Optimal Unemployment and Disability Insurance*

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ABSTRACT.

This paper considers optimal unemployment and disability insurance in an environment in which moral hazard is important. A social planner constructs a history dependent contract without knowledge of an agent’s true state and without observing job search effort. The optimal contract trades off the need for insurance against the moral hazard problems in a way such that the cost of providing a given level of lifetime utility is minimized. In the optimal contract, an agent is indifferent between being unemployed or disabled next period, and prefers to be employed. Consumption is constant during employment, falling during unemployment, and constant during disability. The optimal replacement rate when becoming unemployed is initially above 100 percent, and upon disability consumption should drop. Constant consumption during employment and falling consumption when unemployed means that an agent gets poorer, in every future state, as he goes through life and experiences more and longer unemployment spells. For the same reason inequality between lucky and unlucky agents becomes larger and larger over time.

Keywords: Unemployment Insurance, Disability Insurance, Moral Hazard, Recursive Contracts.

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1 Introduction

Unemployment and disability are important sources of income fluctuations, and individuals facing such risks demand insurance to smooth consumption. Indeed, social insurance programs have become important in most industrialized countries in the last decades. In the United States, the cost of the unemployment insurance system in 2003 was $39 billion (Feldstein, 2005). The outlays on disability insurance even surpasses that amount, being $61 billion already in 2001 (Golosov and Tsyvinski, 2006).

Designing a social insurance system is difficult because of private information. An insurance agency cannot generally infer whether an agent reached a bad state out of bad luck or by choice, or even if he really is in a bad state. Moral hazard problems then limit the extent to which insurance can be achieved. If the insurance agency does not acknowledge the moral hazard problems when designing the insurance system, agents might report a false state, quit a job to collect unemployment benefits, or fail to exert search effort when unemployed.

The purpose of this paper is to study the optimal social insurance system against unemployment and disability risk in an environment where moral hazard is important. I merge Hopenhayn and Nicolini’s (1997, 2004) model of optimal unemployment insurance with a simplified version of Diamond and Mirrlees’s (1978) model of optimal disability insurance and characterize the optimal joint insurance system. A social planner constructs a history dependent contract without knowledge of an agent’s true state and without observing unemployed agents’ job search effort. The optimal contract minimizes the cost of providing a given level of lifetime utility to an agent. In setting up the optimal contract, the planner must trade off insurance motives against the incentive concerns stemming from private information.

The optimal social insurance contract offers considerable consumption smoothing across states compared to the outcome if agents fend for themselves in autarky. In the optimal contract, the future lifetime utility of an agent is equalized between unemployment and disability and the planner offers some excess utility for employment. The planner cannot accomplish more insurance because of the moral hazard problems. For
example, had he offered full insurance, an unemployed would never search for a job. Some differences in consumption across states is instead an integral part of keeping the contract incentive compatible. As an agent is better off employed than unemployed or disabled, the planner need not vary consumption during an employment spell. Because consumption as employed is the same in the first as the last period of the spell, the contract exhibits limited memory. It does not matter for the future if an employment spell lasts one or ten periods, or any other length for that matter. In contrast, as a way of maintaining the agents’ willingness to adhere to the planner’s instructions, consumption should fall during an entire unemployment spell, drop discretely upon disability, and then remain constant. To deliver the same future utility for unemployment and disability while keeping consumption during employment constant, the initial replacement rate in the unemployment insurance becomes above 100 percent.

Declining consumption during unemployment is a way of providing incentives to avoid prolonged unemployment, but also has wider implications. Promised values after gaining employment or if turning disabled is lower the longer the unemployment spell has lasted. The reason is that the planner punishes unsuccessful job search in all future states, which is optimal since agents are risk averse. Long run inequality between lucky agents with few and short unemployment spells, and unlucky ones with the opposite experience, increases over time. Also, the more unemployment an agent has experienced, the lower should his disability benefits be.

Are the moral hazard issues a valid argument for exploring the optimal information constrained social insurance, and why should you analyze unemployment and disability insurance in the same model? There is ample empirical evidence that moral hazard problems are important in social insurance contexts and that unemployment and disability insurance affect each other. Katz and Meyer (1990) find that unemployment insurance increases unemployment. Although hard to measure, previous studies have also found that the disability insurance system reduces labor supply. Gruber (2000) studies a policy change in Canada giving rise to arguably exogenous changes in disability benefits. He

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1See also Carling et al (2001) for European evidence.
finds a significant reduction in labor force participation due to increased benefits, with an implied elasticity of around .3. Gruber’s paper is an argument in its own right for studying unemployment and disability insurance jointly, as one would expect some of those falsely claiming disability to otherwise be unemployed. More generous unemployment insurance would then reduce the disability claims, and vice versa. A more direct test of this conjecture is performed by Autor and Duggan (2003). They study the interaction between aggregate unemployment and the disability insurance explicitly, and attribute as much as one half percentage point of the U.S. mid 1980’s decline in unemployment to the contemporaneous rise in disability benefits.

Despite the clear empirical evidence of interactions between unemployment and disability insurance, previous theoretical research has predominately studied these issues separately. Shavell and Weiss (1979) used a search effort model with an initially unemployed agent exerting search effort to gain unemployment. They show that in this environment, consumption during unemployment should fall over time. The intuition is that the unemployed has to be given incentives to search, and this is done by punishing prolonged unemployment. This assertion is verified by Hopenhayn and Nicolini (1997) that extends the policy space, allowing the planner to tax the agent after gaining employment. They additionally show that the tax upon employment should increase with the length of the unemployment spell. Here, the cost of providing search incentives are minimized by punishment in all states, a feature due to the agent’s strictly concave utility function. Hopenhayn and Nicolini (2004) extends the analysis further by considering a model of multiple employment and unemployment spells. Within this environment consumption falls during unemployment and are either constant or increasing along an employment spell, for low and high effort costs while employed, respectively. In the multiple spells model, Hopenhayn and Nicolini also show that the replacement rate in the optimal unemployment insurance system is always below 100 percent.

The seminal papers on optimal disability insurance is Diamond and Mirrlees (1978, 1986). They study an economy with workers that are hit by stochastic shocks rendering them permanently disabled. The government cannot observe whether a worker that
claims to be disabled actually can work or not, and a moral hazard problem is created by an additive cost of working. In order for an able agent to continue working, the value of continued employment must exceed that of claiming disability. Diamond and Mirrlees show that it is optimal for the planner assign state contingent consumption streams such that a worker is always indifferent between claiming to be disabled or not. The planner would not like to award extra utility for job tenure because marginal utility is higher for a given level of utility when disabled, as disabled has no effort cost. After reporting disabled, an agent’s consumption should be constant. Diamond and Mirrlees also find that consumption and disability claims are optimally increasing with job tenure. The intuition is clear. The planner fears that a worker might stop working too early and awards job tenure by the rising consumption path. Rising disability claims then follows naturally from the fact that agents are risk averse, so it is cheaper to award them in every future state.

2 The Economy

The economy is populated by many risk averse agents. In each point in time an agent can be employed, unemployed or disabled. If employed, the agent earns a constant wage $w > 0$, while if unemployed or disabled he earns no income. Disability is an absorbing state. In contrast, employed and unemployed can transition between employment and unemployment as well as into disability. Specifically, an unemployed becomes disabled with per period probability $\xi$ and can gain employment by exerting search effort. He can either choose $a > 0$ or zero search effort. If search effort is $a$, the probability of gaining employment is $p$, conditional on not becoming disabled. If search effort is zero, an unemployed remain so with probability one, again conditional on not getting disabled. Employed agents face the same probability of becoming disabled as unemployed and maintains employment with exogenous probability $\lambda$, if not becoming disabled. Quits are possible. Effort is assumed to be costly in utility terms. Following Hopenhayn and
Nicolini (1997), the preferences of an agent are ordered by

\[ E \sum_{t=0}^{\infty} \beta^t [u(c_t) - a_t], \quad (1) \]

where \( c_t \) is consumption, \( u(\cdot) \) is strictly increasing, strictly concave, twice continuously differentiable and \( \beta \in (0, 1) \) is the common discount factor, including a probability of dying. Assume also that \( u(0) \) is well defined. No effort is required to maintain employment, so \( a_t \) is zero during employment and disability. All the results remain if we introduce an effort cost of employment, provided it is sufficiently small. The agents can neither save nor borrow. Lack of access to capital markets in combination with risk aversion will constitute scope for improvement upon the autarky equilibrium by a social planner that has the ability to transfer resources over time and across agents. I assume that such a planner has unlimited access to a perfect capital market, where he can borrow and lend at a constant gross interest rate equal to the reciprocal of the agents’ discount factor.

The current state of an agent is unobservable but income is observed by the planner. Hence an employed cannot state unemployment or disability. An employed can quit however, rendering him in a situation identical to one that got laid off. The planner cannot distinguish quits from layoffs. Effort while unemployed is also unobservable. The optimal contract must induce an employed never to quit, unemployed and disabled not to lie, and unemployed to exert effort to become employed.

We proceed by clarifying the problem of an agent by studying the autarky equilibrium and then by stating the recursive version of the planner’s problem.

### 3 Autarky

To highlight the agents’ problem we start the analysis in a situation without a planner. Now the agents lack the opportunity to trade in any way and have to fend for themselves. The autarky values for employed and disabled are:

\[ V_{aut}^e = u(w) + \beta \left\{ \xi V_{aut}^e + (1 - \lambda) V_{aut}^u + \lambda V_{aut}^d \right\}, \]
and
\[ V^d_{aut} = \frac{u(0)}{1-\beta}. \]

The autarky value for an unemployed is the solution to the Bellman equation:
\[
V^u_{aut} = u(0) + \max \left\{ -a + \beta \{ (1 - \xi) [pV^e_{aut} + (1 - p) V^u_{aut}] \} + \beta [ (1 - \xi) V^u_{aut} ] \right\} + \beta \xi V^d_{aut}.
\]

Evidently, an unemployed exerts effort if and only if
\[
\beta (1 - \xi) p (V^e_{aut} - V^u_{aut}) \geq a.
\]

From now on we assume that parameters are such that this condition is fulfilled. Note that then \((V^e_{aut} - V^u_{aut}) > 0\), so an employed agent will never quit. When the planner constructs the optimal contract in the proceeding sections he will, in addition to other incentive constraints, have to see to it that a condition like (2) is fulfilled.

## 4 The Planner’s problem

The planner’s problem is to specify history dependent transfers and effort recommendations to minimize the cost of giving the agent a specified value \(V\). Relying on the revelation principle, I restrict attention to contracts where the planner would like the agent to tell the truth. Furthermore I assume that the primitives of the model, including the initial promised value, are such that the planner would like to induce positive search effort. The planner collects reports on state in each period and makes history dependent transfers in order to induce no quitting, truth telling, and effort if the agent is unemployed. The timing of the contract within each period is:

1. The agent learns his state and reports a state to the planner.

2. Conditional on the history of reports, including the current period, the planner sets up a consumption level and promises for each future state that is possible.
3. If the agent is unemployed he chooses to exert search effort or not which determines the probability distribution over next period’s state.

If the promised value for a given state is $V$, the planner chooses current consumption $c$, and promises $V^e, V^u,$ and $V^d$ for each possible future state. Here, $V^e$ is the expected utility from next period on, should the agent be employed then, and $V^u$ and $V^d$ are defined analogously. In order to deliver the promise the planner has to set $c, V^e, V^u,$ and $V^d$ such that an agent’s expected utility equals $V$. In addition, the contract must involve combinations of $V^e, V^u,$ and $V^d$ that makes it optimal for an agent not to quit, not to lie and to search if unemployed. The value of the planner’s problem when the agent has reported to be employed is $C_e(V)$ and denotes the minimal cost of delivering the value $V$ to an agent currently employed. Corresponding cost functions when an agent is unemployed or disabled, $C_u(\cdot)$ and $C_d(\cdot)$, are defined analogously.

4.1 Employed

Suppose an agent is employed and has promised value equal to $V$. The optimal contract then solves the Bellman equation:

$$C_e(V) = \min_{c, V^e, V^u, V^d} \left\{ c - w + \beta \left\{ (1 - \xi) \left[ \lambda C_e(V^e) + (1 - \lambda) C_u(V^u) \right] + \xi C_d(V^d) \right\} \right\}, \quad (3)$$

where the minimization is subject to

$$V = u(c) + \beta \left\{ (1 - \xi) \left[ \lambda V^e + (1 - \lambda) V^u \right] + \xi V^d \right\}, \quad (4)$$

$$V^e \geq V^u, \quad (5)$$

$$V^u \geq V^d. \quad (6)$$

Here, (4) is the promise keeping constraint, and (5) is the no quitting constraint that makes sure that an agent does not quit if still employed in the beginning of next period. Also, (6) is the truth telling constraint that assures that if unemployed next period, the agent will report so. But what if the agent gets disabled, how do we make sure that he
does not claim to be unemployed and collect (possibly) higher benefits? This can be a serious problem, but it turns out not to be. Later we will see that along the optimal path, it is never optimal to claim unemployment if disabled. The cost function is strictly increasing and strictly convex. Because the agents’ utility function is concave it becomes more and more expensive to deliver an additional util, as promises increase.\(^2\) Attaching nonnegative Lagrange multipliers \(\mu, \beta \theta_e\) and \(\beta \theta_u\) on (4), (5) and (6), respectively, the first order conditions are:

\[
\frac{1}{u'(c)} = \mu \tag{7}
\]

\[
\lambda [C'_e (V^e) - \mu] - \theta_e = 0 \tag{8}
\]

\[
(1 - \lambda) [C'_u (V^u) - \mu] - \theta_u + \theta_e = 0 \tag{9}
\]

\[
\xi [C'_d (V^d) - \mu] + \theta_u (1 - \xi) = 0 \tag{10}
\]

By the envelope theorem,

\[
C'_e (V) = \mu. \tag{11}
\]

### 4.2 Unemployed

If an agent has reported unemployment and his promised value in this state is \(V\), the optimal contract is characterized by the solution to the dynamic program:

\[
C_u (V) = \min_{c, V^e, V^u, V^d} \left\{ c + \beta \left[ (1 - \xi) [p C_e (V^e) + (1 - p) C_u (V^u)] + \xi C_d (V^d) \right] \right\}, \tag{12}
\]

where the minimization is subject to

\[
u(c) - a + \beta \left\{ (1 - \xi) [pV^e + (1 - p) V^u] + \xi V^d \right\} = V, \tag{13}\]

\[
\beta (1 - \xi) p [V^e - V^u] \geq a, \tag{14}\]

\(^2\)As a more formal argument, note that we could rewrite the problem as choosing a utility level \(u\) instead of \(c\). The return function is then the strictly increasing, strictly convex function \(u^{-1}\) and the constraints are linear in \(u, V^e, V^u,\) and \(V^d\). This makes the cost function become strictly increasing and strictly convex. Indeed, the resulting first order conditions with the alternative specification would be identical to those presented here.
\[ V^u \geq V^d. \] (15)

Here, (13) is the promise keeping constraint, (14) is the incentive compatibility constraint inducing effort, and (15) is a truth telling constraint. The truth telling constraint assures that if the agent is unemployed tomorrow, in which case he will get value \( V^u \) from that period on, he is at least as well off as if he reported disabled and got \( V^d \). As we will show later, also in this case a disabled will not lie. The cost function is strictly increasing and strictly convex, for the same reason as in the employment problem. To proceed, attach nonnegative Lagrange multipliers \( \mu, \phi \) and \( \beta (1 - \xi) \theta \) on (13), (14) and (15) respectively. The first order necessary conditions are:

\[
\mu = \frac{1}{u'(c)} \quad (16)
\]

\[
[C^e_e(V^e) - \mu] p - \phi p = 0 \quad (17)
\]

\[
[C^u_u(V^u) - \mu] (1 - p) + \phi p - \theta = 0 \quad (18)
\]

\[
[C^d_d(V^d) - \mu] \xi + \theta (1 - \xi) = 0 \quad (19)
\]

By the envelope theorem,

\[
C^u_u(V) = \mu. \quad (20)
\]

### 4.3 Disabled

Disability is an absorbing state. Therefore, there are no incentive problems between an agent who has claimed disability and the planner. Then it is optimal for the planner to give the agent a constant level of consumption, that delivers the promised value. Let this transfer be \( c^d \), which is implicitly defined by

\[
V^d = \frac{u(c^d)}{1 - \beta}
\]
The cost to the planner is then

\[ C_d(V^d) = \frac{c^d}{1 - \beta} = \frac{u^{-1}(V^d(1 - \beta))}{1 - \beta}. \]

Note that

\[ C'_d(V^d) = \frac{1}{u'(c^d)} \] (21)

is positive and increasing making \( C_d(V^d) \) convex.

5 Optimal insurance

Absent the informational asymmetries, the optimal insurance contract would award an agent the same consumption in every state and period. The planner would simply instruct an unemployed to search, and with effort being observable the planner could punish him without limit, at no cost, should he not adhere to the instruction.\(^3\) Although consumption is the same, the values for different types are not. A disabled will never have to exert search effort and thus enjoys the highest level of utility. An employed faces some risk of having to search if unemployed in the future, but less so than an unemployed. Hence, an employed has higher utility than an unemployed, who is worst off in the full information contract.

5.1 Distribution of values

Under asymmetric information, the state independent allocations from the perfect information contract are not incentive compatible. Regardless of his true state, an agent could always claim disability to enjoy the highest possible utility, or stop searching if unemployed. A natural first step in the inquiry of how the optimal information constrained contract differs from the unconstrained optimum is therefore to find out which information constraints are important. It turns out that the structure of binding incentive constraints is simple and intuitive. An agent is indifferent between becoming or remain-

\(^3\)With an infinite punishment it is never optimal for the agent to deviate from the planner’s instructions. Hence, punishment is costless.
Proposition 1 The truth telling constraints in both the employment and the unemployment problem and the effort incentive constraint in the unemployment problem are binding. The no-quitting constraint is not binding.

It might be useful to compare this to the full information benchmark to appreciate the limitations private information puts on the contract. The structure of binding incentive constraints in the optimal contract is in fact the incentive compatible contract that is closest to the full information contract. A disabled is as well off as an unemployed because this is the highest incentive compatible value the planner can offer a disabled. Also, as the difference in values between an unemployed and an employed in the full information contract is not sufficient to induce job search effort, the planner raises the value for employment no more than what is needed to make it worthwhile to search if unemployed.

At this point it is clear why a disabled has no incentives to claim to be unemployed. By telling the truth, a disabled gets exactly the same expected utility as if truly unemployed. A false claim of unemployment cannot make a disabled better off than a truly unemployed because a disabled has zero probability of gaining employment.

As a final remark of this subsection, note the difference between the distribution of promised values in this paper and in Diamond and Mirrlees (1978). With only employed and disabled, as in their model, the values across employment and disability would be equalized. In this model, where unemployment is present, the values of unemployment and disability are the same, and employed are better off. This difference compared to Diamond and Mirrlees’s model will have further implications of the optimal contract.

5.2 Consumption dynamics

Let us now study the consumption dynamics within a given spell. The assertion by Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) that consumption should fall during an unemployment spell remains valid also in this environment. Furthermore, consumption should not change over time during an employment spell or when disabled.
Proposition 2 Consumption is constant during an entire employment spell, decreasing during an unemployment spell, and constant during disability.

Consumption being constant during employment is a direct consequence of the no-quitting constraint being slack. Given that the employed is not tempted to quit, there is no reason to vary consumption during a spell of employment. On the contrary, the cheapest way of delivering a given value when there are no incentive problems is a constant sequence of consumption, a feature due to the concave utility function. Constant consumption during employment is also a feature shared by Hopenhayn and Nicolini (2004), for their specification with low enough effort cost while employed. Declining consumption during unemployment reflects the planner’s need to induce search from the unemployed. An unemployed must be provided incentives to avoid prolonged spells of unemployment and the planner provides those incentives by gradually lowering consumption during the whole spell. As we have already argued, all incentive problems between the planner and the agent are bygones once an agent has reported to be disabled. Again, concavity of the utility function prescribes constant consumption as the cost minimizing way of delivering a disabled’s promise.

Within a spell there are no qualitative differences in results or intuition between this model and Hopenhayn and Nicolini (1997, 2004), but a clear difference compared to Diamond and Mirrlees (1978). In Diamond and Mirrlees’s analysis, the worker consumes more and more during a spell of employment. The reason in their paper is that the agent is always indifferent between claiming disability or not, so rising consumption is an award for tenure. Such an award is not needed here, because an employed is always strictly better off than an unemployed or a disabled.

We now turn to what happens to consumption at the time of transition between different states. Consumption has to jump at all transitions, partly because different states means different values and partly because of the way the planner delivers these values according to the preceding proposition.

Proposition 3 Consumption rise upon layoff and upon employment, and fall upon disability.
The quite intuitive conclusion that consumption increases upon gaining employment follows naturally from the planner’s need to deliver the high promise upon employment. The rise in consumption upon layoff is harder to explain. It is certainly not a reward for layoff, nor is it a punishment. There is absolutely no need for the planner to punish layoffs since they are exogenous. The answer should instead be sought in the relation between unemployment and disability. Since the truth telling constraint while employed is binding, the agent is as well off getting unemployed as getting disabled. When unemployed we have already seen that consumption should fall, while it remains constant for a disabled. The planner’s need to deliver the same promise to these two distinct states then has to involve consumption first jumping up upon unemployment, to then fall during the spell. The reason that consumption falls upon disability is easy to see if we keep in mind that consumption is constant during employment. If consumption did not fall upon disability then a disabled will be as well off as an employed with zero unemployment risk. This is clearly not incentive compatible, as everyone would then state disability.

The drop in consumption upon disability is in line with Diamond and Mirrlees (1978). The intuition is a bit different. In their model, the drop in consumption is a tool for avoiding that an employed goes into disability too early. In this model instead it is a tool for delivering the same value to disabled as to unemployed, thereby not luring unemployed to lie into disability.

5.3 Dynamics of promised values

It is interesting to study also what happens to agents with different histories. The evolution of promised values is informative in this respect. Naturally, promised values move in the same direction as does consumption.

**Proposition 4** During employment, future promises are constant with tenure. During unemployment, all promises fall by the same amount.

The longer an agent is unemployed the lower is his utility in the future, not just in unemployment but also in future spells of employment, and when he eventually becomes
disabled as well. The intuition is that since agents are risk averse, it is cheaper to maintain incentive compatibility by permanent alterations of values in all future states.

As promises are unaltered during employment, the contract exhibits limited memory. It simply does not matter if an agent’s employment spell was long or short, just the number of spells. For example, the planner awards exactly the same consumption and future promises to an agent that has been employed 100 periods, unemployed for 10 and then gains employment as to one who was employed for one period, unemployed for 10 and then gained employment. Limited memory in this respect means that the common feature of real world social insurance systems that a number of periods of employment is needed to qualify for unemployment insurance does not find support in this model. Here, one period is sufficient.

Even though there is some limited memory in the optimal contract, the evolution of promised values has important implications for the difference across histories. The falling promises during unemployment together with the constant promises during employment suggests that inequality between lucky agents with few and short unemployment spells and unlucky agents, with the opposite experience, gets larger over time. Indeed, an agent will typically consume less and less as he gets older. Moreover, getting disabled early means higher disability benefits than if getting disabled later in life, in complete contrast to Diamond and Mirrlees (1978). The result that as the agent goes through life he sees his future utility gradually falling is reminiscent the immiseration property in Thomas and Worrall (1990). Behind these properties of the optimal contract lurk the inverse Euler equation, that provides intuition for consumption being front loaded (see Rogerson, 1985). To see this, consider an employed agent. Combine the first order conditions in the employment problem with the envelope conditions for all problems to

\[ \text{Other than in these very simple cases it is difficult or impossible to compare exact histories analytically.} \]

\[ \text{The notion of a period in this model has no direct interpretation in form of a length of time. In fact it is the actual gain of employment, not the time working, that is important.} \]

\[ \text{Thomas and Worrall (1990) study a planner providing optimal insurance to an agent hit by identically independently distributed income shocks and show that the agents promised value reaches an arbitrary low level with probability one.} \]
get
\[ \frac{1}{u'(c)} = (1 - \xi) \lambda \frac{1}{u'(c^e)} + (1 - \xi)(1 - \lambda) \frac{1}{u'(c^u)} + \xi \frac{1}{u'(c^d)}. \tag{22} \]

The inverse Euler equation says that the inverse of marginal utility today should equal the expected value of the inverse of marginal utility tomorrow. A similar equation, with \( p \) replacing \( \lambda \), holds for an unemployed. Applying Jensen’s inequality to (22) yields,

\[ u'(c) < (1 - \xi) \lambda u'(c^e) + (1 - \xi)(1 - \lambda) u'(c^u) + \xi u'(c^d), \tag{23} \]

so consumption has a tendency to be front loaded.\(^7\) The reason for this intertemporal wedge is that it is more costly to give consumption tomorrow as it will have to be varied across states to maintain incentive compatibility. In other words, the planner’s marginal rate of transformation exceeds the agent’s marginal rate of substitution.

### 6 Conclusions

There are good reasons to study how to design unemployment and disability insurance in the same model as they are important for each other empirically. When doing so a number of qualitative differences emerge, compared to models in which only one of them can be analyzed.

The introduction of unemployment into the analysis of disability insurance means that employed should not be made indifferent to claiming disability, as this would lure unemployed to not look for a job. Instead, an agent should be made indifferent between unemployment and disability and prefer employment. The ranking of values between states has further implications. First, consumption is constant during a spell of employment. As an employed is in the best possible state, the planner need not vary his consumption within a spell as there are no risks of suboptimal quits. Consumption during unemployment, however, has to fall to punish prolonged unemployment. Maintaining

\(^7\)Rogerson (1985) proves that for a large class of HARA (including Quadratic, CARA and CRRA) utility functions (23) implies that expected consumption is falling over time. As the utility function in this paper is not necessarily in the HARA class, Rogerson’s proof does not go through generically. However the tendency for front loading is clear from (23).
incentive compatibility also call for a surprisingly high initial replacement rate in the unemployment insurance system, and a sharp drop in consumption during disability.

The general need to vary consumption across states to induce incentive compatibility has far reaching long run implications. Inequality tends to increase in the long run between agents with more and longer unemployment spells compared to agents with few and short spells. Also the more an agent has been unemployed, the lower his disability benefits should be.

Concerning future work, I think numerical simulations of the model would be an interesting extension. With a numerical solution to the planner’s problem it would be straightforward to simulate examples of complex histories, which would further reveal the properties of the optimal contract. Numerical simulations would also be appropriate to make quantitative comparisons with Hopenhayn and Nicolini (1997, 2004) and Diamond and Mirrlees (1978).
References


7 Appendix - proofs

7.1 Proof of proposition 1

We prove the proposition by a sequence of Lemmas and a corollary. First we prove that (a) exactly one of either the truth telling constraint or the no quitting constraint is binding in the employment problem. Then we prove that (b), given (a), both constraints in the unemployment problem are binding. Finally, we prove that given (a) and (b), the truth telling constraint is the only binding constraint in the employment problem.

**Lemma A1** In the employment problem, either the no quitting constraint or the truth telling constraint is binding.

**Proof.** This is a proof by contradiction in two steps. First, we show that neither of the constraints being binding cannot be optimal and then we show that both being binding cannot be optimal. Suppose neither are binding. Then, $V^e > V^u > V^d$, and by the first
order conditions (8) - (11),

\[ C'_e (V^e) = C'_u (V^u) = C'_d (V^d) = C'_e (V). \]

This implies \( V^e = V \). Using (21) and (7),

\[ V^d = \frac{u(c)}{1 - \beta}. \]

The promise keeping constraint and the equations above implies

\[ V = V^d (1 - \beta) + \beta \left\{ (1 - \xi) [\lambda V + (1 - \lambda) V^u] + \xi V^d \right\} < V^u (1 - \beta) + \beta \left\{ (1 - \xi) [\lambda V + (1 - \lambda) V^u] + \xi V^u \right\} = V^u + \beta \lambda (1 - \xi) (V - V^u) \implies V^u > V = V^e, \text{ a contradiction.} \]

Next suppose both are binding. Then \( V^e = V^u = V^d \). The first order conditions (8) - (11) now imply that

\[ C'_e (V^e) > C'_u (V) > C'_d (V^d). \]

This implies \( V^e = V^u = V^d > V \) and

\[ V^e = V^u = V^d < \frac{u(c)}{1 - \beta}. \]

The promise keeping constraint is then \( V = u(c) + \beta V^d > u(c) + \beta V \implies V > \frac{u(c)}{1 - \beta} \implies V > V^d = V^e, \text{ a contradiction.} \]

**Lemma A2** If the truth telling constraint in the employment problem is binding when employed in period \( t \), it is also binding if still employed in period \( t + 1 \).

**Proof.** If the truth telling constraint is binding, then \( C_e (V^e) = C_e (V) \implies V^e = V \), so the contract is self generating. ■

**Lemma A3** If the no quitting constraint in the employment problem is binding when employed in period \( t \), it is also binding if still employed in period \( t + 1 \).

**Proof.** Suppose towards a contradiction that no quitting binds at \( t \) but not at \( t + 1 \).
Then

\[ C_u'(V^u_t) < C_e'(V^e_t) = C_e' (V^e_{t+1}) < C_u'(V^u_{t+1}). \]

This implies that

\[ V^u_{t+1} > V^u_t \text{ and } V^e_{t+1} = V^e_t. \]

Since the no quitting constraint binds at \( t \), we have that

\[ V^e_{t+1} = V^e_t = V^u_t. \]

Since the no quitting constraint does not bind at \( t + 1 \), the truth telling constraint binds at \( t + 1 \). Hence, \( V^u_{t+1} < V^e_{t+1} = V^u_t \), a contradiction.

**Corollary A1.** Either the no quitting or the truth telling constraint is binding the entire employment spell.

**Proof.** Follows immediately from the three Lemmas above.

**Lemma A4** If either the effort constraint or the truth telling constraint is binding, then both are.

**Proof.** Suppose that only the truth telling constraint binds. Then \( \theta > 0 \) and \( \phi = 0 \).

Since \( \theta > 0 \),

\[ C_d' (V^d) < C_u' (V) \implies c^d < c \implies V^d = \frac{u(c)}{1 - \beta} < \frac{u(c)}{1 - \beta} \]

By (18) and the fact that the truth telling constraint is binding, \( V^u = V^d > V \). Since \( \phi = 0 \) we have that

\[ V = u(c) - a + \beta \{ (1 - \xi) [pV^e + (1 - p) V^u] + \xi V^d \} > u(c) + \beta \{ (1 - \xi) V^u + \xi V^d \} \]

\[ = u(c) + \beta V^d \]

But then \( V^d > \frac{u(c)}{1 - \beta} \), a contradiction. Hence, if \( \theta > 0 \) then \( \phi > 0 \). Suppose next that \( \phi > 0 \) and \( \theta = 0 \). Then

\[ C_d' (V^d) = C_u' (V) \implies c^d = c \implies V^d = \frac{u(c)}{1 - \beta} \]
Moreover

\[ V = u(c) - a + \beta \left\{ (1 - \xi) [pV^e + (1 - p)V^u] + \xi V^d \right\} = u(c) + \beta \left\{ (1 - \xi) V^u + \xi V^d \right\} = V^d (1 - \beta) + \beta \left\{ (1 - \xi) V^u + \xi V^d \right\} > V^u \]

This implies \( V^d > V^u \), violating the truth telling constraint. ■

**Lemma A5** In the unemployment problem, if the constraints bind in one period, they bind in all subsequent periods during the unemployment spell.

**Proof.** Suppose the constraints bind when unemployed in period \( t \) with promise equal to \( V \) but not if still unemployed in \( t + 1 \). The constraints being binding at \( t \) but not in \( t + 1 \) implies, by () () ()

\[ C^e_t (V^e_t) > C^e_u (V^u_t) > C^e_u (V^u_{t+1}) = C^e_u (V^e_{t+1}) \]

Hence

\[ V^e_t > V^e_{t+1} \text{ and } V^u_t = V^u_{t+1}. \]

The effort constraint is binding in \( t \) but not in \( t + 1 \). Hence

\[ \beta (1 - \xi) p (V^e_t - V^u_t) = a \]

\[ \beta (1 - \xi) p (V^e_{t+1} - V^u_{t+1}) > a \]

But then \( V^e_{t+1} > V^e_t \), a contradiction. ■

**Lemma A6** Consider an agent employed at time \( t \) with a value \( V \) such that the truth telling constraint is binding in the employment problem. Then, if unemployed at \( t + 1 \), the constraints in the unemployment problem are binding.

**Proof.** Suppose towards a contradiction that the truth telling constraint is binding when employed at \( t \) but the constraints in the unemployment problem are not when unemployed in \( t + 1 \). Then \( V^u_t = V^d_t < u (c^e_t) / (1 - \beta) \). Moreover, \( C^e_u (V^u_t) = 1 / u' (c^u_{t+1}) > 1 / u' (c^e_t) \).

Since the constraints are not binding in \( t + 1 \), \( C^e_u (V^u_t) = C^e_u (V^u_{t+1}) = C^d (V^d_{t+1}) \implies \)
\( V_{t+1}^d = u\left(c_{t+1}^u\right) / (1 - \beta) \). Combining these three relations we have that \( V_{t+1}^d > V_t^d = V_t^u \).

The effort constraint is slack in \( t + 1 \) and \( V_{t+1}^u > V_{t+1}^d \) since the truth telling constraint when unemployed in \( t + 1 \) is also slack. Therefore,

\[
V_t^u = u\left(c_{t+1}^u\right) - a + \beta \left\{ (1 - \xi) \left[p V_{t+1}^e + (1 - p) V_{t+1}^u\right] + \xi V_{t+1}^d \right\} < u\left(c_{t+1}^u\right) + \beta \left[V_{t+1}^u + \xi V_{t+1}^d\right] > u\left(c_{t+1}^u\right) + \beta V_{t+1}^d = V_{t+1}^d > V_t^u,
\]

a contradiction. ■

**Lemma A7** Consider an agent employed at time \( t \) with a value \( V \) such that the no quitting constraint is binding in the employment problem. Then, if unemployed at \( t + 1 \), the constraints in the unemployment problem are binding.

**Proof.** Suppose the agent is employed at time \( t \) with the no quitting constraint binding and then unemployed in \( t + 1 \) with no constraint binding. Then

\[ C'_{e} (V_{t+1}^e) > C'_{e} (V) > C'_{u} (V_{t+1}^u). \]

This implies \( V_{t+1}^e = V_{t+1}^u > V \). Since \( C'_{e} (V) > C'_{u} (V_{t+1}^u) \) and \( V_{t+1}^u > V \),

\[ C'_{e} (V_{t+1}^e) > C'_{u} (V_{t+1}^u). \]

No constraints being binding at \( t + 1 \) implies that

\[ C'_{e} (V_{t+1}^e) = C'_{u} (V_{t+1}^u) = C'_{u} (V_{t+1}^u) \implies V_{t+1}^u = V_t^u. \]

Hence \( C'_{e} (V_{t+1}^e) < C'_{e} (V_{t+1}^u) = C'_{u} (V_{t+1}^u) \implies V_{t+1}^u > V_{t+1}^e \), violating the effort incentive constraint while unemployed. ■

**Lemma A8** Only the truth telling constraint is binding in the employment problem.

Proof. Suppose towards a contradiction that the no-quitting constraint binds while employed. Then \( V_{t+1}^e = V_t^u \). Let the promises set in \( t + 1 \) be without hats if unemployed then and with hats if employed then. Then, if unemployed in \( t + 1 \), the truth telling and
effort constraints are binding. Hence,

\[ V_t^u = u(c^u) + \beta V_{t+1}^u = u(c^a) + \beta V_{t+1}^d. \]

If still employed in \( t+1 \), the no quitting constraint is still binding. Hence,

\[ V_t^e = u(c^e) + \beta \left( (1 - \xi) \hat{V}_{t+1}^u + \xi \hat{V}_{t+1}^d \right). \]

It is easy to see that the first order conditions while employed implies that \( c^e > c^a \). Then

\[ u(c^e) - u(c^a) = \beta V_{t+1}^d - \beta \left( (1 - \xi) \hat{V}_{t+1}^u + \xi \hat{V}_{t+1}^d \right) > 0 \]

Since the no quitting constraint is the binding one while employes

\[ \left( (1 - \xi) \hat{V}_{t+1}^u + \xi \hat{V}_{t+1}^d \right) > \hat{V}_{t+1}^d. \]

The last two equations imply

\[ V_{t+1}^d > \hat{V}_{t+1}^d. \]

The first order conditions however state that

\[ C'_d \left( \hat{V}_{t+1}^d \right) = C'_e \left( V_t^e \right) > C'_u \left( V_t^u \right) > C'_d \left( V_{t+1}^d \right). \]

Convexity of the cost functions then imply \( V_{t+1}^d < \hat{V}_{t+1}^d \), a contradiction.

\[ \Box \]

7.2 Proof of proposition 2

The truth telling constraint during employment is binding. Thus

\[ C'_e \left( V^e \right) = C'_e \left( V \right) \implies V^e = V, \]
meaning that consumption is constant during the entire employment spell. During un-
employment, the truth telling constraint is binding implies that \( \theta > 0 \). By (16), (19), and (20),
\[
C'_d(V^d) < C'_u(V) \implies \frac{1}{u'(c^d)} < \frac{1}{u'(c)} \implies V^d < \frac{u(c)}{1 - \beta},
\]
(24)
where \( c^d \) denotes consumption in the next period if disabled then and where the the last
implication follows since
\[
\frac{u(c^d)}{1 - \beta} = V^d.
\]
The incentive constraint for effort implies that
\[
V = u(c) - a + \beta \left\{ (1 - \xi) [pV^e + (1 - p) V^u] + \xi V^d \right\}
\geq u(c) + \beta \left\{ (1 - \xi) V^u + \xi V^d \right\}
= u(c) + \beta V^d,
\]
where the last equality follows since the truth telling constraint is binding, \( V^u = V^d \). Note that by (24)
\[
V \geq u(c) + \beta V^d \implies V > (1 - \beta) V^d + \beta V^d = V^d.
\]
Hence
\[
V > V^d = V^u.
\]
Combining (16) and (20) repeatedly we see that \( V^u < V \) implies that consumption is falling during the entire unemployment spell. Consumption being constant during disability is trivial. □

7.3 Proof of proposition 3

During unemployment, we know that since the effort incentive constraint is binding,
\[
C'_u(V) < C'_e(V^e),
\]
and this implies by the envelope conditions in the two problems and the first order conditions with respect to consumption that \( c < c^e \). The truth telling constraint during employment is binding. Then by (7), (9), (11) and (20),

\[
\frac{1}{u'(c^u)} > \frac{1}{u'(c)},
\]

which means that consumption rise upon layoff. If employed last period before turning disabled, by (7), (10), (11), and (21),

\[
\frac{1}{u'(c^d)} < \frac{1}{u'(c)}.
\]

If unemployed prior to turning disabled the same result is produced using (16), (19), (20), and (21). ■

### 7.4 Proof of proposition 4

During employment, the no quitting constraint is not binding, so \( C'_e (V^e) = C'_e (V) \implies V^e = V \). This means that the problem is self generating, so all promised values remain the same during an employment spell. During unemployment we have already seen in proposition 2 that \( V > V^u \). From (17) and (19) we see that \( V^e \) and \( V^u \) are increasing in \( V \). Hence, as the promise during unemployment falls, so does the promise should the agent turn employed or disabled.