (5)$$

The foreign country's budget constraint and the borrowing constraint are equivalent versions of equations (4) and (5). The borrowing limit for the foreign country is therefore given by  $B_{H,t+1}^*/Q_t + B_{F,t+1}^* \geq -K^*$ .

The domestic household's intratemporal optimization problem of choosing optimal consumptions of the domestic and foreign good give the following intratemporal optimality conditions:

$$C_{H,t} = \gamma p_{H,t}^{-\omega} C_t \quad (6)$$

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<sup>4</sup>In principle, there is also a 'natural debt' limit as in Aiyagari (1994) according to which both countries will not borrow more than the minimum value that the endowment can take at period  $t+1$  discounted to period  $t$  prices. To compute the natural debt limit in a two country model, where the interest rate is endogenous, is more difficult than in a partial equilibrium model where the interest rate is exogenous. In addition if one of the constraint binds for one of the economies the interest rate generally differs for each agent (for a detailed discussion see Anagnostopoulos (2006)). However, the debt limits we impose here are generally stricter than the natural debt limit.

$$C_{F,t} = (1 - \gamma) p_{F,t}^{-\omega} C_t \quad (7)$$

Denote with  $\lambda_t$  the Lagrange multiplier on the domestic household's budget constraint, and with  $\mu_t$  the multiplier on the borrowing constraint. The representative household's first order conditions to the intertemporal optimization problem can then be stated as:

$$C_t^{-\sigma} = \lambda_t \quad (8)$$

$$\lambda_t - \mu_t R_t = \beta R_t E_t [\lambda_{t+1}] \quad (9)$$

$$\lambda_t Q_t - \mu_t R_t^* Q_t = \beta R_t^* E_t [\lambda_{t+1} Q_{t+1}] \quad (10)$$

$$\frac{B_{H,t+1}}{R_t} + \frac{Q_t B_{F,t+1}}{R_t^*} = B_{H,t} + Q_t B_{F,t} + p_{H,t} Y_t - C_t \quad (11)$$

$$0 = \mu_t [B_{H,t+1} + Q_t B_{F,t+1} + K] \quad (12)$$

Equation (8) relates Lagrange multiplier  $\lambda_t$  to the marginal utility of consumption. Equation (9) is the Home country's Euler equation w.r.t. the bond paying in domestic goods, which is obtained from the combination of the first order conditions for  $C_t$  and  $B_{H,t+1}$  and states that the marginal benefit from using debt to increase consumption at time  $t$  must be greater than or equal to the expected marginal loss at time  $t + 1$  arising from the additional debt. Equation (10) is the corresponding Euler equation w.r.t. the bond that pays in units of foreign goods,  $B_{F,t+1}$ . Equation (11) is the Home country's budget constraint, stating that current consumption and outstanding debt have to be financed either from current output or by issuing new debt. Equation (12) is the complementary slackness condition with  $\mu_t$  being the multiplier on inequality constraint (5). The foreign country faces an equivalent problem, which results in a set of optimality conditions similar to equations (6) to (12).

In the following, in order to avoid having to solve a portfolio problem, I assume that only one of the bonds are internationally traded. In particular, the domestic bond  $B_{H,t}$  can be internationally traded while the foreign currency bond  $B_{F,t}$  is only held within the foreign country (and as a result of the representative agent assumption, is therefore not traded,  $B_{F,t}^* = 0$ ).

The equilibrium of this economy is defined as a path of prices  $\{R_t, R_t^*, Q_t\}_{t=0}^{\infty}$  together

with consumption plans  $\{C_{H,t}, C_{F,t}, C_t\}_{t=0}^{\infty}$  and  $\{C_{H,t}^*, C_{F,t}^*, C_t^*\}_{t=0}^{\infty}$  and debt plans  $\{B_{H,t}\}_{t=0}^{\infty}$  and  $\{B_{H,t}^*\}_{t=0}^{\infty}$  such that:

1.  $C_{H,t}$  and  $C_{F,t}$  minimize the expenditure needed to buy one unit on domestic consumption good,  $C_t$ , given by (2). Similarly,  $C_{H,t}^*$  and  $C_{F,t}^*$  minimize the expenditure needed to buy one unit on foreign consumption good,  $C_t^*$ .
2.  $C_t$  and  $B_{H,t+1}$  maximize (1) subject to (4)-(5), for all  $t$ , and  $B_{H,0}$  given,
3.  $C_t^*$ ,  $B_{H,t+1}^*$  and  $B_{F,t+1}^*$  maximize the foreign version of (1) s.t. the foreign versions of (4)-(5), for all  $t$ , and  $B_{H,0}^*$  and  $B_{F,0}^*$  given,
4. the real interest rates clears the bond markets,  $B_{H,t} + B_{H,t}^* = 0$ ,  $B_{F,t}^* = 0$ , for all  $t$ ,
5. the goods markets also clear, that is  $C_{H,t} + C_{H,t}^* = Y_t$ ,  $C_{F,t} + C_{F,t}^* = Y_t^*$ , and  $\frac{B_{H,t+1}}{R_t} = B_{H,t} + p_{H,t}Y_t - C_t$  for all  $t$ .

The complete list of equilibrium conditions is summarized in Appendix A.

## 2.2 Comparison Model Economies

For easier interpretation of the workings of the incomplete market model with borrowing constraints, and to put the results into perspective, it is of interest to compare the baseline model of section 2.1 to a number of benchmark cases studied previously in the literature (see, e.g. Baxter and Crucini (1995), Heathcote and Perri (2002)), which are briefly summarized in this section.

The first of these benchmark cases is one of complete international financial markets (CM). In this model economy there exists a complete set of state-contingent assets for each possible state of world tomorrow. It is well known, that this financial market structure leads to full international risk sharing, that is, to an equalization of the the ratio of marginal utilities across countries to the real exchange rate:

$$Q_t = \frac{C_t^{*-\sigma}}{C_t^{-\sigma}}. \quad (13)$$

The second benchmark case is the exact opposite of the above, the case in which no assets can be traded internationally and the two economies are in financial autarky (FA). As a result,

there is no room for intertemporal trade and the current consumption expenditure has to be fully financed from current income in both economies:

$$p_{H,t}Y_t = C_t. \tag{14}$$

Finally, a third interesting reference case is the incomplete-markets bond-economy without the presence of additionally imposed ad hoc borrowing constraints (IM-unconstrained). In order to avoid a well-known non-stationarity problem I assume that there is a (small) convex cost on portfolio adjustments as in Schmitt-Grohé and Uribe (2003). The resulting Euler equation under this financial market assumption is given by:

$$1 - \phi_B \frac{B_{H,t+1}}{Q_t} = \beta R_t E_t \left[ \frac{C_{t+1}^{*-\sigma}}{C_t^{*-\sigma}} \frac{Q_t}{Q_{t+1}} \right]$$

### 3 Parameterization and Solution Technique

In this section, I outline the baseline parameterization of the model, which is also summarized in Table 1. Both countries are symmetric in all structural parameters. The discount factor,  $\beta$  is set to 0.99, corresponding to an annualized interest rate of about 4%. The degree of home bias in consumption, that is the weight of domestic goods in the consumption basket,  $\gamma$ , is set to 0.75, a value that implies a an average degree of openness.

Table 1: PARAMETERIZATION

Discount factor	$\beta$	0.99
Coeff. of relative risk aversion	$\sigma$	1
Degree of home bias	$\gamma$	0.75
Trade elasticity	$\omega$	0.44
Autocorrelation of AR(1)	$\rho$	0.95
Standard Deviation of AR(1)	$\sigma_\epsilon$	0.01

In line with most of the international business cycle literature, I assume a rather persistent exogenous forcing process, taking the coefficient of autocorrelation of the endowment process,  $\rho$ , to be 0.95 and a standard deviation of  $\sigma_\epsilon = 0.01$ . I abstract from spillovers or cross-country correlation of the shocks. The trade elasticity  $\omega$ , that is, the elasticity of substitution

between domestic and foreign good, is a crucial parameter in the class of open economy macro models, and its value varies largely in the literature. While estimates from the trade literature suggest values of around 6 or higher, the macro literature suggests substantially lower values, especially when stylized facts on international relative prices are matched (see, e.g. Heathcote and Perri (2002), Corsetti, Dedola and Leduc (2008), Enders and Mueller (2009), Thoenissen (2010)). Movements in the real exchange rate and the terms of trade will be larger in an economy where the trade elasticity is lower. Since the present paper’s aim is to address exchange rate related puzzles, it makes sense to follow the macro literature in choosing a low trade elasticity. More precisely, to match the relative volatility of the real exchange rate to GDP, which is about 3% for US quarterly data,  $\omega$  is taken to be 0.44. This value also implies that wealth effects from any underlying shock are large and the incompleteness of financial markets is of great importance.

Finally, I want to comment briefly on the model solution method. To address the questions we are interested in, local approximation techniques like log-linearization around the non-stochastic steady state cannot be used. Instead, a global solution technique needs to be used, that can explicitly account for the influence of second moments and occasionally binding inequality constraints on agent’s policy functions. Further details about the solution technique are provided in the appendix.

## 4 Results

This section illustrates the workings of the model with borrowing constraints, contrasting it throughout to the reference cases of complete financial markets (CM), financial autarky (FA) and the case where a single bond is internationally traded but trade is not restricted through ad-hoc borrowing constraints (IM-unconstrained). I start by presenting some sample time paths from a model simulation. I then turn right away to the main results, presenting the implied UIP coefficients from simulated model data, as well as some additional model moments documenting the properties of exchange rate variations, interest rate differentials, and the risk premium, and some business cycle stylized facts. To shed further light on the mechanism underlying these results, I continue the comparison in terms of policy functions of interest, and the implied stationary distribution.

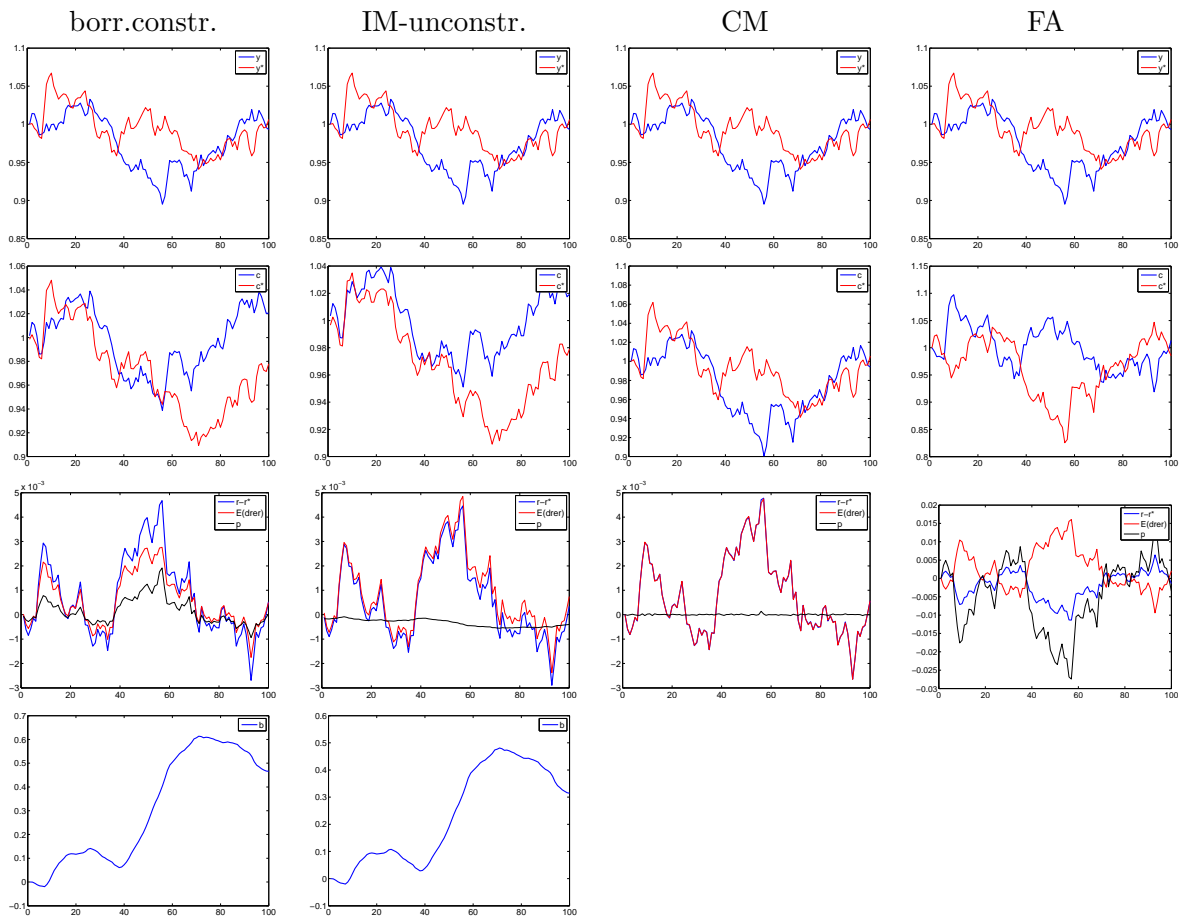
## 4.1 Model Simulation

Figure 1 portrays the behavior of the economy in response to a random sequence of shocks. The first row displays the time paths of the exogenous output endowment processes, the second row turns to the implied consumption paths, in the baseline model with borrowing constraints as well as in the comparison cases with different international financial market assumptions. As can be expected, the consumption time paths differ substantially across financial market structures. The basic mechanism driving these results can be understood as follows. Whenever the domestic country experiences a positive output realization (relative to Foreign), this lowers the price of the domestic good relative to the foreign good, and, as a result, depreciates the terms of trade and the real exchange rate. This leads to two effects, a *substitution effect* towards the relatively cheaper good, and a (negative) *income effect*, as the decrease in the domestic good price also changes the value of its current income,  $p_{H,t}Y_t$ . The relative strength of these two effects depends crucially on the chosen value of the intratemporal elasticity of substitution (see Corsetti et al. (2008) for a discussion). Calibrating the elasticity of substitution such as to match the exchange rate variability of the data implies a relatively low value for this parameter, such that small changes in relative quantities lead to large changes in international relative prices. As a result the implied substitution described above effect is very small; at the same time the implied negative income effect is very large.

In the case of complete international financial markets, the presence of state contingent assets implies that the economies are relatively sheltered from large negative income effects and that the consumption paths are unaffected by them. In contrast, in the other extreme case of financial autarky, no assets can be traded internationally, which means consumption paths are strongly dominated by the negative income effect: a positive output shock that lowers the price of its good actually hurts the country, since it lowers the value of its resources from which to finance current consumption. The cases of incomplete financial markets where a bond can be traded to borrow and lend internationally can be understood as intermediate cases between CM and FA, where some of the negative income effect can – not be averted as in the case of state-contingent payments, but – be alleviated by borrowing internationally.

As can be seen in Figure 1, initially, in the first 40 periods, when bond holdings are not too different from zero, the consumption paths in the economy with borrowing constraints do not

Figure 1: Sample time paths in the log-linear model and in the model with borrowing constraints



behave too differently from the behavior under CM. Once bond holdings grow substantially and move closer to one of the countries' constraint (in this case Foreign's), this country's precautionary motives increase drastically. As a result, the consumption paths start to diverge from their respective equivalents under CM and – because of large negative income effects that become harder to counteract by borrowing – behave somewhat closer like under financial autarky. It is important to emphasize that this does happen not only once a borrowing constraint becomes actually binding, but long before, as a result of the fear of becoming constrained in the future and the increasingly strong precautionary motives.

The third row of Figure 1 turns to the cyclical behavior of the variables of main interest, displaying the behavior of the (log) interest rate differential and its decomposition into (log) expected exchange rate variations and a (log) risk premium. This decomposition follows Fama (1984), and, stated for real variables, is given by

$$r_t - r_t^* = E_t [q_{t+1} - q_t] + p_t. \quad (15)$$

As seen by the panels in the third row, the interest rate differential in the case of CM always move together almost one-to-one. When consumption is low in the Home country its interest rate is high. The domestic interest rate relates domestic marginal utility of consumption today to marginal utility of consumption tomorrow, while the foreign interest rate relates foreign marginal utility of consumption today to marginal utility of consumption tomorrow. In addition, in each and every period the real exchange rate is efficient, such that it always is equated to the ratio of marginal utilities of consumption. As a result, the foreign asset is an equivalently good asset once transformed into the domestic countries own currency, the interest rate differential and expected exchange rate variations move one to one, and the implied risk premium is (close to) zero.

On the other hand, there are very large deviations of interest rate differential and expected exchange rate variations under financial autarky – a finding that is not surprising as the UIP condition is not an equilibrium condition of the FA model, and the exchange rate is entirely determined in the goods market. Under FA the domestic interest rate is low when consumption is low and the domestic bond provides bad insurance.<sup>5</sup> Moreover, the real

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<sup>5</sup>That is, the inverse shadow price of a bond that would be traded only domestically.

exchange rate fluctuates much more strongly and leads to an inefficiently high relative price when domestic consumption is relatively low.

Finally, turning to our baseline model with borrowing constraints, there are also substantial deviations from the UIP relation. The intuition herefore can be summarized as follows. As explained previously, when bond holdings are centered around zero and precautionary motives are relatively weak for both countries, the implied consumption paths behave comparably similar to the case of CM. As a result even in the baseline borrowing constraints model, interest rate differential and expected exchange rate changes move closely together, and the implied risk premium is close to zero. The changes markedly when a state is reached in which one of the countries has accumulated a substantial amount of debt. In such case, the indebted country increasingly has the incentive to reduce its borrowing somewhat, thereby trying to avoid becoming constrained and accepting that it has to reduce consumption already somewhat earlier. As a result, the interest rates and expected exchange rate start behave somewhat closer like under financial autarky and a wedge between the two appears. The asset of the country that has a stronger precautionary motive – that is, the country that has become a debtor because of a history of relatively worse income situations – has to carry a risk premium.

Table 2 reports the results from the uncovered interest parity regression, from the data and from a model simulation. Column 1 and 2 report the UIP coefficients found in empirical studies, based on either the nominal UIP regression – based on the regression of nominal exchange rate variations on the nominal interest rate differential – or the real UIP regression, – based on an equivalent regression in terms of real variables. As has been argued in the literature, it would seem intuitive to expect the currency of a country that currently pays a higher interest rate to depreciate, such that no excess returns could be made in expectations. Equating the nominal interest rate differential to the expected change in the exchange rate would imply a UIP slope coefficient of one. The empirical evidence for this type of regression for advanced economies is in stark contrast to this. Coefficients very different from one have been found empirically, for advanced economies these are typically even negative, suggesting that currency prices for high interest rates tend, instead, to appreciate. Backus, Foresi and Telmer (2001) report values of  $\hat{\beta}$  between  $-1.84$  and  $-0.74$ , based on exchange rates of the US

Table 2: UIP REGRESSION COEFFICIENTS

	nom. UIP regress. (data)	real UIP regress. (data)	borr.constr. model (baseline)	IM-unconstr. model	CM model	FA model
$\hat{\alpha}$	(-0.006, 0.008)		0.000	-0.000	0.000	0.000
$\hat{\beta}$	(-1.940, 0.320)	(0.133, 0.215)	0.683	0.997	1.002	-1.488

with The United Kingdom, Germany, and Japan. Based on a sample of 43 countries Iwata and Tanamee (2009) report UIP regression coefficients of  $-0.57$  for developed economies, and  $0.32$  for developing countries<sup>6</sup>.

Much less empirical work has been dedicated on estimating the relationship in real terms. Iwata and Tanamee (2009) find UIP regression coefficients of  $0.22$  for developed economies, and  $0.13$  for developing countries based on the regression of (log) real exchange rate variations on the (log) real interest rate. The puzzle of negative coefficients appears to be gone, nevertheless large deviations from interest parity remain. As argued originally by Fama (1984), the empirical findings of large deviations from interest rate parity can be reconciled if one accounted for the possible presence of an unobserved variable, a time-varying risk premium.

Macroeconomic models have a hard time generating such a time-varying risk premium. In a theoretical model, the risk premium can be understood to capture the conditional covariance of the stochastic discount factor (the intertemporal marginal rate of substitution) with future exchange rates variations, which in principle disturbs the comovement of the interest rate differential with expected exchange rate changes. However, there are two reasons why the exchange rate risk premium in theoretical models are generally small. One concerns the solution methods applied – modern macro models are often solved with (log) linear approximation methods, that impose certainty equivalence and thereby ignore the influence of a covariance term between future marginal utility and future exchange rates. This problem is explicitly accounted for in the present paper, as the model – including all comparison

<sup>6</sup>As found by Bansal and Dahlquist (2000) the UIP puzzle is less puzzling among developing countries than among developed countries. A possible reason is that when inflation is low – that is, for the developed world – exchange rate adjustments tend to be slow because adjustment is costly (see, e.g. Alvarez et al. (2009) and Bacchetta and van Wincoop (2010)).

Table 3: PROPERTIES OF INTEREST RATE DIFFERENTIAL, EXPECTED EXCHANGE RATE VARIATIONS AND RISK PREMIUM

	borr.constr. model (baseline)	IM-unconstr. model	CM model	FA model
$cov(E\Delta q, rp)$	0.000002	-0.000000	-0.000000	-0.000079
$cov(E\Delta q, r - r^*)$	0.000008	0.000004	0.000004	-0.000032
$var(r - r^*)$	0.000011	0.000004	0.000004	0.000022
$var(E\Delta q)$	0.000006	0.000004	0.000004	0.000047
$var(rp)$	0.000001	0.000000	0.000000	0.000133

versions – is solved using a global solution approach. Nevertheless the covariance term of future marginal utility and future exchange rates typically is of minor importance, implying a strongly positive, almost one-to-one relationship between the interest rate differential and expected exchange rate changes. Columns 4 and 5 of Table 2 show that, in the case of complete financial markets (CM) and the incomplete-markets bond economy without borrowing constraints (IM-unconstrained) this fact is manifested in regression coefficients of (close to) 1.

Figure 1 has shown that the implied dynamics of the model with borrowing constraints differ markedly from the case of complete markets and the unconstrained incomplete-markets bond-economy scenario. The presence of the time-varying risk premium drives a wedge between interest rate differential and expected exchange rate changes, which is successful in lowering the UIP regression coefficient,  $\hat{\beta}$ , to 0.68, moving it substantially closer to its empirical counterpart.

To understand the sources of the regression results in Table 2 it is useful to again consider the decomposition of the (log) interest rate differential into (log) expected exchange rate variation plus the risk premium. As pointed out by Fama (1984) from the decomposition in equation (15) it can be seen that in a regression of actual changes in the (log) exchange rate on the (log) interest rate rate differential,  $\Delta q_{t+1} = \alpha + \beta(r_t - r_t^*) + u_{t+1}$ , the sign and size of the regression coefficient  $\hat{\beta}$  depends on the variance and covariance of the variables in equation (15). In particular, to replicate the negative UIP coefficient  $\hat{\beta} = \frac{Cov\{E_t\Delta q_{t+1}, E_t\Delta q_{t+1} + p_t\}}{Var(E_t\Delta q_{t+1} + p_t)} = \frac{Cov(E_t\Delta q_{t+1}, p_t) + Var(E_t\Delta q_{t+1})}{Var(E_t\Delta q_{t+1} + p_t)}$ ,  $p_t$  and  $E_t\Delta q_{t+1}$  need to have a negative covariance, and  $p_t$  needs to have greater variance than  $E_t\Delta q_{t+1}$ . For match-

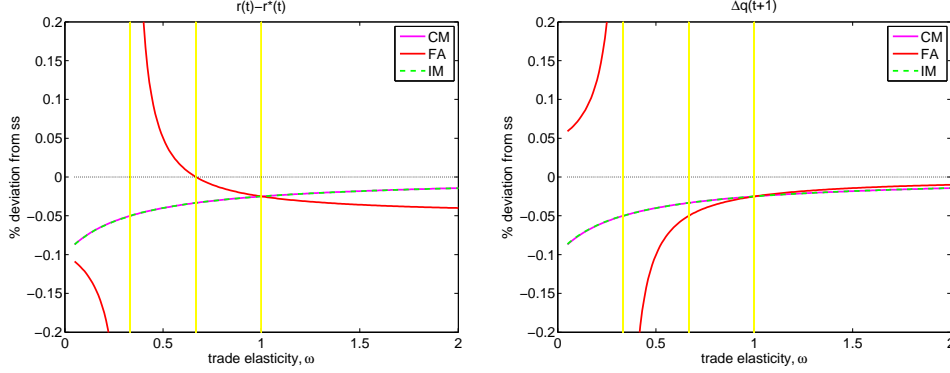
Table 4: BUSINESS CYCLE STATISTICS

	borr.constr. model (baseline)	IM-unconstr. model	CM model	FA model
$\sigma_c/\sigma_y$	1.45	1.69	0.93	1.65
$\sigma_q/\sigma_y$	3.12	3.43	1.22	4.44
$\sigma_r/\sigma_y$	0.10	0.05	0.05	0.08
$\rho_{c,y}$	0.24	0.20	1.00	-0.33
$\rho_{q,y}$	0.56	0.51	0.71	0.70
$\rho_{r,y}$	-0.63	-0.99	-1.00	0.32
$\rho_{\Delta(c-c^*),\Delta q}$	0.29	0.87	1.00	-1.00

ing the empirical coefficients based on real UIP regressions, the theoretical conditions on the comovement properties of expected exchange rate variations and the risk premium are less stringent. In particular, to match a slope coefficient that is positive but less than one, the covariance between the interest rate differential and expected exchange rate variations,  $Cov\{E_t\Delta q_{t+1}, r_t - r_t^*\}$ , is required to be lower than the variance of the interest rate differential,  $Var(E_t\Delta q_{t+1} + p_t)$ , or, equivalently, that the risk premium is sufficiently volatile. Table 3 summarizes the volatility and comovement properties of the interest differential, expected exchange rate changes and the risk-premium term.

Table 4 presents a summary of some additional second moments of the simulated model data. While the degree of exchange rate volatility was matched by calibrating the trade elasticity correctly, it is noteworthy that the model is successful at addressing also another long-standing puzzle in international economics. A large literature documents that exchange rate data display a negative correlation with cross-country consumption ratios, that is, that there is an apparent lack of efficient risk sharing (see Backus and Smith (1993)). The presence of international borrowing constraints increases the incompleteness of the financial markets which decouples consumption paths from their relative price and thus help realign the consumption-real exchange rate anomaly with respect to the data.

Figure 2: Impact Impulse Response to a Domestic Output Increase under various International Financial Market Assumptions

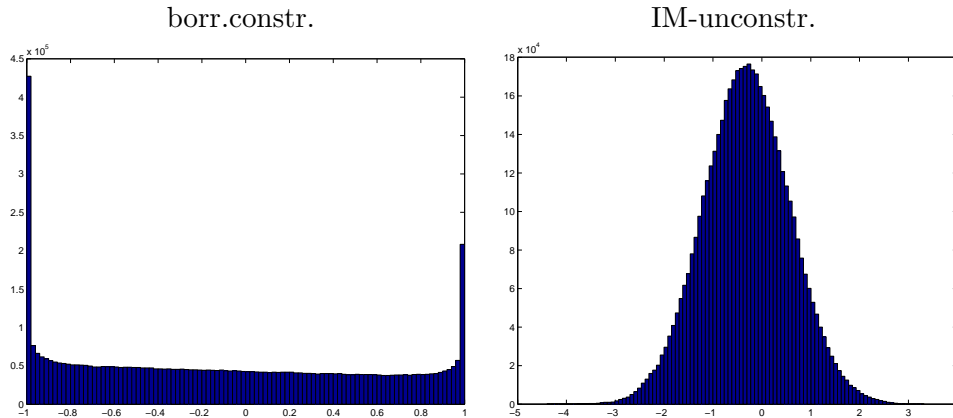


## 4.2 The Role of the Intra-temporal Substitution Elasticity

As mentioned previously, the value of the intra-temporal substitution elasticity is crucial for the results, because of its large influence on how international price movements impinge on the relative wealth of both countries. I therefore present some intuition for how various assumptions on the international financial market structure and the value of the trade elasticity affect the comovement of the interest rate differential with the change in exchange rate, in the three comparison cases of section 2.2 .

Figure 2 captures these comovement properties under the three financial market assumptions, displaying the first period – impact – response to a 1 % domestic output increase as a function of the trade elasticity  $\omega$ , which is on the horizontal axis. It can be seen that under complete markets the impact responses of the interest rate differential and subsequent exchange rate changes always comove one-to-one (as can be seen by the purple firm line representing CM). Interestingly, turning to the incomplete-markets bond-economy, so the scenario in which a bond is traded internationally but without the presence of borrowing constraints, this comovement is virtually identical. In fact, inspecting Figure 2, the impact responses in the IM-unconstrained scenario (given by the dashed green lines) seem to almost coincide with the case of CM. Finally, we can observe that it is only under the assumption of financial autarky, that the model is able to generate a less than one-to-one comovement, or even a negative comovement of the interest rate differential and subsequent exchange rate changes. In

Figure 3: Stationary Distribution of Bond Holdings

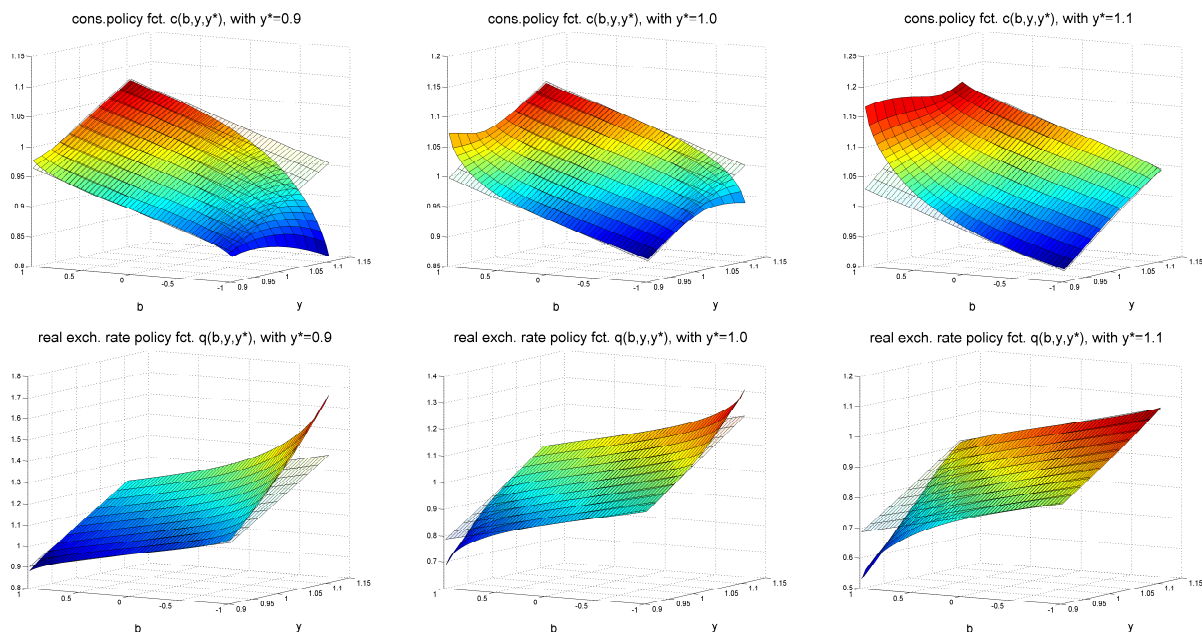


this case, the exchange rate is determined by the relative quantities of domestic versus foreign goods, that is, solely in the goods market (and not, like in the other financial market scenarios, jointly in the goods and asset market). As Figure 2 documents, to realign the empirical finding of deviations from uncovered interest parity in a model of the international economy two ingredients are crucial, that have been employed in the paper: sufficiently incomplete financial markets, and a low substitution elasticity that, because of large income effects that transfer wealth through relative price movements, makes the incompleteness matter a lot.

### 4.3 Stationary Distribution of Bond Holdings, and Nonlinearity of Policy Functions

As has been argued, the presence of borrowing constraints makes the incompleteness of the standard bond-economy case worse: we could expect the comovement behavior of interest rate differential and exchange rate changes to be somewhat closer to the case of financial autarky. In fact, one can interpret the scenario of financial autarky as the limit case of the model presented in section 2, with the constraints  $K$ ,  $K^*$  being set to zero. As a result, we can expect that the interest rate differential does not move one-to-one when agents of the two countries face international borrowing constraints. It is crucial, however, to note that this wedge in the interest rate parity relationships generally does not stem from international trade in the bond actually stopping because a borrowing constraint is hit, which happens less

Figure 4: Consumption and Real Exchange Rate Policy Function



than 4% of the times. As long as bond holdings are often enough close to either country's borrowing constraint, they will have a significant influence on the behavior of the model economy's variables.

Figure 3, which plots the stationary distribution of bond holdings in its unconstrained and constrained version, helps to illustrate this fact, and shows that this is indeed the case. A lot of mass of bond holdings is situated far away from the center when the nonlinear effects from increased precautionary motives are strongest.

This effect of the presence of the international borrowing constraints and strength of precautionary motive can also be seen by directly inspecting the policy functions. Figure 4 plots the policy functions for consumption and the real exchange rate, as functions of bond holdings and the domestic output endowment, keeping the level of the foreign output endowment at a constant (low, average and high) level. As can be seen, the presence of the borrowing constraints lead to a very different behavior of all variables compared to the standard incomplete-markets bond-economy case without borrowing constraints, which are

included as the transparent surfaces in Figure 4.

## 5 Conclusions

This paper presented a stylized two-country model in which precautionary motives – that arise from the presence of international borrowing constraints – help generate a time-varying exchange rate risk premium. It has long been argued that a time-varying risk premium on foreign exchange may help explain the empirical failure of uncovered interest rate parity, yet few theoretical models have had success in generating such a premium. In the model of the present paper, the asset of the country that has become more indebted as a result of worse economic times carries a risk premium. This introduces a time-varying wedge between the interest rate differential and expected exchange rate changes, thereby lowering the coefficient in a UIP regression and bringing it somewhat closer to the data.

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## A Appendix: The Model's Equilibrium Conditions

The model's equilibrium equations can be listed as follows:

$$Q_t = \frac{[\gamma^* + (1 - \gamma^*) \tau_t^{1-\omega}]^{\frac{1}{1-\omega}}}{[\gamma + (1 - \gamma) \tau_t^{1-\omega}]^{\frac{1}{1-\omega}}} \quad (16)$$

$$C_t = \left[ \gamma^{\frac{1}{\omega}} C_{H,t}^{\frac{\omega-1}{\omega}} + (1 - \gamma)^{\frac{1}{\omega}} C_{F,t}^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \quad (17)$$

$$C_t^* = \left[ \gamma^{*\frac{1}{\omega}} C_{H,t}^{*\frac{\omega-1}{\omega}} + (1 - \gamma^*)^{\frac{1}{\omega}} C_{H,t}^{*\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \quad (18)$$

$$\frac{1 - \gamma}{\gamma} \frac{C_{H,t}}{C_{F,t}} = \tau_t^\omega \quad (19)$$

$$\frac{1 - \gamma^*}{\gamma^*} \frac{C_{H,t}^*}{C_{F,t}^*} = \tau_t^\omega \quad (20)$$

$$C_t^{-\sigma} - \mu_t R_t = \beta R_t E_t [C_{t+1}^{-\sigma}] \quad (21)$$

$$C_t^{*-\sigma} - \mu_t^* R_t^* = \beta R_t^* E_t [C_{t+1}^{*-\sigma}] \quad (22)$$

$$\frac{C_t^{*-\sigma}}{Q_t} - \mu_t^* \frac{R_t}{Q_t} = \beta R_t E_t \left[ \frac{C_{t+1}^{*-\sigma}}{Q_{t+1}} \right] \quad (23)$$

$$\frac{B_{H,t+1}}{R_t} + \frac{Q_t B_{F,t+1}}{R_t^*} = B_{H,t} + Q_t B_{F,t} + p_{H,t} Y_t - C_t \quad (24)$$

$$Y_t = C_{H,t} + C_{H,t}^* \quad (25)$$

$$Y_t^* = C_{F,t} + C_{F,t}^* \quad (26)$$

$$0 = \mu_t [B_{H,t+1} + K] \quad (27)$$

$$0 = \mu_t^* \left[ \frac{B_{H,t+1}^*}{Q_t} - K \right] \quad (28)$$

## B Appendix: Solution Technique

The model is solved by an iterative algorithm to find the conditional expectations of the model's equilibrium conditions. Below I briefly outline the steps of the algorithm used:

- In the following, denote  $t + 1$  variables with a prime, e.g.  $B = B_t$ ,  $B' = B_{t+1}$ , and accordingly,  $B'' = B_{t+2}$ . I construct a 3-dimensional grid over the model's state variables at time  $t$ , that is, over  $B, y = \log Y, y^* = \log Y^*$  consisting of  $n_b n_y n_{y^*}$  grid points. The grid in dimension of  $b$  ranges from  $[-K, K]$ . The gridpoints in dimensions of  $y, y^*$  are obtained by discretizing the continuous AR processes by following the method of

Rouwenhorst (1995) . As a recent contribution by Kopecky and Suen (2010) shows, the Rouwenhorst discretization has proven to lead to substantially better approximations than more conventional discretizations, e.g. Tauchen and Hussey (1991), particularly when the underlying process is very persistent, as is the case here. The number of gridpoints was chosen to be  $n_b = 51$  and  $n_y = n_{y^*} = 11$ .

- Set counter equal to 1. I make initial guesses on the model's conditional expectations by using the log-linear solution as starting point. In the endowment economy guesses are made for the conditional expectations of the bond Euler equations,  $CE_{BH}(B; y, y^*) \equiv E\{u_{C'}\}$ ,  $CE_{BH^*}(B; y, y^*) \equiv E\left\{\frac{u_{C'^*}}{Q'}\right\}$  and  $CE_{BF^*}(B; y, y^*) \equiv E\{u_{C'^*}\}$ , at each gridpoint  $(B; y, y^*)$ .
- Using the guesses for the conditional expectations, the endogenous variables  $B'$ ,  $C_H$ ,  $C_F$ ,  $C$ ,  $C_H^*$ ,  $C_F^*$ ,  $C^*$ ,  $R$ ,  $R^*$ ,  $\mu$ ,  $\mu^*$ ,  $\tau$ , and  $Q$  can be computed at each gridpoint, by solving the set of equilibrium' conditions outlined in Appendix A, equations (16). This is done, by initially assuming that the borrowing constraints do not bind, i.e.  $\mu = 0$  and  $\mu^* = 0$ . In case that at any gridpoint  $(B; y, y^*)$  one of the borrowing constraints is hit, the bond holding is set to the constraint and system (16) is solved again for the respective (positive) value of the multiplier on the constraint. Having in hand the guesses of the conditional expectations as functions of  $(B; y, y^*)$ , I use interpolation methods to find  $CE'_{BH}$ ,  $CE'_{BH^*}$  and  $CE'_{BF^*}$ , as functions of  $(B'; Ey', Ey'^*) = (B'; \rho y, \rho y^*)$ .  $CE'_{BH}$ ,  $CE'_{BH^*}$  and  $CE'_{BF^*}$  can in turn be used to obtain  $B''$ ,  $C'_H$ ,  $C'_F$ ,  $C'$ ,  $C'^*_H$ ,  $C'^*_F$ ,  $C'^*$ ,  $R'$ ,  $R'^*$ ,  $\mu'$ ,  $\mu'^*$ ,  $\tau'$ , and  $Q'$  for any  $(B; y, y^*)$ .
- Having in hand solutions for  $C'$ ,  $C'^*$ , and  $Q'$ , together with the discretized states and transition matrix,  $\pi$ , for the exogenous processes, the guesses of the conditional expectations can be updated by computing:

$$\begin{aligned}
CE_{BH}(B; y, y^*) &\equiv E_t [C_{t+1}^{-\sigma}] = \sum \pi(y', y'^* | y, y^*) C'(B; y', y'^*)^{-\sigma} \\
CE_{BH^*}(B; y, y^*) &\equiv E_t [C_{t+1}^{*\sigma} / Q_{t+1}] = \sum \pi(y', y'^* | y, y^*) [C'^*(B; y', y'^*)^{-\sigma} / Q'(B; y', y'^*)] \\
CE_{BF^*}(B; y, y^*) &\equiv E_t [C_{t+1}^{-\sigma}] = \sum \pi(y', y'^* | y, y^*) C'^*(B; y', y'^*)^{-\sigma}
\end{aligned}$$

- The above steps are repeated until convergence is achieved.