Growth and Nonemployment
A Search Model with Vintage Human Capital*

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Abstract

This paper extends the standard vintage capital/search model of Aghion and Howitt (1994) and Mortensen and Pissarides (1998) to incorporate vintage human capital and studies the impact of capital-embodied growth on equilibrium unemployment and nonparticipation. In addition to the capital obsolescence (or creative destruction) effect that tends to raise unemployment in standard vintage models, vintage human capital introduces a skill obsolescence effect of faster growth that has the opposite sign. Faster skill obsolescence reduces the value of unemployment, hence wages and leads to more job creation and less job destruction, unambiguously reducing unemployment. The paper also characterizes the impact of capital and skill obsolescence on non-participation decisions.

Keywords: Growth, skill transferability, unemployment, nonparticipation

JEL Classification: E24, J21, J23, J24, J31, J64

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1 Introduction

Economic growth involves continuous restructuring and factor reallocation. Technological change, embodied in new generations of capital, renders earlier vintages of machines obsolete and eliminates jobs associated to these technologies. New jobs are created to operate the new technology vintages and workers from destroyed jobs need to reallocate to these jobs. The standard result in vintage capital/search models, commonly use to study the relation between growth and unemployment, is that a faster rate of technological growth tends to increase equilibrium unemployment (e.g. Aghion and Howitt (1994), Mortensen and Pissarides (1998)). In a vintage capital model, new more productive technology vintages enter the economy, making older generations of capital relatively less productive. As workers operating old technology vintages may potentially change to a job with a more productive and higher paying technology vintage, jobs of older technology vintages have to pay wages that take account of this outside option. Eventually the increasing wage makes old capital vintages unprofitable and associated jobs are destroyed. A faster rate of growth increases this capital obsolescence effect that tends to shorten job durations and increase unemployment incidence. A serious shortcoming of these models is that they implicitly assume that workers skills are fully transferable from obsolete technologies to the technological frontier, which is clearly at odds with reality.

This paper extends the standard vintage capital/search model of Aghion and Howitt (1994) and Mortensen and Pissarides (1998) to incorporate vintage human capital and studies the impact of capital-embodied growth on equilibrium unemployment and nonparticipation. In addition to vintage capital and capital obsolescence present in earlier models we introduce skill obsolescence of workers into the vintage framework. Workers learn through learning-by-doing while employed, but their learning is specific to the vintage of technology they operate. Upon separation, workers become unemployed and search for a new job. The amount of skills they can transfer on the new jobs (of the leading edge vintage) is proportional to the technological distance between the old and the new capital they work with. While unemployed, skills keep becoming obsolete. Our model allows to study questions pertaining to the impact of capital-embodied technological change on important labor market equilibrium outcomes such as the skill distribution and inequality in the wage distribution, equilibrium unemployment and labor market participation.

We show that the comparative static results of the standard vintage capital are reversed when skill depreciation is fast enough. In addition to the capital obsolescence (or creative destruction) effect that tends to raise unemployment, vintage

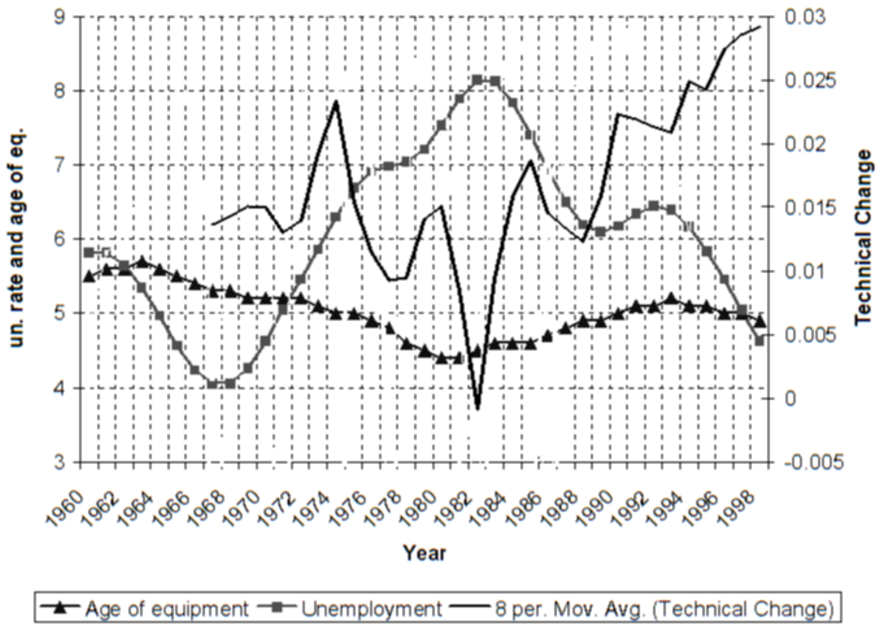


Figure 1: Unemployment, age of equipment, and technical change, U.S. 1960-1998.

human capital introduces a skill obsolescence effect of faster growth that has the opposite sign. Faster skill obsolescence reduces the value of unemployment, hence wages and leads to more job creation and less job destruction, unambiguously reducing unemployment.

We study the determination of unemployment following the long tradition of works on growth and unemployment. We highlight a new channel for this interaction: as the rate of productivity growth increases, we not only have a capital obsolescence effect that tends to shorten job durations and increase unemployment incidence, but we also have a skill obsolescence effect that reduces the value of unemployment and tends to offset the former force. We analyze job creation and destruction on the assumption of skill obsolescence being independent of the rate of technical change, and on the assumption that the rate of skill obsolescence and the rate of technical change are related so that transferability is decreasing in the productivity growth parameter.

The relationship between capital age and unemployment is illustrated in figure 1. The figure draws the rate of unemployment, the age of capital equipment, which also measures job duration in the vintage model, and rate of capital-embodied technical change for capital equipment.[1] According to the standard vintage capi-

[1]Unemployment data are available from the website of the U.S. Bureau of Labor Statistics, age of equipment data are available from the website of the U.S. Bureau of Economic Analysis, and the data for technical change is from Cummins and Violante (2002).

tal/search model, when the rate of technical change rises, the age of capital falls (job durations fall) and unemployment increases. According to the present model, if skill obsolescence dominates, when the rate of technical change increases, the age of capital rises and unemployment falls. As the figure illustrates, the data of age of capital and unemployment are consistent with both the standard vintage capital model and the model with vintage human capital presented in this paper However, when the rate of technical change is considered the model of the present paper seems more plausible. Furthermore, it can be noted that wages in the U.S. stagnated over the period 1975-1995 as well, which is consistent with a skill obsolescence story.

In addition to the seminal papers on growth and unemployment (Aghion and Howitt (1994) and Mortensen and Pissarides (1998)), a number of recent papers have studied unemployment and wage inequality in vintage/search models. In a series of related papers, Hornstein, Krusell and Violante (2005, 2007a, 2007b) use the vintage framework to study the heterogenous labour market outcomes between workers. Their approach differs from ours, as their focus is largely on modelling capital of firms, whereas we concentrate on the specificity of human capital of workers.[2] Our model relates to studies that focus on the interactions of technological change, skills and specificity. Caballero and Hammour (1997, 1998a, 1998b) study the implications of specificity and appropriability of job specific rents in the context of growth and job reallocation. Embodiment of technology in capital vintages and job specific human capital gives rise to appropriable quasi rents that distort job reallocation in the process of creative destruction. Ljungqvist and Sargent (1998) study the determination of equilibrium unemployment in a model where workers accumulate skills on the job and lose skills during unemployment.[3] Skill dynamics of this type are also present in Laing et al. (2003), but they incorporate growth of the stock of public knowledge in the economy. They introduce overlapping generations (vintages) of workers who differ in productivities s.t. knowledge growth is embodied in new entrant workers. In this setup the entry of new (younger) workers renders

[2]In Hornstein et al. (2007a) irreversible investment in new vintages of capital creates heterogeneity in productivity among firms, and they show that capital-embodied technological change reduces labor demand and raises equilibrium unemployment and unemployment durations. Hornstein et al. (2005) examine how technological change affects wage inequality and unemployment, distinguishing between two polar cases : a “creative destruction” economy where new machines enter chiefly through new matches and an “upgrading” economy where machines in existing matches are replaced by new machines. Hornstein (2007b) study the implications of investment specific technological change for the matching model, where they take as the defining feature of vintage capital the fact that the capital content of a machine vintage cannot be adjusted after the vintage has been introduced.

[3]Pissarides (1992) studies unemployment persistence in a model where skill loss during unemployment has an adverse effect on vacancy creation.

the knowledge of older workers obsolete. Galor and Moav (2000) build a growth model which is characterized by ability biased technological transition where technological acceleration leads to a rise in wage inequality both between and within skill groups. Their study is close in spirit to the present paper, as technological progress is assumed to reduce the adaptability of existing human capital for the new technological environment. While superior technologies increase productivity, they also erode existing human capital that is adaptable to the new technologies, thus reducing productivity. A characteristic of their model is that able individuals have a comparative advantage in adapting to new technology. Violante (2002) considers workers with two-dimensional skills. An employed worker accumulates skills by learning-by-doing, but when changing jobs to one of a more recent technological vintage, only a fraction of the workers skills are transferable. Technological acceleration reduces workers' capacity to transfer skills from old to new machines, generating cross-sectional variance of skills and therefore wages. Ljungqvist and Sargent (1998) and Violante (2002) do not study the 'creative destruction' -nature of technological change as their models do not embed the vintage capital structure. Carré and Drouot (2004) introduce learning-by-doing into a vintage capital framework but do not consider transferability of skills between technology vintages.

We adopt the two-dimensional nature of skills (Violante 2002) into the vintage matching framework. There are two technological parameters in the model, the speed of technical change which measures the rate of capital obsolescence and skill transferability which measures the rate of skill obsolescence. One can also conjecture that the rate of skill obsolescence and the rate of technical change are related so that transferability is decreasing in the productivity growth parameter. We think of the productivity growth parameter as capturing the degree to which new features of the technologies are embodied in capital, and therefore the extent to which capital is different across vintages.

First, assuming that skill transferability is independent of the rate of technical change, an increase in the latter increases both unemployment incidence (the age of jobs decreases) and unemployment duration (labor market tightness decreases). Therefore the equilibrium unemployment rate unambiguously increases as in the standard vintage model. An increase in the transferability parameter has the opposite effect, both unemployment incidence and unemployment duration decrease (job age and labor market tightness both increase) and equilibrium unemployment decreases.

Next, we study the impact of an increase in the rate of technical change when skill transferability is decreasing in the productivity growth parameter. We derive a condition to distinguish between two cases: when skills depreciate sufficiently little with the rate of technical change technological so that obsolescence dominates and when skills depreciate fast enough with the rate of technical change so that skill obsolescence dominates. For the former unemployment increases (job age and labor market tightness both decrease) with an increase in the rate of technical change and for the latter unemployment decreases (job age and labor market tightness both increase). We conclude that the comparative static results of the standard vintage capital are reversed when skill depreciation is fast enough.

In the second (preliminary and incomplete) part of the paper we study the impact of capital and skill obsolescence on non-participation decisions. In the presence of an income from home-production, or a welfare payment unconditional on search, the model implies that some workers will choose optimally to exit the labor force because their skills have depreciated so much that their value of searching is below what is offered as non-participants. Murphy and Topel (1998) show that in synchrony with the rise of inequality, a larger fraction among the low-skilled agents have left the labor force for good. They have argued that the same labor demand shift is responsible for both. In our model a rise in the productivity growth of new capital increases the rate of obsolescence of skill, so it might lead in equilibrium to a larger fraction of agents whose skills end up commanding a wage lower than the value of nonparticipation. We characterize nonparticipation associated to skill depreciation for various cases.

First we consider the case where the value of nonparticipation is always below that of unemployment for employed workers. In other words the relevant threat point for the employed worker in the bargaining process is unemployment. A sufficient condition for this to happen is that the skill level of an employed worker, expressed in terms of her productivity on the newest technology grows with tenure: in this case unemployed workers would start their jobs from skill levels above the threshold skill level for participation and their skill would always remain above that level as long as they are employed. We determine the threshold skill level of participation which depends on the value of unemployment. With a faster rate of technical change jobs last less and they are fewer, so the value of unemployment falls and therefore the threshold skill level increases. Skill obsolescence has a direct and an indirect effect. The direct effect makes skills get obsolete faster and thus the threshold skill level increases. Skill obsolescence increases labor market tightness and this indirect effect implies more job creation and therefore the threshold skill level decreases (higher job creation increases the value of unemployment).

If the skill level of an employed worker, expressed in terms of her productivity on the newest technology depreciates with tenure we have three cases: i) unemployment is the outside option of the worker for the whole duration of the job, ii) the outside option switches from unemployment to nonparticipation during the lifetime of the job, iii) the workers skills are so low that the payoff from participating to the labor market is too low for the worker to seek a job. The workers outside option and participation decision depend on the initial skills of a worker, job duration (time for skill depreciation) and labor market tightness. We characterize the determination of the equilibrium values.

The paper is organized as follows. Section 2 presents the structure of the economy and the evolution of skills and productivity. In section 3 we characterize the balanced growth equilibrium of the model and derive the two key equilibrium conditions: the job creation and destruction conditions. Section 4 describes the behavior of the economy wrt. growth and the key parameters of the model. Section 5 characterizes the impact of capital and skill obsolescence on non-participation decisions. Section 6 concludes.

2 The Economy

We first describe the structure of the economy and the asset value equations that characterize the flow returns to the firms and workers when participating in a match and when idle. The key element in the structure of the economy is "vintage human capital" which interacts with technological vintage capital present in earlier studies.

Demographics and preferences– Time is continuous. The economy is populated by a measure one of ex-ante equal workers, who are infinitely lived, risk-neutral and discount the future at rate $r > 0$. Every period, workers can be either employed or unemployed. Workers retire exogenously at rate $\rho > 0$ and are replaced by a new inflow of unemployed workers of measure ρ.

Technology– New technologies are embodied in physical capital, and the leading edge technology in the economy advances at an exogenous rate $\gamma > 0$. A firm (or production unit, or machine) created at time t embeds a fixed amount of capital (normalized to unity) of vintage t with productivity $e^{\gamma t}$. Production is decentralized across different firms and takes place when the firm is matched with a worker. Every period, firms can be either vacant or matched. As standard in the literature, we assume that all vacant firms at time t embody the best available technology.

Skills– Human capital is partially vintage specific. New workers entering the labor market at time t have skill level $\bar{z}_t$ specific to the newest technology of vintage t, i.e. the skill level $\bar{z}_t$ measures the productivity of a worker operating capital of vintage t. Over time these skills become obsolete with respect to more recent technology vintages. We will assume that at time t, an unemployed worker with skill level z_τ of vintage τ (s.t. $\tau < t$) can transfer skills $z_t < z_\tau$ on a new job with a machine of the latest vintage t, where z_t is determined by

$$z_t = z_\tau e^{-\phi(t-\tau)}. \tag{1}$$

One may think of the parameter ϕ as a measure of the vintage specificity of the skills of a worker. A worker with a given productivity z_τ in operating capital of vintage τ is less productive in operating capital of a more recent vintage t. The larger is the distance between t and τ, the smaller is the fraction of skills that are relevant or transferable to a more recent vintage t. Given that all vacancies at any time t embody the best vintage of technology, it is convenient to express the skill level of a worker z_τ always in terms of the newest vintage t. This convention and equation (1) imply that for unemployed workers, skills evolve (relative to the best vintage of technology) according to the law of motion $\dot{z} = -\phi z$, where the subscript t is omitted to lighten the notation.

When employed, workers cumulate skills through learning-by-doing at the instantaneous constant learning rate $\lambda > 0$, therefore during employment the law of motion for skills (expressed in terms of the newest vintage t) is $\dot{z} = (\lambda - \phi)\, z$. The rate of skill obsolescence relative to the leading edge technology is thus lower for employed workers than for unemployed workers, as skills not only depreciate relative to new technology vintages, but also accumulate when employed.[4] To avoid that the skill space expands infinitely, we make the convenient assumption $\lambda \leq \phi$. With this assumption, the domain of the skill distribution is bounded between $(0, \bar{z}]$.[5]

At time t, output produced by a match between a machine of vintage τ and a worker of skills z (always expressed in terms of the newest vintage t) is

$$y\,(t, \tau, z) = e^{\gamma\tau} z e^{\phi(t-\tau)}, \tag{2}$$

where $e^{\gamma\tau}$ is the fixed productivity of capital of vintage τ, z is the productivity of the worker expressed in terms of the newest vintage t including learning by doing up to date t, and $e^{\phi(t-\tau)}$ converts worker productivity to productivity in operating

[4] Separating the skill evolution parameters into the learning by doing λ and depreciation ϕ parameters is to simplify intuition. Alternatively one could simply assume that skill obsolescence ϕ_i $(i = e, u)$ is smaller for employed workers than for unemployed workers, because employed accumulate skills (although their skills also depreciate relative to the most recent vintage).

[5] It is important to make the distinction of *vintage* specificity of skills as opposed to *match* specific skills. In this study, skills are specific to technology vintage, not to an individual match. Therefore, as workers have an initial skill level $\bar{z}_t$ when entering the labor market, even a worker who has never been matched loses skills relative to the leading edge vintage. Match specific skills, aquired in a match, would require a worker to have been matched before any skills could be lost.

capital of vintage τ.[6]

Matching– Firms observe workers' skills perfectly and they can direct their search towards any type z, hence the matching markets will be segmented by skill level. Firms pay a search cost

$$c(t,z) = ze^{\gamma t}c \tag{3}$$

to search in market z. The search cost is proportional to the productivity of new jobs in market z, reflecting the realistic feature that job creation and recruiting costs of high productivity jobs are larger than those of low productivity jobs. Each of these matching markets has the same constant returns to scale matching function $m(u(z), v(z))$, where $u(z)$ and $v(z)$ are respectively unemployed workers and vacancy firms in market z. We denote by $\theta(z)$ the vacancy-unemployment ratio (or market tightness) in market z, $p(\theta(z))$ the meeting probability for an unemployed worker and $q(\theta(z))$ the meeting probability for a vacant firm in market z. The meeting rates imply an expected duration of search for firms of $1/\ q(\theta(z))$ and an expected duration of unemployment of $1/\ p(\theta(z))$ in market z. Matches dissolve exogenously at rate δ.

Worker's income– The fractions of output going to wages and to profits along the match are determined through Nash bargaining between the firm and the worker. The pair rebargains every instant on the new discounted stream of output. Denote the wage of a match between an employed worker with skill level z on a machine of vintage τ at time t as $w(t,\tau,z)$. Unemployed workers spend all their time endowment searching, hence having no utility from leisure. They receive an unemployment income $b(z,t) = ze^{\gamma t}b$ which is proportionate to the growth rate of aggregate income and the evolution of worker's skills. Interestingly, this assumption implies that benefits fall with unemployment duration, as in most of the real-world unemployment insurance systems.

2.1 Value Functions and Match Surplus

The values of participating to the market for the firms and workers are described by a set value equations. At time t, denote the value of vacant firms searching in market z by $V(t,z)$; the value of firms of vintage τ matched with workers of type z by $J(t,\tau,z)$; the value of employed workers in the same type of match by

[6] If the workers productivity is z on the leading edge capital vintage t, the term $e^{\phi(t-\tau)}$ is included to reverse the skill depreciation over the period $t-\tau$. The workers skills become obsolete wrt. new technology vintages, but they do not depreciate wrt. to the current job of vintage τ.

$W(t,\tau,z)$; and the value of unemployed workers with skills z by $U(t,z)$. Then, it is easy to derive that[7]

$$rV(t,z) = \max\left\{c(t,z) + q(\theta(z))\left[J(t,t,z) - V(t,z)\right] + \frac{dV(t,z)}{dt}, 0\right\} \quad (4)$$

$$\begin{aligned} rJ(t,\tau,z) &= \max\Big\{y(t,\tau,z) - w(t,\tau,z) - (\delta+\rho)\Big[J(t,\tau,z) - \max_z V(t,z)\Big] \quad (5) \\ &\quad + \frac{dJ(t,\tau,z)}{dt}, r\max_z V(t,z)\Big\} \end{aligned}$$

$$\begin{aligned} rW(t,\tau,z) &= \max\{w(t,\tau,z) - \delta\left[W(t,\tau,z) - U(t,z)\right] - \rho W(t,\tau,z) \quad (6) \\ &\quad + \frac{dW(t,\tau,z)}{dt}, rU(t,z)\Big\} \end{aligned}$$

$$\begin{aligned} rU(t,z) &= \max\{b(z,t) + p(\theta(z))\left[W(t,t,z) - U(t,z)\right] - \rho U(t,z) \quad (7) \\ &\quad + \frac{dU(t,z)}{dt}, 0\Big\} \end{aligned}$$

where $\frac{dV(t,z)}{dt} = V_t(t,z)$, $\frac{dJ(t,\tau,z)}{dt} = J_t(t,\tau,z) + \dot{z}J_z(t,\tau,z)$ and $\frac{dW(t,\tau,z)}{dt} = W_t(t,\tau,z) + \dot{z}W_z(t,\tau,z)$ $s.t.$ $\dot{z} = (\lambda - \phi)z$ and $\frac{dU(t,z)}{dt} = U_t(t,z) + \dot{z}U_z(t,z)$ $s.t.$ $\dot{z} = -\phi z$.[8]

A vacant job in the market for skills z costs $c(t,z)$ per unit time and the firm matches with a worker at rate $q(\theta(z))$. The change of state yields a return of $J(t,t,z) - V(t,z)$ to the firm. An occupied job yields the return $y(t,\tau,z) - w(t,\tau,z)$ per unit time, which is the productive output of the match minus the wage paid to the worker. The job may be terminated due to either an exogenous shock or retirement at the respective rates δ and ρ. The value of the match depends on the skills of the worker, which evolve according to the law of motion $\dot{z} = (\lambda - \phi)z$. The worker accumulates skills at rate λ as long as the firm and worker are matched and the workers skills depreciate relative to the newest technology vintage at rate ϕ.

When employed a worker earns the wage $w(t,\tau,z)$ per unit time. The match may be destroyed due to an exogenous shock at rate δ in which case the worker

[7] A typical derivation of a value equation (as limit of a discrete time model economy) is described in the Appendix.

[8] Observe that whereas z evolves in time for a filled job, an employed worker and an unemployed worker, it is constant in the value equation of a vacancy and $\frac{dV(t,z)}{dt}$ does not involve the effect through z.. This is because a firm posts a vacancy with a specific skill requirement z, for example fresh high school graduates, and although the individuals in the pool of unemployed workers with skills z come and go (new graduates with skills z enter and unemployed graduates of an earlier vintage exit the pool as their skills depreciate over time), the pool where the firm searches and therefore z remains the same.

becomes unemployed and loses the difference between the two labor market states $W(t,\tau,z) - U(t,z)$. In the case of retirement which occurs at rate ρ the worker loses $W(t,\tau,z)$. The evolution of skills for an employed worker follows the law of motion $\dot{z} = (\lambda - \phi)z$. An unemployed worker receives an unemployment income $b(z,t)$ and matches with a firm at rate $p(\theta(z))$ which yields a return of $W(t,t,z) - U(t,z)$. In the case of retirement the worker loses $U(t,z)$. When unemployed, skills depreciate according to the law of motion $\dot{z} = -\phi z$ as no skill accumulation takes place. Note that the value of unemployment $U(t,z)$ for an employed worker in equation (6) evolves according to $\dot{z} = (\lambda - \phi)z$. This is so because as long as a worker is employed, also the value of eventual unemployment is affected by skill accumulation on the job.

3 Balanced Growth Equilibrium

We now characterize the equilibrium of the model, along a balanced growth path where all the values above grow at rate γ. Once we stationarize all values by the growth factor $e^{\gamma t}$ all the relevant equilibrium objects are only a function of the difference $(t-\tau)$ which is "age" and we denote as a. The balanced growth path versions of the value equations are[9]

$$(r-\gamma)V(z) = \max\{-zc + q(\theta(z))[J(0,z) - V(z)], 0\} \tag{8}$$

$$\begin{aligned}(r-\gamma)J(a,z) &= \max\Big\{e^{-\gamma a}ze^{\phi a} - w(a,z) - (\delta+\rho)\Big[J(a,z) - \max_z V(z)\Big] \\ &\quad + J_a(a,z) + (\lambda-\phi)zJ_z(a,z), (r-\gamma)\max_z V(z)\Big\}\end{aligned} \tag{9}$$

$$\begin{aligned}(r-\gamma)W(a,z) &= \max\{w(a,z) - \delta[W(a,z) - U(z)] - \rho W(a,z) \\ &\quad + W_a(a,z) + (\lambda-\phi)zW_z(a,z), (r-\gamma)U(z)\}\end{aligned} \tag{10}$$

$$\begin{aligned}(r-\gamma)U(z) &= \max\{bz + p(\theta(z))[W(0,z) - U(z)] - \rho U(z) \\ &\quad - \phi z U_z(z), 0\}\end{aligned} \tag{11}$$

Intuitively, these equations are expressed in terms of age relative to the leading edge technology vintage of age zero. For example, technological productivity $e^{-\gamma a}$ in equation (9) decreases with age relative to the leading edge technology.

The key object for the characterization of the model is the "surplus function", defined as the value of the match for a worker and a firm, net of their respective outside options. The surplus of a match of age a is given by

[9] See appendix for details.

$$S(a,z) = J(a,z) + W(a,z) - V(z) - U(z). \tag{12}$$

The worker and the firm divide the match surplus according to the Nash bargaining solution and the first-order condition is

$$\beta \left[J(a,z;w(a,z)) - V(z) \right] = (1-\beta) \left[W(a,z;w(a,z)) - U(a,z) \right]. \tag{13}$$

where β represents the worker's share of match surplus. The division of the match surplus is continuously renegotiated s.t. it reflects the evolution of match surplus over time. Substitution of the value equations into the first-order condition yields the wage equation[10]

$$w(a,z) = \beta z e^{(\phi-\gamma)a} + (1-\beta) z \left(b + \frac{\beta}{1-\beta} c\theta(z) \right). \tag{14}$$

The first term is the worker's share of match output. Relative to productivity in leading edge technology jobs, technological productivity decreases with age a. As z measures the productivity of a worker in a leading edge job, $e^{\phi a}$ reverses skill depreciation, as skill depreciation does not take place as long as the worker continues working with the current technology of age a.[11] The second term reflects the workers outside option and is expressed in terms of the leading edge vintage. It depends on the unemployment compensation received by unemployed workers of skills z and labor market tightness.

It is useful to formally define a stationary equilibrium for this economy.

Definition: *A stationary (stationarized balanced growth) equilibrium is a list of: (i) values* $\{V(z), U(z), J(a,z), W(a,z), S(a,z)\}$, *(ii) market tightness function* θ, *(iii) optimal destruction age* $\bar{a}$, *(iv) wage function* $w(a,z)$ *such that:*

1. *The values* $\{V(z), J(a,z), W(a,z), U(z)\}$ *satisfy equations* (8)−(12) *above;*
2. *There is free entry of vacancies in each market* z, *thus equilibrium market tightness* θ *satisfies the condition* $V(z) = 0$;
3. *The optimal destruction age* $\bar{a}$ *satisfies the condition* $S'(\bar{a}, z) = 0$;
4. *The wage function* $w(a,z)$ *solves the Nash bargaining equation* (13).

[10] See appendix for detailed derivation.

[11] Recall that z includes learning by doing over the age of the match.

3.1 Characterization

Substituting the value equations (9), (10) and the value of unemployment for an *employed* worker (with the law of motion $\dot{z} = (\lambda - \phi) z$) into (12) and using $V(z) = 0$ produces[12]

$$\begin{aligned}(r-\gamma+\delta+\rho) S(a,z) &= \max\left\{ze^{(\phi-\gamma)a} - bz - p(\theta(z))\beta S(0,z)\right. \\ &\left. + S_a(a,z) + (\lambda-\phi) z S_z(a,z)\right\}.\end{aligned} \tag{15}$$

To see that the surplus (15) is a first order differential equation as a function of t, observe that $S(a,z) = S(a(t), z(t))$ and consequently we can express the derivatives $\frac{dS(a(t),z(t))}{dt} = S_a(a,z) + (\lambda - \phi) z S_z(a,z)$, recall that $\dot{z} = (\lambda - \phi) z$.

From the definition of the surplus and the Nash condition we obtain $S(0,z) = \frac{1}{1-\beta} J(0,z)$ and using the free entry condition in (8) we get $J(0,z) = \frac{cz}{q(\theta(z))}$. Then (15) reduces to[13]

$$(r-\gamma+\delta+\rho) S(a,z) - \frac{dS(a,z)}{dt} = \max\left\{ze^{(\phi-\gamma)a} - bz - \frac{\beta}{1-\beta} cz\theta(z), 0\right\}. \tag{16}$$

This is a first order differential equation for the surplus as a function of t. The max operator implies that we have a boundary condition $S(\bar{a}, z) = 0$. The particular solution for this differential equation, once we impose the boundary condition, is

$$S(a,z) = e^{(\phi-\gamma)a} z \int_a^{\bar{a}} \left[1 - \omega(\theta) e^{-(\phi-\gamma)\tilde{a}}\right] e^{-(r+\delta+\rho-\lambda)(\tilde{a}-a)} d\tilde{a} \tag{17}$$

where $\omega(\theta) = b + \frac{\beta}{1-\beta} c\theta$ is the outside option of the worker. From the optimality condition $S'(\bar{a}) = 0$, it follows that the destruction age $\bar{a}$ for the matched pair satisfies the following rule:

$$e^{(\phi-\gamma)\bar{a}} = \omega(\theta) \tag{18}$$

This condition is very intuitive and can be explained in two alternative ways. First, let us rewrite the equation above as

$$ze^{\lambda\bar{a}} = e^{\gamma\bar{a}} \omega(\theta) ze^{(\lambda-\phi)\bar{a}}.$$

The left hand side of that expression is the output flow on a machine of age $\bar{a}$ matched with a worker who had skills z upon matching; the right hand side is the flow value of the outside option of the same worker (recall the outside option of the firm V is zero in equilibrium): her skills have increased at the pace λ but, at the same time, became obsolete at rate ϕ. The term $e^{\gamma\bar{a}}$ represents the higher value

[12] See appendix for details.

[13] Here we have also used the properies of the matching function $\frac{p(\theta(z))}{q(\theta(z))} = \theta$.

of job opportunities today compared to $\bar{a}$ periods ago thanks to the growth of the leading edge technology at rate γ.[14]

Second, one can provide an interpretation in terms of the wage $w(a)$. Using (18) in the wage equation (14) it is now immediate to derive that $w(\bar{a}) = e^{(\phi-\gamma)\bar{a}}$. In other words, at age $\bar{a}$ all output is claimed by the worker as wage bill and no more profits can be generated, so the job is destroyed endogenously.

It is important to remark that (18) implies that in order to have a meaningful economic problem, we need to assume throughout that $\gamma > \phi$. If $\gamma \leq \phi$, then jobs are never destroyed endogenously, as the surplus will rise with age (instead of declining as usual in this class of models) because the value of unemployment falls faster than output, due to skill obsolescence.[15]

Now that we have derived a solution for the surplus function, we can show that the model can be reduced to two equations into the pair of unknowns $(\theta, \bar{a})$. The first equation, the job creation condition, is derived from free entry in equilibrium; the second equation, the job destruction condition, is derived from the optimal separation rule for a match.

3.2 Job Creation condition

The first equation follows directly from the free entry condition $V(z) = 0$, which implies $J(0) = c/q(\theta)$. Using $J(a,z) = (1-\beta) S(a,z)$ and the definition of the surplus in (17) evaluated at $a = 0$, together with the destruction rule –that we use to eliminate $\omega(\theta)$ from (**??**)–we arrive at

$$\frac{c}{(1-\beta) q(\theta)} = \int_0^{\bar{a}} \left[1 - e^{-(\gamma-\phi)(\bar{a}-\tilde{a})}\right] e^{-(r+\delta+\rho-\lambda)\tilde{a}} d\tilde{a} \tag{19}$$

which characterizes the optimal entry decisions of firms in each of the z markets. Notice that it is an equation in both $\bar{a}$ and θ and that it is positively sloped in the $(\theta, \bar{a})$ space: $q'(\theta) < 0$ and the right-hand-side of the expression above is increasing in $\bar{a}$. To see that, solve explicitly the right-hand-side to obtain

$$\begin{aligned} S(0;\bar{a}) &= \frac{1}{r+\delta+\rho-\lambda}\left(1 - e^{-(r+\delta+\rho-\lambda)\bar{a}}\right) \\ &\quad - \frac{1}{r+\delta+\rho-\lambda-\gamma+\phi}\left[e^{-(\gamma-\phi)\bar{a}} - e^{-(r+\delta+\rho-\lambda)\bar{a}}\right] \end{aligned} \tag{20}$$

[14]Note that since learning by doing increases both the productivity in the current job and the outside option of the employed worker, it cancels out and does not feature in the job destruction condition (18).

[15]Jobs would still be destroyed endogeneously if the worker had a positive return from nonparticipating which is independent of her skill level z. The model would generate direct flows from employment to out-of-the labor force. This is discussed in Vanhala and Violante (2005) in detail.

and differentiate wrt. $\bar{a}$, which gives

$$\frac{\partial S(0;\bar{a})}{\partial \bar{a}} = \frac{1}{r+\delta+\rho-\lambda-\gamma+\phi}(\gamma-\phi)e^{-(\gamma-\phi)\bar{a}}\left[1-e^{-[r+\delta+\rho-\lambda-(\gamma-\phi)]\bar{a}}\right] > 0,$$

thus the surplus at age zero is increasing in the destruction threshold $\bar{a}$. The intuition for why the job creation curve is positively sloped is simple: the surplus function is increasing in $\bar{a}$ thus an increase in $\bar{a}$ makes the marginal job created profitable, and more vacancies will be opened to restore the zero-profit condition (and reduce the meeting probability for the firm), which will increase θ.[16]

Let us study the behavior of the curve as $\theta \to 0$ (i.e. $q(\theta) \to \infty$): the left-hand-side of (19) goes to zero, and the equation states that $S(0;\bar{a}) = 0$, i.e. $\bar{a}^{\min} = 0$. In other words, as the hiring friction disappears, and the firm instantaneously finds new workers, machines will be constantly updated.[17] To see what happens as $\bar{a} \to \infty$, use the solution for $S(0;\bar{a})$ in (20) to obtain

$$\frac{c}{(1-\beta)q(\theta^{\max})} = \frac{1}{r+\delta+\rho-\lambda} \Rightarrow \theta^{\max} = q^{-1}\left(\frac{c(r+\delta+\rho-\lambda)}{1-\beta}\right).$$

In other words, even if the job tenure is infinitely long, the firm needs a minimum meeting rate to recoup the flow vacancy cost c. Figure 2 shows the job creation curve in the $(\theta,\bar{a})$ space.

3.3 Job Destruction condition

The outside option in the optimal separation condition (18) is equal to the value of unemployment for an employed worker and may be expressed in terms of the match surplus. The separation rule may thus be expressed as[18]

$$e^{-(\gamma-\phi)\bar{a}} = b + p(\theta)\beta S(0,\bar{a}), \tag{21}$$

Substituting the expression for the surplus (17) evaluated at $a = 0$, and using (18) we obtain the job destruction condition

$$e^{-(\gamma-\phi)\bar{a}} = b + p(\theta)\beta\int_0^{\bar{a}}\left[1-e^{-(\gamma-\phi)(\bar{a}-\tilde{a})}\right]e^{-(r+\delta+\rho-\lambda)\tilde{a}}d\tilde{a}, \tag{22}$$

[16] A second reason often cited in the literature, namely that a longer job duration increases the length over which a positive flow surplus accrues to the pair is not true. This is because $\bar{a}$ is chosen optimally by the firm, and by the envelope theorem this effect is zero.

[17] Recall that in this model the cost of a new machine is zero, but scrapping an old machine to buy a new one implies separation from the worker. Hence insofar as looking for a new worker takes time, upgrading is costly.

[18] See appendix for details.

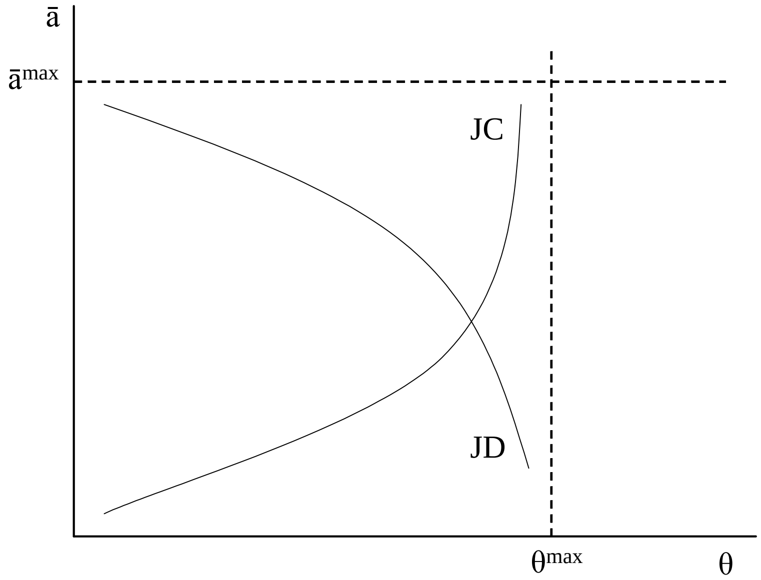


Figure 2: Job creation and destruction

which traces a negatively sloped curve in the $(\theta, \bar{a})$ space. The intuition for the negative slope is that a rise in $\bar{a}$ reduces the productivity of the marginal job, but it also raises the value of unemployment, as the value of search rises, thus for the destruction rule to be restored, the workers' meeting rate has to fall, so θ has to fall.

Consider the situation where $\theta \to 0$ (i.e. $p(\theta) \to 0$), then we have an upper bound for the destruction age

$$\bar{a}^{\max} = -\frac{\ln b}{\gamma - \phi}$$

which is a positive number since $b < 1$.[19] In other words, when the worker's meeting rate is extremely small, the option value of search does not affect the joint destruction decision, only the welfare benefit does.

3.4 Equilibrium Unemployment

In steady state unemployment is constant and the flow of job creation must equal the flow of job destruction. Job creation is given by the flow of new matches

$$JC = m(u, v) = \int_0^{\bar{z}} m(u(z), v(z))\, dz.$$

[19] This has to be true in a viable labor market where the normalized productivity of the best vintage is 1.

Jobs are destroyed either endogenously or exogenously

$$JD = \delta \left(1 - \int_0^{\bar{z}} u(z)\, dz\right) + e^{-\delta \bar{a}} \int_0^{\bar{z}} m(u(z), v(z))\, dz$$

where we have normalized the labor force to one. A fraction δ of employed workers become unemployed due to exogenous job destruction. A fraction $e^{-\delta \bar{a}}$ of matches survive until the age of obsolescence and are then destroyed endogenously.

To derive the aggregate unemployment rate denote $u = u(z), v = v(z)$ and $m(u, v) = \int_0^{\bar{z}} m(u(z), v(z))\, dz$. In steady state $JC = JD$ so we have

$$m(u, v) = \delta (1 - u) + e^{-\delta \bar{a}} m(u, v) \tag{23}$$

where $1 - u = e$. Rearranging produces

$$p(\theta)\, u \left(1 - e^{-\delta \bar{a}}\right) = \delta (1 - u) \tag{24}$$

where $p(\theta)\, u = m(u, v)$ by the properties of the matching function. Rearranging gives the equilibrium unemployment rate

$$u = \frac{\delta}{\delta + p(\theta)\left(1 - e^{-\delta \bar{a}}\right)} \tag{25}$$

The equilibrium unemployment rate is decreasing in age $\bar{a}$ and labor market tightness θ. This is intuitive as higher $\bar{a}$ implies lower unemployment incidence and higher θ implies shorter unemployment duration.

4 Equilibrium Comparative Statics

There are two key technological parameters in the model, the speed of technical change γ which measures the rate of capital obsolescence and the rate of skill obsolescence ϕ. Recall that the parameter ϕ should be interpreted as a measure of specificity of skills: a large value of ϕ implies vintage specific skills, whereas a low value of ϕ means general skills that are largely transferable to other job vintages. We start by the simplest case, whereby ϕ is independent of γ. Alternatively, one can reasonably conjecture that ϕ and γ are related by the function $\phi(\gamma)$, with $\phi' > 0$. In other words, transferability is decreasing in the productivity growth parameter γ as, for example, in Galor and Moav (2000), Violante (2002), and Gould, Moav and Weinberg (2001).[20] We proceed by first examining the effect of the technological parameters on market tightness and the destruction age, then we consider the effects on equilibrium unemployment.

[20]We think of γ as capturing the degree to which new features of the technologies are embodied in capital, and therefore the extent to which capital is different across vintages. Note that γ is often measured through quality-adjusted relative prices, consistently with this view. This effect is called in the literature the "human capital erosion effect" due to faster growth .

4.1 The Effects on $(\theta, \bar{a})$

The impact of γ– Let's start with the effect of the rate of technical change γ on the surplus function $S(0, \bar{a})$. Differentiating (20), we obtain

$$\frac{\partial S(0,\bar{a})}{\partial \gamma} = \frac{e^{-(\gamma-\phi)\bar{a}}\left\{[r+\delta+\rho-\lambda-(\gamma-\phi)]\,\bar{a}+e^{-(r+\delta+\rho-\lambda-\gamma+\phi)\bar{a}}-1\right\}}{(r+\delta+\rho-\lambda-\gamma+\phi)^2} > 0$$

where the inequality follows from the fact that

$$\frac{1-e^{-(r+\delta+\rho-\lambda-\gamma+\phi)\bar{a}}}{r+\delta+\rho-\lambda-\gamma+\phi} = \int_0^{\bar{a}} e^{-(r+\delta+\rho-\lambda-\gamma+\phi)x}dx < \bar{a}$$

as the argument of the integral is strictly less than 1 everywhere on the domain $[0, \bar{a}]$. Therefore, the job creation curve (19) shifts downward: for a given contact rate θ, the faster technical change increases the value of the surplus, thus $\bar{a}$ has to fall to rebalance the job creation condition.

It is easy to see that the job destruction curve (22) shifts downwards as well: the rise in the surplus raises the value of unemployment, for given θ. At the same time, the marginal value of a job $e^{-\gamma\bar{a}}$ falls, thus the meeting probability has to decline as well to rebalance this condition. In conclusion, we have an unambiguous rise in unemployment incidence ($\bar{a}$ decreases) but ambiguous effects on unemployment duration (expected unemployment duration is given by $1/p(\theta)$).

The impact of ϕ– Consider now a change in the transferability parameter ϕ. From the surplus function

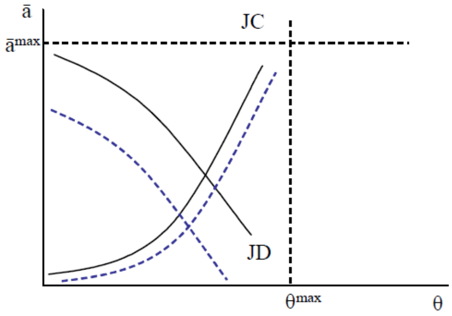
$$\frac{\partial S(0,\bar{a})}{\partial \phi} = \frac{-e^{-(\gamma-\phi)\bar{a}}\left\{[r+\delta+\rho-\lambda-\gamma+\phi]\,\bar{a}+e^{-(r+\delta+\rho-\lambda-\gamma+\phi)\bar{a}}-1\right\}}{(r+\delta+\rho-\lambda-\gamma+\phi)^2} < 0,$$

thus following the same logic we have used above, it is easy to show that both curves shift upward, leading to a rise in $\bar{a}$ (and a fall in unemployment incidence) and ambiguous effects on unemployment duration.

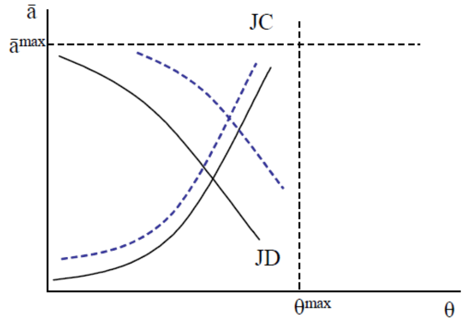
The impact of γ **when** ϕ **depends on** γ– The derivative of the surplus becomes

$$\frac{\partial S(0,\bar{a})}{\partial \gamma} = \frac{[1-\phi'(\gamma)]\,e^{-(\gamma-\phi)\bar{a}}\left\{[r+\delta+\rho-\lambda-(\gamma-\phi(\gamma))]\,\bar{a}+e^{-[r+\delta+\rho-\lambda-(\gamma-\phi(\gamma))]\bar{a}}-1\right\}}{[r+\delta+\rho-\lambda-\gamma+\phi(\gamma)]^2}$$

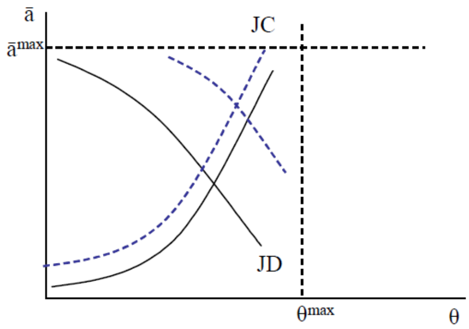
which is positive when $\phi'(\gamma) < 1$ and negative when $\phi'(\gamma) > 1$. For example, suppose that $\phi(\gamma) = \phi\gamma$. Then the assumption $\phi(\gamma) < \gamma$ implies that $\phi < 1$ and $\phi'(\gamma) < 1$. Thus, when ϕ is linear, the comparative statics are like in the case where



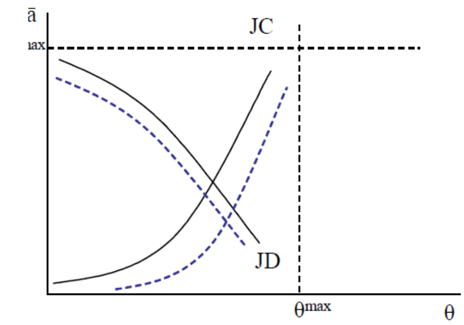
Technological acceleration γ



Skill depreciation Φ



Technological acceleration, when Φ'(γ) > 1



Learning by doing, λ

ϕ is independent of γ. However, when nonlinearities are present and $\phi'(\gamma) > 1$, we can have a situation where $\frac{\partial S(0,\bar{a})}{\partial \gamma} < 0$ and a rise in γ has the same comparative statics as a rise in ϕ. In this case the specificity of skills increases and consequently the transferability of skills is reduced with the rate of technical change.

The impact of λ**–** Finally consider a change in the learning rate λ. The derivative of the surplus function is

$$\begin{aligned}\frac{\partial S(0;\bar{a})}{\partial \lambda} &= \frac{-\bar{a}e^{-(r+\delta-\lambda)\bar{a}}(r+\delta+\rho-\lambda)+1-e^{-(r+\delta+\rho-\lambda)\bar{a}}}{(r+\delta+\rho-\lambda)^2} \\ &\quad -e^{-(\gamma-\phi)\bar{a}}\times \\ &\quad \frac{1-e^{-[r+\delta+\rho-\lambda-(\gamma-\phi)]\bar{a}}-\bar{a}e^{-[r+\delta+\rho-\lambda-(\gamma-\phi)]\bar{a}}[r+\delta+\rho-\lambda-(\gamma-\phi)]}{[r+\delta+\rho-\lambda-(\gamma-\phi)]^2}\end{aligned}$$

which is easily seen to be positive. Again, following the same logic we have used above, we can show that both the job creation and destruction curves shift downwards. Unemployment incidence increases ($\bar{a}$ decreases) and the effect on unemployment duration is ambiguous.

4.2 The Effects on Equilibrium Unemployment

The job creation and destruction conditions derived above produce unambiguous comparative statics for $\bar{a}$ and unemployment incidence. However the comparative statics with respect to θ and unemployment duration remain ambiguous. To determine the effects of (γ, ϕ, λ) on θ express the equilibrium job creation condition $c/q(\theta) = (1-\beta) S(0)$ as

$$\frac{c}{q(\theta)} = (1-\beta) \int_0^{\bar{a}} \left[1 - e^{(\gamma-\phi)\tilde{a}} \left(b + \frac{\beta}{1-\beta} c\theta\right)\right] e^{-(r+\delta+\rho-\lambda)\tilde{a}} d\tilde{a}. \qquad (26)$$

This equation allows us to perform simply the comparative statics on θ since it is (locally) independent of $\bar{a}$ due to the envelope theorem. It is therefore easy to show that

$$\frac{d\theta}{d\gamma} < 0, \qquad \frac{d\theta}{d\phi} > 0, \qquad \frac{d\theta}{d\lambda} > 0.$$

Thus we conclude that an increase in the rate of technical change γ raises both unemployment incidence ($\bar{a}$ decreases) and unemployment duration (θ decreases), and the equilibrium unemployment rate unambiguously increases. An increase in the transferability parameter ϕ has the opposite effect: both unemployment incidence and unemployment duration decrease ($\bar{a}$ and θ both increase), and equilibrium unemployment decreases. Finally, an increase in the learning rate λ increases unemployment incidence by reducing $\bar{a}$, and reduces unemployment duration by increasing θ with ambiguous effect on the equilibrium unemployment rate.

To see the impact of γ when ϕ depends on γ, it is enough to recognize from (26) that $\frac{d\theta^*}{d\gamma}$ and $\frac{d\theta^*}{d\phi}$ are equal in absolute value. When $\phi'(\gamma) > 1$, the indirect effect of γ through skill obsolescence dominates and a technological acceleration reduces unemployment. When $\phi'(\gamma) < 1$ the direct effect of γ through technological obsolescence dominates and we have the standard result that a technological acceleration increases unemployment.

These results have a very intuitive interpretation. A higher rate of technical change increases the value of frontier jobs relative to older vintages. In the standard vintage capital-search model this increases the worker's outside option. The consequent higher wage demands of workers and less profitable firms leads to less job creation and more job destruction, leading ultimately to higher equilibrium unemployment. The skill obsolescence effect counteracts this mechanism, by reducing the outside value of the worker relative to the current job. This is because skills are not fully transferable to new jobs. When the skill obsolescence effect dominates, workers settle for lower wages, which implies more profitable firm's and ultimately lower equilibrium unemployment.

We conclude that the comparative static results of the standard vintage capital are reversed when skill depreciation is fast enough. In addition to the capital obsolescence (or creative destruction) effect that tends to raise unemployment, vintage human capital introduces a skill obsolescence effect of faster growth that has the opposite sign. Faster skill obsolescence reduces the value of unemployment, hence wages and leads to more job creation and less job destruction, unambiguously reducing unemployment.

5 Nonparticipation

THIS SECTION IS PRELIMINARY AND INCOMPLETE

In the presence of an income from home-production, or a welfare payment unconditional on search, the model implies that some workers will choose optimally to exit the labor force because their skills have depreciated so much that their value of searching is below what is offered as non-participants. Murphy and Topel (1998) show that in synchrony with the rise of inequality, a larger fraction among the low-skilled agents have left the labor force for good. They have argued that the same labor demand shift is responsible for both. In our model a rise in the productivity growth of new capital increases the rate of obsolescence of skill, so it might potentially lead in equilibrium to a larger fraction of agents who are "discouraged" and quit. We characterize nonparticipation associated to skill depreciation for various cases.

5.1 Nonparticipation when $\gamma > \lambda > \phi$

We first characterize nonparticipation for a simple case where the value of nonparticipation is always below that of unemployment for employed workers, in other words the relevant threat point for the employed worker in the bargaining process is still $U(z) > N$. A sufficient condition for this to happen is $\lambda > \phi$ i.e. the skill level of an employed worker, expressed in terms of her productivity on the newest technology grows with tenure: in this case unemployed workers would start their jobs from levels of z above z^* and their skill would always remain above that level as long as they are employed.

Then, the only relevant thing to consider is the threshold age/skill level z^* at which the unemployed worker switches to nonparticipation. There will be a level z^* such that $(r-\gamma)N = (r-\gamma)U(z^*)$. From equation (11) and the surplus splitting Nash rule, we obtain

$$\eta = \frac{(r-\gamma)\, z^*}{r-\gamma+\phi} p(\theta)\, \beta S(0;\bar{a}) \Rightarrow z^* = \frac{r-\gamma+\phi}{r-\gamma} \frac{\eta}{p(\theta)\, \beta S(0;\bar{a})}$$

which using the job destruction condition (22) implies

$$z^* = \frac{r-\gamma+\phi}{r-\gamma} \eta e^{(\gamma-\phi)\bar{a}} = \frac{r-\gamma+\phi}{r-\gamma} \eta \frac{1-\beta}{\beta c \theta} \tag{27}$$

Here we have

$$\frac{\partial z^*}{\partial \gamma} > 0$$

as $\frac{\partial \theta}{\partial \gamma} < 0$. The threshold skill level z^* for participation increases with the rate of technical change: faster technical change reduces job duration and the number of jobs so the threshold participation level increases. In other words, when jobs last less and they are fewer, the value of unemployment falls and so the threshold level to opt out of the labour market increases.

The effect of skill obsolescence on the threshold skill level z^* is

$$\frac{\partial z^*}{\partial \phi} \gtreqless 0.$$

$\frac{\partial z^*}{\partial \phi}$ is ambiguous as an increase in ϕ has a direct and an indirect (general equilibrium) effect. First, the direct effect of an increase in ϕ makes skills get obsolete faster and thus z^* increases. Second, an increase in ϕ increases θ and this indirect effect implies more job creation and therefore z^* decreases (higher job creation increases the value of unemployment).

In general, the unemployment rate is not affected by z^* as

$$p(\theta)\, u(z) = \delta \left[l(z) - u(z) \right] + p(\theta)\, u(z)\, e^{-\delta \bar{a}}$$

where $l(z)$ is the measure of labor with skills z. We have

$$p(\theta)\, u = \delta \left[l - u \right] + p(\theta)\, u e^{-\delta \bar{a}}$$

which produces

$$u = \frac{\delta}{\delta + p(\theta) \left[1 - e^{-\delta \bar{a}} \right]} \tag{28}$$

i.e. the equilibrium unemployment rate does not depend on z^*. The measure of nonparticipants is

$$n = \int_0^{z^*} l(z)\, dz. \tag{29}$$

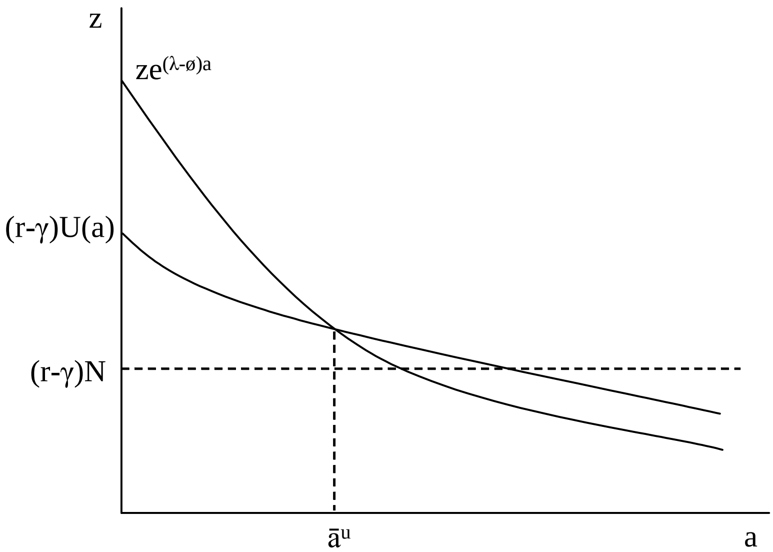


Figure 3: Outside option is U.

5.2 Nonparticipation when $\gamma > \phi > \lambda$

We now proceed to the consider the situation $\phi > \lambda$, where skill depreciation ϕ is faster than learning on the job λ.Now the workers outside option and participation decision depends on the initial skills of a worker. We have three possible cases.

In the first case, case U, unemployment is the outside option of the worker for the whole duration of the job. This requires that the initial skills of the worker at the starting time of the match are above a threshold level, such that even accounting for the depreciation of skills during the match (by $\lambda - \phi$) the value of unemployment remains higher than the value of nonparticipation $(r - \gamma) N = \eta$. We denote this threshold initial skill level by z^U and the duration of a job in this case as $\bar{a}^U$. (figure 3)

In the second case, case N, the outside option switches from $U(a)$ to N during the lifetime of the job. This situation arises when the initial skills of the worker are lower than z^U but high enough that the worker decides to participate (unemployment is the outside option at the time of meeting a firm) and can thus find a job. We denote the participation threshold by z^N and the duration of a job in this case as $\bar{a}^N$.(figure 4)

The third case arises when the workers skills are below z^N. In this case the workers skills are so low that the payoff from participating to the labor market is too low for the worker to seek a job. Therefore, when the workers skills are below

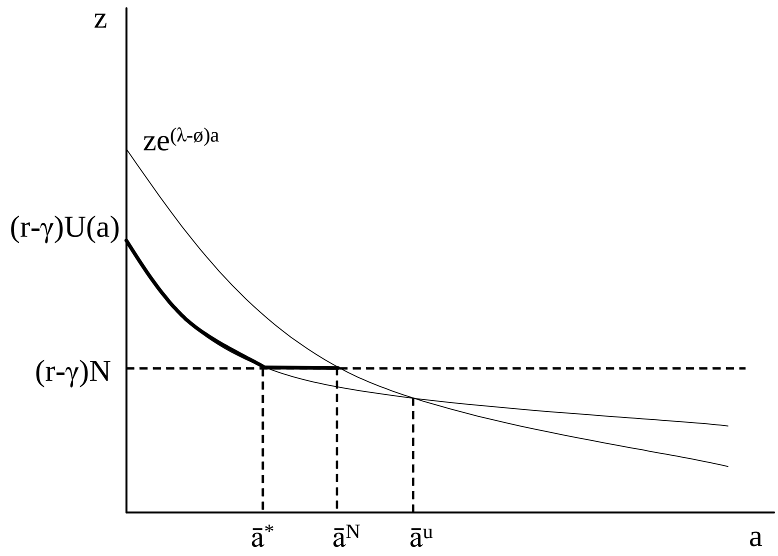


Figure 4: Outside option switches from U to N.

z^N, the worker chooses nonparticipation with payoff $(r-\gamma)N=\eta$.

We can express z^U as

$$z^U = z^N e^{(\phi-\lambda)\bar{a}^U}$$

since a match started at skill level z^U can last until $\bar{a}^U$ without the outside option changing. As stated above, z^U is the minimum skill level of an unemployed worker such that if she finds a job, unemployment is the outside option for the whole duration of the job.

TO BE COMPLETED

(SEE APPENDIX FOR PRELIMINARY SKETCH OF DERIVATIONS)

6 Concluding Remarks

Earlier models on growth and unemployment (e.g. Aghion and Howitt 1994, Mortensen and Pissarides 1999) have focused on the 'creative destruction'-nature of technical change that takes place as new capital vintages replace old ones. The capital obsolescence effect present in these models governs job creation and destruction, as jobs are destroyed along with the obsolete capital they are matched with and new jobs are created at the technological frontier where new capital is introduced. This paper extends the standard vintage capital/search model to incorporate *vintage human capital* by introducing skill obsolescence. The novel feature of the present model is that workers skills are two-dimensional: skill accumulation and depreciation take place simultaneously. During the lifetime of a job workers accumulate skills that are relevant to the capital they currently operate, but these skills are only partially transferable to jobs of more recent capital vintages. Therefore, as job tenure increases so does the distance to the technological frontier where new jobs are created.

In addition to the capital obsolescence (or creative destruction) effect that tends to raise unemployment, vintage human capital introduces a skill obsolescence effect of faster growth that has the opposite sign. Faster skill obsolescence reduces the value of unemployment, hence wages and leads to more job creation and less job destruction, unambiguously reducing unemployment.

In the (preliminary and incomplete) second part of the paper we study the labor force participation decision of workers when there is an income from home production, or a welfare payment unconditional on search. Workers with sufficiently depreciated skills will choose optimally to exit the labor force if the payoff of searching for a job is below the value of nonparticipation. The participation decision depends on the threat point of the employed worker, whether it is unemployment or nonparticipation. Also, the outside option may switch from unemployment to nonparticipation during the lifetime of a job if skills depreciate in time. We characterize the impact of capital and skill obsolescence on non-participation decisions for various cases.

In addition to unemployment and participation, the search model with vintage human capital presented in this study provides a framework to study questions pertaining to the impact of capital-embodied technological change on other important labor market equilibrium outcomes such as: (i) the *skill distribution*, and consequently the *inequality in the wage distribution*. The rise in US inequality has been linked by a number of authors to a productivity acceleration. This model suggests that vintage human capital provides an important theoretical link between productivity growth and wage distribution, through the equilibrium skill distribution; (ii) the impact of *retraining policies*. The model can allow to evaluate the impact of policies that retrain the worker by moving him upward on the skill ladder on aggregate output.

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A Appendix

A.1 Derivation of a typical value equation

A typical derivation of the value equations goes as follows. Consider the value of a vacant job. The strategy is to write the functions in discrete time, with interval length dt and take the limit as dt approaches zero.

$$\begin{aligned} V(t,z) &= \max\left\{\frac{1}{1+rdt}\left[\int_t^{t+dt} -y(x,z)\,c\,dx + q(\theta)\,dtJ(t+dt,t+dt,z)\right.\right. \\ &\quad \left. + (1-q(\theta)\,dt)\,V(t+dt,z)\right],0\} \end{aligned}$$

$$\begin{aligned} (1+rdt)\,V(t,z) &= \max\left\{\left[\int_t^{t+dt} -y(x,z)\,c\,dx + q(\theta)\,dtJ(t+dt,t+dt,z)\right.\right. \\ &\quad \left. + (1-q(\theta)\,dt)\,V(t+dt,z)\right],0\} \end{aligned}$$

Multiplying both sides by $(1+rdt)$, subtracting $V(t,z)$ from both sides and dividing by dt, we obtain

$$\begin{aligned} rV(t,z) &= \max\left\{\left[\frac{\int_t^{t+dt} -y(x,z)\,c\,dx}{dt} + q(\theta)\left[J(t+dt,t+dt,z) - V(t+dt,z)\right]\right.\right. \\ &\quad \left.\left. + \frac{V(t+dt,z)-V(t,z)}{dt}\right],0\right\} \end{aligned} \tag{30}$$

where

$$\frac{\int_t^{t+dt} -y(x,z)\,c\,dx}{dt} = \frac{-\int_t^{t+dt} e^{\gamma x}zc\,dx}{dt} = \frac{-e^{\gamma t}zc}{\gamma}\left(\frac{e^{\gamma dt}-1}{dt}\right).$$

We now take limits as $dt \to 0$ for each component of (30). To begin with,

$$\lim_{dt\to 0}\left(\frac{-e^{\gamma t}zc}{\gamma}\right)\left(\frac{e^{\gamma dt}-1}{dt}\right) = \frac{-e^{\gamma t}zc}{\gamma}\gamma = -e^{\gamma t}zc = -y(t,z),$$

where we have made use of De L'Hopital rule for the term $\left(\frac{e^{\gamma dt}-1}{dt}\right)$. Moreover,

$$\lim_{dt\to 0}\left[J(t+dt,t+dt,z) - V(t+dt,z)\right] = J(t,t,z) - V(t,z).$$

Furthermore, we have

$$V_1(t,z) = \lim_{dt\to 0}\frac{V(t+dt,z)-V(t,z)}{dt}.$$

Therefore, $rV(t,z)$ in (30) simplifies to

$$rV(t,z) = \max\left\{\left[-y(t,z) + q(\theta)\left[J(t,t,z) - V(t,z)\right] + V_1(t,z)\right],0\right\}.$$

A.2 Wage equation

The wage is given by the Nash bargaining solution and maximizes

$$w(a,z) = \arg\max \left[J(a,z) - V(z)\right]^{1-\beta} \left[W(a,z) - U(z)\right]^{\beta}. \tag{31}$$

Observe that in this condition $U(z)$ is the value of unemployment for an employed worker and is thus affected by both learning by doing λ and skill depreciation ϕ. Therefore the law of motion for $U(z)$ in (31) is $\dot{z} = (\lambda - \phi) z$.

The first order condition, given $V(z) = 0$, is

$$\begin{aligned} & (1-\beta) \frac{\partial J(a,z)}{\partial w(a,z)} \left[J(a,z) - V(z)\right]^{-\beta} \left[W(a,z) - U(z)\right]^{\beta} \\ & +\beta \frac{\partial W(a,z)}{\partial w(a,z)} \left[J(a,z) - V(z)\right]^{1-\beta} \left[W(a,z) - U(z)\right]^{\beta-1} \\ = & \ 0 \end{aligned} \tag{32}$$

Divide both sides by $\left[J(a,z) - V(z)\right]^{-\beta} \left[W(a,z) - U(z)\right]^{\beta-1}$ and use $\frac{\partial W(a,z)}{\partial w(a,z)} = -\frac{\partial J(a,z)}{\partial w(a,z)}$ to get

$$\beta J(a,z) = (1-\beta)\left[W(a,z) - U(z)\right]. \tag{33}$$

By substitution from equations (9), where $y(a,z) = ze^{(\phi-\gamma)a}$, and (10) and using the free-entry condition $V(z) = 0$, we obtain

$$\begin{aligned} & \beta \left[\frac{ze^{(\phi-\gamma)a} - w(a,z) + J_t(a,z) + \dot{z} J_z(a,z)}{r+\delta+\rho}\right] \\ = & \ (1-\beta)\left[\frac{w(a,z) + \delta U(z) + W_t(a,z) + \dot{z} W_z(a,z)}{r+\delta+\rho} - U(z)\right], \end{aligned} \tag{34}$$

which by cancelling terms and rearranging gives

$$\begin{aligned} w(a,z) = & \ \beta z e^{(\phi-\gamma)a} + (1-\beta)(r+\rho) U(z) \\ & +\beta\left[J_t(a,z) + \dot{z} J_z(a,z)\right] - (1-\beta)\left[W_t(a,z) + \dot{z} W_z(a,z)\right] \end{aligned} \tag{35}$$

Substitute the value equation for unemployment for an employed worker to get

$$\begin{aligned} w(a,z) = & \ \beta e^{\gamma\tau} z e^{\phi(t-\tau)} + (1-\beta)\left\{bze^{\gamma t} + p(\theta(z))\left[W(0,z) - U(z)\right]\right\} \\ & +\beta\left[J_t(a,z) + \dot{z} J_z(a,z)\right] \\ & -(1-\beta)\left[W_t(a,z) + \dot{z} W_z(a,z) - U_t(z) - \dot{z} U_z(z)\right] \end{aligned} \tag{36}$$

As the value of unemployment in the wage bargain is that of an employed worker, observe that all the values $J(a,z), W(a,z)$ and $U(z)$ have the law of motion $\dot{z} = (\lambda - \phi) z$.

Using the first order condition (33) to cancel the derivative terms we obtain

$$w(a,z) = \beta z e^{(\phi-\gamma)a} + (1-\beta)\{bz + p(\theta(z))[W(0,z) - U(z)]\} \tag{37}$$

By the first order condition $W(0,z) - U(z) = \frac{\beta}{1-\beta} J(0,z)$, and $V(z) = 0$ in (8) implies $J(0,z) = \frac{zc}{q(\theta)}$. Substituting into (37) and using the properties of the matching function the wage equation reduces to

$$w(a,z) = \beta z e^{(\phi-\gamma)a} + (1-\beta) z \left(b + \frac{\beta}{1-\beta} c\theta \right). \tag{38}$$

A.3 Match Surplus

To derive the equations $(8) - (11)$ given in the text, substitute (2), (3) and (14) into the value equations $(4) - (7)$ and stationarize the equations. Along the balanced growth path all the values above grow at rate γ. Let $a = t - \tau$ denote the age of the production unit. Hence, it follows that $V(t,z) = e^{\gamma t} V(z)$, with $V_t(t,z) = \gamma e^{\gamma t} V(z)$; $J(t,\tau,z) = e^{\gamma t} J(a,z)$, with $J_t(t,\tau,z) = \gamma e^{\gamma t} J(a,z) + e^{\gamma t} J_a(a,z)$; $W(t,\tau,z) = e^{\gamma t} W(a,z)$, with $W_t(t,\tau,z) = \gamma e^{\gamma t} W(a,z) + e^{\gamma t} W_a(a,z)$; $U(t,z) = e^{\gamma t} U(z)$, with $U_t(t,z) = \gamma e^{\gamma t} U(z)$. Finally, we have $w(t,\tau,z) = e^{\gamma t} w(a,z) = e^{\gamma t} z w(a)$.

The balanced growth path versions of the value equations are (after some manipulation)

$$(r-\gamma) V(z) = \max\{-zc + q(\theta(z))[J(0,z) - V(z)], 0\} \tag{39}$$

$$\begin{aligned}(r-\gamma) J(a,z) \quad = \quad & \max\Big\{e^{-\gamma a} z e^{\phi a} - w(a) z - (\delta+\rho)\Big[J(a,z) - \max_z V(z)\Big] \\ & + J_a(a,z) + (\lambda-\phi) z J_z(a,z), (r-\gamma)\max_z V(z)\Big\}\end{aligned} \tag{40}$$

$$\begin{aligned}(r-\gamma) W(a,z) \quad = \quad & \max\{w(a) z - \delta[W(a,z) - U(z)] - \rho W(a,z) \\ & + W_a(a,z) + (\lambda-\phi) z W_z(a,z), (r-\gamma) U(z)\}\end{aligned} \tag{41}$$

$$\begin{aligned}(r-\gamma) U(z) \quad = \quad & \max\{bz + p(\theta(z))[W(0,z) - U(z)] - \rho U(z) \\ & -\phi z U_z(z), 0\}\end{aligned} \tag{42}$$

Substituting the value equations $(40), (41)$ and the value of unemployment for an *employed* worker (with the law of motion $\dot{z} = (\lambda - \phi) z$) into (12), using $V(z) = 0$ and the definition of surplus produces

$$\begin{aligned}(r-\gamma+\delta+\rho) S(a,z) \quad = \quad & \max\{e^{(\phi-\gamma)a} z - bz - p(\theta(z))[W(0,z) - U(z)] \\ & + J_a(a,z) + (\lambda-\phi) z J_z(a,z) \\ & + W_a(a,z) + (\lambda-\phi) z W_z(a,z) \\ & - (\lambda-\phi) z U_z(z), 0\}.\end{aligned} \tag{43}$$

Use the the Nash first order condition to substitute $[W(0,z)-U(z)]=\beta S(0,z)$ and definition of surplus to combine the derivative terms to obtain

$$\begin{aligned}(r-\gamma+\delta+\rho)S(a,z) &= \max\left\{e^{(\phi-\gamma)a}z-bz-p(\theta(z))\beta S(0,z)\right. \\ &\quad \left.+S_a(a,z)+(\lambda-\phi)zS_z(a,z)\right\}.\end{aligned} \tag{44}$$

To see that (44) is a first order differential equation as a function of t, observe that $S(a,z)=S(a(t),z(t))$ and consequently we can express the derivatives $S_a(a,z)+(\lambda-\phi)zS_z(a,z)=\frac{dS(a(t),z(t))}{dt}$, recall that $\dot{z}=(\lambda-\phi)z$. Furthermore, use the Nash condition $S(0,z)=\frac{1}{1-\beta}J(0,z)$ and the free entry condition $V(z)=0$ in the value equation for $J(0,z)$ to get $J(0,z)=\frac{cz}{q(\theta(z))}$to obtain

$$(r-\gamma+\delta+\rho)S(a,z)-\frac{dS(a,z)}{dt}=\max\left\{e^{(\phi-\gamma)a}z-bz-\frac{\beta}{1-\beta}cz\theta(z),0\right\} \tag{45}$$

where we have used the properties of the matching function to produce $\frac{p(\theta(z))}{q(\theta(z))}=\theta(z)$. This equation is a first order differential equation for the surplus as a function of t.

Note that z represents the skills of a worker on the current leading edge technology, in terms of which the unemployment compensation b and search cost c are expressed. However, as the worker in a match of age a operates capital of that age, z is augmented by the factor $e^{\phi a}$ in the first right-hand-side term.

The max operator implies that we have a boundary condition $S(\bar{a},z)=0$. The general solution for the differential equation, using $\omega(\theta)z=bz+\frac{\beta}{1-\beta}cz\theta(z)$, for the surplus is

$$\begin{aligned}S(a,z) &= Ae^{-(r-\gamma+\delta+\rho)a}\times \\ &\quad +\int_a^{\bar{a}}\left[e^{-\gamma\tilde{a}}e^{\phi a}ze^{\lambda(\tilde{a}-a)}-\omega(\theta)ze^{(\lambda-\phi)(\tilde{a}-a)}\right]e^{-(r-\gamma+\delta+\rho)(\tilde{a}-a)}d\tilde{a}\end{aligned}$$

Note that in the first term in the integral, worker productivity is constant over time $(e^{\phi a}z)$, except for learning by doing from age a onwards $\left(e^{\lambda(\tilde{a}-a)}\right)$. In the second term in the integral, $e^{(\lambda-\phi)(\tilde{a}-a)}$ accounts for the workers skills depreciating at rate ϕ relative to the newest technology and accumulating by learning by doing at rate λ from age a onwards, from the initial level z. Divide and multiply by $e^{(\lambda-\phi)(\tilde{a}-a)}$ to get

$$\begin{aligned}S(a,z) &= Ae^{-(r-\gamma+\delta+\rho)a}\times \\ &\quad +\int_a^{\bar{a}}\left[e^{-\gamma\tilde{a}}e^{\phi a}ze^{\lambda(\tilde{a}-a)}e^{-(\lambda-\phi)(\tilde{a}-a)}-\omega(\theta)z\right]e^{-(r-\gamma+\delta+\rho+\phi-\lambda)(\tilde{a}-a)}d\tilde{a}\end{aligned}$$

and cancel terms

$$\begin{aligned} S(a,z) &= Ae^{-(r-\gamma+\delta+\rho)a} \times \\ & + \int_a^{\bar{a}} \left[e^{(\phi-\gamma)\tilde{a}} z - \omega(\theta) z \right] e^{-(r-\gamma+\delta+\rho+\phi-\lambda)(\tilde{a}-a)} d\tilde{a} \end{aligned}$$

Using the boundary condition $S(\bar{a},z) = 0$ implies that we must have $A = 0$ as the integral term in the differential equation is equal to zero at $\bar{a}$. Therefore after rearranging, we obtain,

$$S(a,z) = e^{(\phi-\gamma)a} z \int_a^{\bar{a}} \left[e^{(\phi-\gamma)(\tilde{a}-a)} - \omega(\theta) e^{-(\phi-\gamma)a} \right] e^{-(r-\gamma+\delta+\rho+\phi-\lambda)(\tilde{a}-a)} d\tilde{a}$$

Divide and multiply by $e^{(\phi-\gamma)(\tilde{a}-a)}$ to get

$$S(a,z) = e^{(\phi-\gamma)a} z \int_a^{\bar{a}} \left[1 - \omega(\theta) e^{-(\phi-\gamma)\tilde{a}} \right] e^{-(r+\delta+\rho-\lambda)(\tilde{a}-a)} d\tilde{a}. \tag{46}$$

A.3.1 Job Creation

Using the Nash first order condition and the free entry condition we obtain the job creation condition from (39)

$$\frac{c}{q(\theta)} = e^{(\phi-\gamma)a} \int_a^{\bar{a}} \left[1 - \omega(\theta) e^{-(\phi-\gamma)\tilde{a}} \right] e^{-(r+\delta+\rho-\lambda)(\tilde{a}-a)} d\tilde{a}$$

where z cancels out from both sides which is independent of z. This equation implicitly solves θ, and is a function of the destruction age $\bar{a}$ and the parameters of the model. z does not feature in this equation.

Furthermore, using $\omega(\theta) = b + \frac{\beta}{1-\beta} c\theta$ the job destruction condition is given by

$$e^{(\phi-\gamma)\bar{a}} = b + \frac{\beta}{1-\beta} c\theta$$

or

$$\bar{a} = -\frac{1}{\gamma-\phi} \ln\left(b + \frac{\beta}{1-\beta} c\theta \right) \tag{47}$$

Substitute this for $\bar{a}$ in the job creation condition. The resulting equation implicitly solves θ as a function of the parameters of the model and this equation is independent of z.

A.3.2 Job Destruction

The outside option in the optimal separation condition (18) is equal to the value of unemployment for an employed worker

$$\omega(\theta) = b + \frac{\beta}{1-\beta} c\theta = b + \frac{\beta}{1-\beta} p(\theta) \frac{c}{q(\theta)}$$

where the last equality follows the properties of the matching function. Use $J(0,\bar{a}) = \frac{c}{q(\theta)}$ and the Nash first-order condition to get

$$\omega(\theta) = b + p(\theta)\left[W(0,z) - U(z)\right] \tag{48}$$

or

$$\omega(\theta) = b + p(\theta)\beta S(0,\bar{a}). \tag{49}$$

The value equation for an employed is given by

$$(r - \gamma + \rho + \phi - \lambda)U(z) = b + p(\theta)\beta S(0), \tag{50}$$

hence the outside option is equal to the value of unemployment for an employed worker.[21]

Substituting (49) into (18) gives

$$e^{-(\gamma-\phi)\bar{a}} = b + p(\theta)\beta S(0,\bar{a}). \tag{51}$$

Substituting the expression for the surplus (17) evaluated at $a = 0$, and using (18) we obtain the job destruction condition

$$e^{-(\gamma-\phi)\bar{a}} = b + p(\theta)\beta\int_0^{\bar{a}}\left[1 - e^{-(\gamma-\phi)(\bar{a}-\tilde{a})}\right]e^{-(r+\delta+\rho-\lambda)\tilde{a}}d\tilde{a}, \tag{52}$$

A.4 Nonparticipation when $\gamma > \phi > \lambda$

A.4.1 Value equations

The free-entry condition is

$$y(t,z)c = q(\theta(z))J(t,t,z) \tag{53}$$

The value of nonparticipation

$$(r-\gamma)N(t) = y(t)\eta + N_1(t) \tag{54}$$

Notice that the value of nonparticipation is independent of z.

Unemployment

$$\begin{aligned} rU(t,z) &= \max\left\{p(\theta(z))\left[W(t,t,z) - U(t,z)\right] + \frac{dU(t,z)}{dt}, rN(t,z)\right\} \\ s.t.\ \dot{z} &= -\phi z \end{aligned} \tag{55}$$

[21] The value of unemployment for an employed worker includes learning by doing as discussed above, hence the term λ.

where $\frac{dU(t,z)}{dt} = U_1(t,z) + \dot{z}U_2(t,z)$ s.t. $\dot{z} = -\phi z$ and where we have dropped the unemployment income b to simplify notation.

The value of a job

$$rJ(t,\tau,z) = \begin{cases} y(t,\tau,z) - w^U(t,\tau,z) - \delta J(t,\tau,z) + J_1(t,\tau,z), & t \leq \tau + a^*(z) \\ y(t,\tau,z) - w^N(t,\tau,z) - \delta J(t,\tau,z) + J_1(t,\tau,z), & t > \tau + a^*(z) \end{cases} \tag{56}$$

The value of being employed

$$rW(t,\tau,z) = \begin{cases} \max\left\{w^U(t,\tau,z) - \delta\left[W(t,\tau,z) - U(t,\chi(t,\tau,z))\right] \right. \\ \left. \quad +W_1(t,\tau,z), rU(t,\chi(t,\tau,z))\right\}, \quad t \leq \tau + a^*(z) \\ \\ \max\left\{w^N(t,\tau,z) - \delta\left[W(t,\tau,z) - U(t,\chi(t,\tau,z))\right] \right. \\ \left. \quad +W_1(t,\tau,z), rN(t)\right\}, \quad t > \tau + a^*(z) \end{cases} \tag{57}$$

s.t. $\dot{z} = \lambda z$.

A.4.2 Stationarization

Guess

$$\begin{aligned} J(t,\tau,z) &= e^{\gamma t}J(t-\tau,z) = e^{\gamma t}J(a,z) \\ W(t,\tau,z) &= e^{\gamma t}W(t-\tau,z) = e^{\gamma t}W(a,z) \\ U(t,z) &= e^{\gamma t}U(z) \\ N(t) &= e^{\gamma t}N \end{aligned}$$

Moreover

$$\begin{aligned} y(t,\tau,z) &= e^{\gamma t}e^{(\lambda-\gamma)(t-\tau)}z = e^{\gamma t}e^{(\lambda-\gamma)a}z \\ \chi(t,\tau,z) &= ze^{-(\phi-\lambda)(t-\tau)} = ze^{-(\phi-\lambda)a} = \chi(t-\tau,z) = \chi(a,z) \end{aligned}$$

Now the value equations become:

The Free-entry condition

$$zc = q(\theta(z))\,J(0,z) \tag{58}$$

The value of nonparticipation

$$\begin{aligned} rN &= \eta + \gamma N \\ &\Rightarrow (r-\gamma)N = \eta \end{aligned} \tag{59}$$

For unemployment we have two value equations, one for an unemployed worker and one for an employed worker. For the unemployed worker

$$(r-\gamma)\,U(z)=p(\theta(z))\,[W(0,z)-U(z)]-\phi z U_z(z) \tag{60}$$

and for the employed worker

$$\begin{aligned}(r-\gamma)\,U(\tilde z) &= p(\theta(\tilde z))\,[W(0,\tilde z)-U(\tilde z)] \\ &\quad +\chi_1(a,z)\,U_z(z) \\ \text{where } \chi_1(a,z) &= -(\phi-\lambda)\,z e^{-(\phi-\lambda)}=-(\phi-\lambda)\,\chi(a,z) \\ &= -(\phi-\lambda)\,\tilde z\end{aligned} \tag{61}$$

The value of a job

$$(r-\gamma)\,J(a,z)=\begin{cases} y(a,z)-w^U(a,z)-\delta J(a,z)+J_1(a,z)\,, & a\le a^*(z) \\ y(a,z)-w^N(a,z)-\delta J(a,z)+J_1(a,z)\,, & a> a^*(z)\end{cases} \tag{62}$$

The value of being employed

$$(r-\gamma)\,W(a,z)=\begin{cases} \max\left\{w^U(a,z)-\delta\,[W(a,z)-U(\chi(a,z))] \right. \\ \quad \left. +W_1(a,z)\,,(r-\gamma)\,U(\chi(a,z))\right\}, & a\le a^*(z) \\ \\ \max\left\{w^N(a,z)-\delta\,[W(a,z)-U(\chi(a,z))] \right. \\ \quad \left. +W_1(a,z)\,,rN\right\}, & a> a^*(z)\end{cases} \tag{63}$$

A.4.3 Nash bargaining

The Nash first-order condition is

$$\begin{aligned}\beta J(a,z) &= (1-\beta)\,[W(a,z)-U(\chi(a,z))]\,, \qquad a\le a^*(z) \\ \beta J(a,z) &= (1-\beta)\,[W(a,z)-N]\,, \qquad a> a^*(z)\end{aligned} \tag{64}$$

The surplus is

$$S(a,z)=\begin{cases} J(a,z)+W(a,z)-U(\chi(a,z))\,, & a\le a^*(z) \\ J(a,z)+W(a,z)-N, & a> a^*(z)\end{cases} \tag{65}$$

A.4.4 Determination of z^U

For skills z^N we have

$$\begin{aligned} N &= U\left(z^N\right) \\ (r-\gamma) N &= (r-\gamma) U\left(z^N\right) \end{aligned}$$

from which it follows that $U_1\left(z^N\right) = 0$ and thus the value of unemployment for a worker with skills z^N is

$$(r-\gamma) U\left(z^N\right) = p\left(\theta\left(z^N\right)\right) \beta S\left(0, z^N\right)$$

Hence z^N solves

$$p\left(\theta\left(z^N\right)\right) \beta S\left(0, z^N\right) = \eta \tag{66}$$

As a result

$$z^U = z^N e^{(\phi-\lambda)\bar{a}^U} \tag{67}$$

since a match started at z^U can last until $\bar{a}^U$ without the outside option changing.

A.4.5 Determination of $a^*(z)$

$a^*(z)$ is the age of a match started at skill level z such that the outside option switches from U to N.

For all z, $a^*(z)$ solves

$$z e^{-(\phi-\lambda)a^*} = z^N \tag{68}$$

from which we solve

$$a^*(z) = \frac{1}{(\phi-\lambda)} \ln\left(z/z^N\right) \tag{69}$$

A.4.6 Surplus function

The surplus of a match is now

$$S(a,z) = S^U(a,z) + S^N(a,z). \tag{70}$$

For $a \leq a^*(z)$

$$\begin{aligned} (r-\gamma) S^U(a,z) &= \max\left\{z e^{(\lambda-\gamma)a} - \delta S(a,z) + S_1(a,z) - (r-\gamma) U(\chi(a,z))\right. \\ &\quad \left. + \chi_1(a,z) U(\chi(a,z)), 0\right\} \end{aligned} \tag{71}$$

and for $a > a^*(z)$

$$(r-\gamma) S^N(a,z) = \max\left\{z e^{(\lambda-\gamma)a} - \delta S(a,z) + S_1(a,z) - (r-\gamma) N, 0\right\} \tag{72}$$

where $(r-\gamma) N = \eta$.

We first solve the second term $S^N(a,z)$ of the surplus. This is given by (72) which is a first order differential equation and gives the general solution

$$S^N(a,z) = \kappa e^{(r-\gamma+\delta)a} + \int_a^{\bar{a}^N} \left[z e^{(\lambda-\gamma)\tilde{a}} - \eta \right] e^{-(r-\gamma+\delta)(\tilde{a}-a)} d\tilde{a} \tag{73}$$

We can use a boundary condition, since $\bar{a}^N$ is such that $S^N\left(\bar{a}^N, z\right) = 0$. Then $\kappa = 0$. The destruction rule is

$$z e^{(\lambda-\gamma)\bar{a}^N} = \eta \tag{74}$$

from which we can solve

$$\bar{a}^N = \frac{1}{(\gamma-\lambda)} \ln(z/\eta). \tag{75}$$

where $z > \eta$ (if $z < \eta$, at z we have $N > U$ and we are not in this case $\rightarrow z > \eta$). We have $\frac{\partial \bar{a}^N}{\partial z} > 0$.

A.4.7 Conjectured solution

For $z > z^U$ only the first part of the surplus is relevant because $\bar{a}^U(z) < a^*$. Therefore the second term in the surplus function irrelevant, and the solution is the same as without nonparticipation.

Given that we have $\bar{a}^N(z)$ for all z, all we need to determine is z^N and $\theta(z)$ in the region $\left(z^N, z^U\right)$. In fact, z^U is a function of $\left(z^N, \bar{a}^U(z)\right)$ and $a^*(z)$ is a function of z^N.

A.4.8 Determination of z^N

From the indifference condition of the unemployed worker at z^U, we have

$$p\left(\theta_{z^N}\right) \beta S\left(0, z^N\right) = \eta \tag{76}$$

From the free-entry condition at z^N we have

$$q\left(\theta_{Z^N}\right)(1-\beta) S\left(0, z^N\right) = z^N c \tag{77}$$

Putting together these two expressions we obtain

$$\theta_{z^N} = \left(\frac{1-\beta}{\beta} \frac{\eta}{c}\right) \frac{1}{z^N}. \tag{78}$$

Furthermore using the Cobb-Douglas functional form for the matching function

$$p\left(\theta_{z^N}\right) = p\left(\frac{1-\beta}{\beta} \frac{\eta}{c} \frac{1}{z^N}\right) = \left(\frac{1-\beta}{\beta} \frac{\eta}{c} \frac{1}{z^N}\right)^{\alpha} \tag{79}$$

We need to solve

$$S\left(0, z^N\right) = \frac{1}{\beta} \frac{\eta}{p\left(\theta_{z^N}\right)} \tag{80}$$

where the LHS is linearly increasing in z^N and the RHS is concave in z^N as

$$\frac{1}{\beta}\frac{\eta}{p\left(\theta_{z^N}\right)} = \frac{\eta}{\beta}\left(\frac{\beta}{1-\beta}\frac{c}{\eta}\right)^{\alpha}\left(z^N\right)^{\alpha} \tag{81}$$

Since

$$\begin{aligned} S\left(0, z^N\right) &= \int_0^{\bar{a}^N}\left[z^N e^{(\lambda-\gamma)\tilde{a}} - \eta\right] e^{-(r-\gamma+\delta)\tilde{a}} d\tilde{a} \\ &= \int_0^{\bar{a}^N}\left[z^N e^{-(r-\lambda+\delta)\tilde{a}} - \eta e^{-(r-\gamma+\delta)\tilde{a}}\right] d\tilde{a} \end{aligned}$$

and using the optimality condition (74)

$$\begin{aligned} S\left(0, z^N\right) &= \frac{z^N}{r-\lambda+\delta}\left(1-e^{-(r-\lambda+\delta)\bar{a}^N}\right) - \frac{z^N e^{(\lambda-\gamma)\bar{a}^N}}{r-\lambda+\delta}\left(1-e^{-(r-\gamma+\delta)\bar{a}^N}\right) \qquad (82) \\ &= z^N\left[\frac{1-e^{-(r-\lambda+\delta)\bar{a}^N}}{r-\lambda+\delta} - e^{(\lambda-\gamma)\bar{a}^N}\left(\frac{\left(1-e^{-(r-\gamma+\delta)\bar{a}^N}\right)}{r-\gamma+\delta}\right)\right] \end{aligned}$$

Here,

$$\frac{dS\left(0, z^N, \bar{a}^N\right)}{dz^N} = \frac{\partial S}{\partial z} + \frac{\partial S}{\partial \bar{a}^N}\frac{\partial \bar{a}^N}{\partial z}$$

But given that we have substituted in the job destruction rule, we have that

$$\frac{\partial S}{\partial \bar{a}^N} = 0,$$

so we conclude that the 'maximized surplus' is linear in z^N.

Comparative statics of z^N wrt. γ :

As the second term in the square brackets of (82) does not depend on z^N and by the envelope theorem we don't need to differentiate wrt. $\bar{a}^N$, hence

$$sign\left(\frac{\partial S}{\partial \gamma}\right) = sign\frac{\partial}{\partial \gamma}\left[-e^{(\lambda-\gamma)\bar{a}^N}\int_0^{\bar{a}^N} e^{-(r-\gamma+\delta)\tilde{a}} d\tilde{a}\right]. \tag{83}$$

so we have, denoting $I = \int_0^{\bar{a}^N} e^{-(r-\gamma+\delta)\tilde{a}} d\tilde{a}$,

$$\begin{aligned} & -\left[-\bar{a}^N e^{(\lambda-\gamma)\bar{a}^N} I + e^{(\lambda-\gamma)\bar{a}^N}\int_0^{\bar{a}^N} \tilde{a} e^{-(r-\gamma+\delta)\tilde{a}} d\tilde{a}\right] \qquad (84) \\ &= e^{(\lambda-\gamma)\bar{a}^N}\left[\bar{a}^N I + \int_0^{\bar{a}^N} \tilde{a} e^{-(r-\gamma+\delta)\tilde{a}} d\tilde{a}\right] > 0 \end{aligned}$$

as $\tilde{a}$ in the integral always satisfies $\tilde{a} < \bar{a}^N$.

We thus have

$$\frac{\partial z^N}{\partial \gamma} < 0 \tag{85}$$

and

$$\frac{\partial \theta_{z^N}}{\partial \gamma} > 0. \tag{86}$$

The comparative statics are opposite to those of case U.

A.4.9 Determination of $\theta(z)$

Substituting the wages (derived in the appendix) for $w^U(a,z)$ we have the value of an occupied job as

$$(r-\gamma) J(a,z) = \begin{cases} (1-\beta) z e^{(\lambda-\gamma)a} - \beta z c \theta(\tilde{z}) - \delta J(a,z) + J_1(a,z), & a \leq a^*(z) \\ \hat{J}(a) z, \quad a > a^*(z) \end{cases} \tag{87}$$

where $\tilde{z} = z e^{-(\phi-\lambda)\tilde{a}}$. We have

$$J(a,z) = z \int_a^{a^*(z)} \left[(1-\beta) e^{(\lambda-\gamma)\tilde{a}} - c\theta\left(z e^{-(\phi-\lambda)\tilde{a}}\right)\right] e^{-(r-\gamma+\delta)(\tilde{a}-a)} d\tilde{a} \tag{88}$$

$$+ e^{-(r-\gamma+\delta)(a^*(z)-a)} \hat{J}\left(a^*(z), \bar{a}^N\right) z \tag{89}$$

Hence

$$J(0,z) = z \int_0^{a^*(z)} \left[(1-\beta) e^{(\lambda-\gamma)\tilde{a}} - c\theta\left(z e^{-(\phi-\lambda)\tilde{a}}\right)\right] e^{-(r-\gamma+\delta)\tilde{a}} d\tilde{a} + e^{-(r-\gamma+\delta)a^*(z)} \hat{J}\left(a^*(z), \bar{a}^N\right) z \tag{90}$$

So the free-entry condition is

$$\frac{c}{q(\theta(z))} = \int_0^{a^*(z)} \left[(1-\beta) e^{(\lambda-\gamma)\tilde{a}} - c\theta\left(z e^{-(\phi-\lambda)\tilde{a}}\right)\right] e^{-(r-\gamma+\delta)\tilde{a}} d\tilde{a} + e^{-(r-\gamma+\delta)a^*(z)} \hat{J}\left(a^*(z), \bar{a}^N\right) z \tag{91}$$