Wealth and Unemployment: Do Borrowing Constraints Matter?

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Abstract

It is well known that risk averse workers who are imperfectly insured against shocks to their labor income set reservation wages that depend on their asset holdings. Much less is known about the quantitative importance of this effect on the aggregate outcomes of the labor market. Does the distribution of wealth matter for a country’s unemployment rate? We address this question using a general equilibrium model of frictional unemployment, where all transitions back and forth between employment and unemployment are endogenous. The results show that interactions of the distribution of wealth and of human capital increase the unemployment rate, although the effect is modest in size.

PRELIMINARY AND INCOMPLETE: PLEASE DO NOT CITE.

Acknowledgement 1 The author is grateful to Martin Flodén for his continuous support and help throughout this project.
1 Introduction

Risk aversion and binding borrowing constraints affect the optimal search policy of unemployed workers. Whereas a risk neutral agent will choose a search policy that maximizes the expected discounted sum of future income, the reservation wage of a risk averse worker will, in general, depend also on her ability to cushion the income loss associated with unemployment using her own savings. Unemployed workers who are financially poor and borrowing constrained will be forced to lower their level of consumption while unemployed, making the utility cost of search high. For this reason, search models generally predict that workers’ reservation wages increase in wealth holdings. (Danforth 1979)\textsuperscript{1} Three empirical studies have tested these predictions, finding a negative and significant effect of wealth on the hazard rate.\textsuperscript{2} What is much less clear, however, is whether or not this effect is quantitatively important for aggregate outcomes. As far as we are aware, Chetty (2006) is the only existing study to explicitly address this question. Using data on unemployment spell durations, wealth holdings and on lump-sum severance payments in the United States, he estimates that 70\% of the increase in durations that follow from an increase in the level of unemployment benefits, can be attributed to this wealth effect. Though Chetty cautions that this estimate relies on comparisons across small samples, his study nonetheless implies that borrowing constrained households may potentially

\textsuperscript{1}In models where the variable of choice is a search intensity instead of a reservation wage, a similar result obtains: wealthy workers choose to exert less effort in search than do poor workers (Lentz and Tranas 2005).

\textsuperscript{2}See Alexopoloues and Gladden (2002), Algan et al. (2003) and Bloemen and Stancanelli (2001).
play a non-negligible roll in determining unemployment dynamics.

This paper uses a general equilibrium model of frictional unemployment to investigate whether or not wealth matters for the aggregate unemployment rate. Previous general equilibrium search models that feature endogenous savings, such as Gomes et al. (2001), have been unable to realistically reproduce the high degree of dispersion that characterizes the wealth distribution in the United States and in other industrialized countries. However, given the question we address in this paper, it is crucial that the model can achieve a relatively realistic rendering of the distribution of wealth. This is particularly important at the left tail of the distribution, where agents are close to the borrowing constraint and therefore lead a hand-to-mouth existence.

The existing literature on saving gives valuable clues to why wealth distributions exhibit considerable skewness. In particular, Hugett and Ventura (2000) show that three features of their calibrated life cycle model conspire to produce higher saving rates among high income households than among low income households. These key features are 1) the life cycle motive of saving, 2) relatively permanent differences in earnings abilities across agents, and 3) the structure of the (U.S.) public pension system. Based on these findings, our strategy will be to embed heterogeneity in agents’ productivities and a life cycle motive of saving into a general equilibrium search model. More precisely, we build on the model presented in Gomes et al. (2001), and extend it in two basic dimensions. First, we introduce two types of differences across agents’ earnings abilities, namely a permanent difference between agents with different
educational attainments, and transitory but relatively persistent differences in acquired skills. Second, we allow for a life cycle motive of saving by introducing a state of retirement.

Given these extensions to the original model, some changes to the government policy of that model arises naturally. In Gomes et al. (2001) the tax schedule is regressive in income. Because our alterations of the model will reinforce the wage differences across agents, we replace their quasi lump-sum tax schedule with a flat tax rate on labor, so that the absolute value of the tax is proportional to income. A second change to the government policy of the original model is the introduction of a public pension scheme that has redistributive characteristics. As pointed out by Hugett and Ventua (2000), the redistributive nature of the American Social Security system is an important reason why households with high income have higher savings rates than do low-income households.

Simulations of the model reveal that the distribution of wealth does affect the unemployment rate, though the size of the effect is moderate. A bit surprisingly, the distribution of wealth interacts with the distribution of skills in a way that increases the unemployment rate. When skill dynamics are present, agents’ reservation wages may be both increasing and decreasing in wealth. In particular, low skills and relatively generous unemployment benefits induce poor workers to increase their reservation wages, causing an increase in the unemployment rate.

In the following two sections, we present the model setup and the equilibrium
definition used to solve the model. Section three deals in some detail with the issue of calibrating the model. The results are presented in section five, with a thorough discussion of the benchmark (American) equilibrium. To answer the paper’s main question, we perform an out-of-equilibrium simulation that assesses the impact of the wealth distribution on the equilibrium unemployment rate. Details on the numerical methods used for the solution and simulations of the model are deferred to the Appendix.
2 Model

An economy is populated by a continuum of agents, all of which are either workers or retired citizens. The total mass of these agents is normalized to unity. With per-period probability \( \lambda \) an active agent will start out the following period as retired. When this happens, an agent can no longer earn any labor income, and must therefore live off past savings and government transfers. The only choice of a retired agent is how much of disposable income she will consume and how much she will save to the next period. Death occurs to retired agents with probability \( \kappa \). Deceased agents are immediately replaced with an offspring who inherits any assets that are left behind and who enters active life as unemployed. All agents enjoy consumption and dislike labour. They maximize the expected, discounted sum of future utility, with per period utility defined as follows:

\[
 u(c_t, 1 - l_t) = \left( c_t - \frac{l_t^{\frac{1}{1+\theta}}}{1+\theta} \right)^{1-\sigma} - 1, \quad \theta > 0, \quad \sigma > 0 \tag{1}
\]

Here, \( (1 - l_t) \) represents the amount of leisure the agent enjoys in period \( t \), \( \sigma \) is the coefficient of relative risk aversion, and \( 1/\theta \) designates the elasticity of labor supply. All agents discount future streams of utility at the constant factor \( \beta \in (0, 1) \).

The productivities of working agents depend on the level of their education, the amount of skills they have accumulated while working and the attractiveness of the job at which they are currently employed. A worker’s educational attain-
ment is determined before she enters the labour market and does not change during the course of her working life. There is thus an \textit{ex ante} difference between worker’s productivities that is permanent in nature. On the other hand, workers are \textit{ex ante} identical with respect to their skill level: all agents enter active life at the lowest possible level of skills. The skill level of a worker then evolves stochastically as she moves through active life. The probability of losing and gaining skills varies depending on her employment status. Finally, workers differ with respect to the attractiveness of the jobs at which they are employed.

Let us denote by \((\phi + \gamma_t)\) the level of productivity specific to an individual agent at time \(t\). Here, \(\phi\) takes on the value \(\phi_{nc}\) if the agent does not have a college education, and \(\phi_c\) if she does. At any point in time, her skill level \(\gamma_t\) can take on one out of \(G\) different levels, depending on her work history. Following Ljungqvist and Sargent (1998), we assume that skills evolve differently for employed and for unemployed agents, so that \(H_e(\gamma, \gamma') = \text{prob} \{ \gamma_{t+1} \leq \gamma' | \gamma_t = \gamma \}\) represents the distribution function of \(\gamma\) conditional on employment in period \(t\). If an agent is unemployed in period \(t\), her skill level in period \((t + 1)\) will be distributed according to \(H_u(\gamma, \gamma')\). At the beginning of each period, active agents have at hand a job opportunity that is characterized by an idiosyncratic level of productivity, \(\varepsilon_t\). \(\varepsilon_t\) is the realization of a continuous random variable whose support is the set of real numbers. The job opportunity allows the agent to work in the present period with production technology \(O:\)

\[
O(k_t, l_t; \varepsilon_t + \phi + \gamma_t) = \exp(\varepsilon_t + \phi + \gamma_t)k_t^\alpha l_t^{1-\alpha} \tag{2}
\]
where \( \alpha \in (0, 1) \) and where \( l_t \) and \( k_t \) are, respectively, the inputs of labour and capital. A worker supplies her own labour to the job and rent capital from a competitive spot market. There is one homogenous type of good, which can be used either for consumption or as capital in production. When the good is used as capital, the rate of depreciation is \( \delta \).

When a new period begins, active agents observe their own productivity \( (\phi + \gamma_t) \) and that of the job currently at hand, \( \varepsilon_t \). Given their level of savings, they then decide whether to work at the available job or to become unemployed and search for a new job. If an agent chooses to work, she can rent capital in the competitive capital market at rental rate \((r + \delta)\). Agents who decide to work choose optimal levels of \( k_t \) and \( l_t \), and they pay for the capital used in production and are subject to a per period flat tax rate on labour income, \( \tau \). Output net of rental and tax payments is divided between consumption and savings. In the following period, a new realisation of the job-specific productivity, \( \varepsilon_{t+1} \), is drawn from the distribution \( I(\varepsilon; \varepsilon') \). Together with a new realisation of the individual productivity, \( (\phi + \gamma_{t+1}) \), \( \varepsilon_{t+1} \) will determine the attractiveness to the agent of keeping the job. If the agent decides not to retain the job, she becomes unemployed in period \( t + 1 \) and can then start to search for new jobs.

Agents who decide not to work in period \( t \) are considered to be unemployed in that period. This means they have no income besides the interest on her savings and the unemployment benefits. Unemployed agents pay no taxes. Like other agents, they decide how much to consume and how much to save. After each period of search, unemployed agents receive a new job opportunity with
the level of productivity, $\varepsilon_{t+1}$, drawn from $J(\varepsilon)$. Just like employed agents, the unemployed then decide either to retain the new job opportunity and to work or to remain unemployed for one more period, waiting to draw a new productivity $\varepsilon_{t+2}$ in the period after that. The search technology specified here follows Gomes et al. (2001). Unemployed agents cannot decide how much search effort to exert; their only choice is an optimal reservation productivity.

The government pays unemployment benefits as a lump sum transfer $b$ to all unemployed agents in every period. The government also runs a pay-as-you-go pension scheme, with a transfer $s$ paid to all retired agents in all periods. These undertakings are financed by a proportional income tax $\tau$ levied on all labor earnings. In every period, the government balances its budget.

## 3 Recursive formulation

Define $Y_j(\varepsilon, \gamma)$ as the income net of rental and tax payments and of the disutility of work of an employed agent whose permanent productivity is $\phi_j$:

$$Y_j(\varepsilon, \gamma) = \max_{k,l} \left\{ (1 - \tau) \left[ O(k, l; \varepsilon + \phi_j + \gamma) - (r + \delta)k \right] - D(l) \right\} \quad (3)$$

where $D(l) = l^{1+\theta}$. Further, let $\tilde{c}$ refer to an agents’ consumption net of the disutility of work. This notation is convenient since the agent’s optimization over $k$ and $l$ is independent of her intertemporal optimization over $c$ and $a'$. The
functional form of the per-period utility implies that agents make no distinction
between units of consumption and utils derived from leisure. Workers are thus
indifferent between two consumption baskets with the same level of \( \tilde{c} \) but with
different compositions of consumption, \( c \), and leisure. For future reference,
let \( K_j(\varepsilon, \gamma) \) and \( L_j(\varepsilon, \gamma) \) be the policy functions for capital and labour inputs
that solve 3, given that the agent’s permanent productivity is \( \phi_j \). In order to
keep track of tax payments to the government, we also define \( \bar{Y}_j(\varepsilon, \gamma) \) to be the
income, net of rental payments, of a working agent with permanent productivity
\( \phi_j \):

\[
\bar{Y}_j(\varepsilon, \gamma) = O \{ K_j(\varepsilon, \gamma), L_j(\varepsilon, \gamma); \varepsilon + \phi_j + \gamma \} - (r + \delta)K_j(\varepsilon, \gamma)
\] (4)

Now, let \( W^j(a, \varepsilon, \gamma) \) be the value of a working agent whose permanent, in-
dividual productivity is \( (\phi_j + \gamma) \), who has at hand a job opportunity with
productivity level \( \varepsilon \) and whose level of savings is \( a \). Further, let \( S^j(a, \gamma) \) be the
value function of an unemployed worker and denote by \( R(a) \) the value function
of a retired agent. The worker’s problem is then:
\[ W^j(a, \varepsilon, \gamma) = \max_{\tilde{c}, a'} \left\{ U(\tilde{c}) + \lambda \beta R(a') + (1 - \lambda) \beta \int \sum_{\gamma'} \max [W^j(a', \varepsilon', \gamma'), S^j(a', \gamma')] h_e(\gamma' | \gamma) dI(\varepsilon, \varepsilon') \right\} \]

\[ \text{(P1)} \]

s.t.

\[ \tilde{c} + a' = Y_j(\varepsilon, \gamma) + (1 + r)a \]

where \( h_e(\gamma' | \gamma) \) is the conditional probability of \( \gamma' \), given \( \gamma \), and where \( \bar{a} \) is a borrowing limit.

The searcher’s problem is:

\[ S^j(a, \gamma) = \max_{\tilde{c}, a'} \left\{ U(\tilde{c}) + \lambda \beta R(a') + (1 - \lambda) \beta \int \sum_{\gamma'} \max [W^j(a', \varepsilon', \gamma'), S^j(a', \gamma')] h_a(\gamma' | \gamma) dJ(\varepsilon') \right\} \]

\[ \text{(P2)} \]

s.t.

\[ \tilde{c} + a' = (1 + r)a + b \]

\[ a' \geq \bar{a}, \]

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Finally, the recursive problem of a retired agent reads:

\[ R(a) = \max_{c,a'} \{ U(c) + (1 - \kappa)\beta R(a') \} \]  

s.t.

\[ c + a' = (1 + r)a + s \]

\[ a' \geq \bar{a}, \]

The solution to P1 is an optimal policy function for savings, \( a' = A_j^{W}(a, \varepsilon, \gamma) \).

Similarly, the policy functions that solve the searchers’s and the retiree’s programs are \( A_j^{S}(a, \gamma) \) and \( A^{R}(a) \), respectively. In the beginning of a period, an agent with permanent productivity \( \phi_j \) and state \( (a, \varepsilon, \gamma) \) will choose to work if \( W_j^{j}(a, \varepsilon, \gamma) \geq S_j^{j}(a, \gamma) \) and will search otherwise. The optimal policy of an active agent with respect to the employment decision will be represented by the function \( \Omega_j(a, \varepsilon, \gamma) \), where:

\[
\Omega_j(a, \varepsilon, \gamma) = \begin{cases} 
1 & \text{if } W_j^{j}(a, \varepsilon, \gamma) \geq S_j^{j}(a, \gamma) \\
0 & \text{otherwise} 
\end{cases} \]  

(5)

Given the employment policy \( \Omega \) of active agents, an optimal savings policy for active agents, \( A_j(a, \varepsilon, \gamma) \), reads:

\[
A_j(a, \varepsilon, \gamma) = \Omega_j(a, \varepsilon, \gamma) A_j^{W}(a, \varepsilon, \gamma) + [1 - \Omega_j(a, \varepsilon, \gamma)] A_j^{S}(a, \gamma)
\]
In a steady state equilibrium, the distribution of agents across different states of productivity and wealth is time invariant. Define \( z^A_j(a, \varepsilon, \gamma) \) and \( z^R(a) \) to be measures of active and retired agents over the state space. \( z^A_j(a, \varepsilon, \gamma) \) is then the measure of active agents with permanent productivity \( \phi_j \) who enjoy the transitory productivity \((\varepsilon + \gamma)\) and who holds \( a \) units of savings. With these definitions, the market clearing condition of the capital market reads:

\[
\int \left\{ \sum_j \sum_a z^A_j(a, \varepsilon, \gamma) A_j(a, \varepsilon, \gamma) + z^R(a) A^R(a) - \sum_j \sum_a \Omega_j(a, \varepsilon, \gamma) z^A_j(a, \varepsilon, \gamma) K_j(\varepsilon, \gamma) \right\} \text{d} \varepsilon = 0
\]

(6)

The government runs a balanced budget, with tax receipts exactly offsetting the payments to the unemployed and the retired. A balanced policy \( \{b, s, \tau\} \) satisfies:

\[
\tau \int \sum_j \sum_a \Omega_j(a, \varepsilon, \gamma) z^A_j(a, \varepsilon, \gamma) \bar{Y}_j(\varepsilon, \gamma) \text{d} \varepsilon = \int \left\{ \sum_j \sum_a [1 - \Omega_j(a, \varepsilon, \gamma)] b z^A_j(a, \varepsilon, \gamma) + s z^R(a) \right\} \text{d} \varepsilon
\]

(7)
A *recursive equilibrium* consists of:

1) A collection of value functions \( \{W_j(a, \varepsilon, \gamma), S_j(a, \gamma)\} \) \( j = \{nc, c\} \) and \( R(a) \), and associated policy functions such that:

\[
W_j(a, \varepsilon, \gamma) \text{ and } A^W_j(a, \varepsilon, \gamma) \text{ solves P1 for } j = nc, c.
\]

\[
S_j(a, \gamma) \text{ and } A^S_j(a, \gamma) \text{ solves P2 for } j = nc, c.
\]

\[
R(a) \text{ and } A^R(a) \text{ solves P3}
\]

2) A policy function \( \Omega_j(a, \varepsilon, \gamma) \) as defined in (5).

3) An interest rate \( r \) such that (6) holds.

4) A government policy \( \{b, s, \tau\} \) such that (7) is satisfied.

We solve the model numerically. As a benchmark for further analysis, the following section outlines a calibration to American data. A brief explanation of the solution method can be found in the appendix.

### 4 Calibration

The functional form of agents’ utility and several parameters of that function are the same as in Gomes et al. (2001). Specifically, the coefficient of relative risk aversion, \( \sigma \), is set to 2 and the elasticity of labour supply, \( \frac{1}{\beta} \), to 0.1. Further, a time period of the model is half a quarter. The parameters governing the life cycle dynamics (\( \lambda \) and \( \kappa \)) are fixed so as to make the average agent be active for
50 years and retired for 20 years. These life cycle dynamics adds considerably to agents’ impatience. For this reason, the discount factor $\beta$ is fixed at a higher value than in Gomes et al. (2001).

### 4.1 Human Capital and Technology

According to OECD (2004), 38% of American workers held some form of college education in 2002. In the calibrated model, then, 38% of all active agents are assigned the higher of the two values of permanent productivity, $\phi_c$. In what follows, these agents will be referred to as ‘workers with college’ or ‘educated workers’. Furthermore, in 2003, workers with tertiary education earned an hourly wage that on average was 70% higher than workers without such education (OECD 2005). Assuming that the two different values of $\phi$ are symmetric around zero, this statistic identifies the two values of $\phi$.

Turning to the transitory component of worker-specific productivity, $\gamma$, this variable evolves over time as the individual worker gains experience and suffers spells of unemployment. Here, the calibration of $\gamma$ roughly follows that of the corresponding skill variable in Ljungqvist and Sargent (1998). In particular, it is assumed a) that an agent with the highest (transitory) skill level earns exactly twice as much as an agent with the lowest skill level, and b) that working agents face a certain probability that their skills will appreciate, while unemployed agents are subject to stochastic skill losses.\(^3\)

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\(^3\)Ljungqvist and Sargent (1998) focus their analysis on the interaction of human capital accumulation and unemployment insurance. They therefore allow for a fine grid of 21 different skill levels. In the present model, where much analytical effort is spent on precautionary savings, the grid for transitory skill levels must necessarily be much coarser. Thus, $\gamma$ can take
The last part of a worker’s productivity state, \( \varepsilon \), is specific to the job opportunity that the agent currently has at hand. For employed agents, \( \varepsilon \), is assumed to be determined as the outcome of an AR(1) process:

\[
\varepsilon' = \rho \varepsilon + \xi, \quad \xi \sim N(0, \sigma^2_{\varepsilon}) \quad \text{and} \quad \rho \in (0, 1) \tag{8}
\]

For searching agents, on the other hand, \( \varepsilon \) is determined as a random draw from a normal distribution:

\[
\varepsilon = v, \quad v \sim N(\mu_v, \sigma^2_v) \tag{9}
\]

For comparability, the benchmark calibration has the same value of \( \rho \), 0.9, as the one chosen by Gomes et al. (2001). In that paper, \( \sigma^2_\varepsilon \) and \( \sigma^2_v \) are chosen so as to make the model match the average rate of unemployment in the U.S. (5.9%) and the average duration of unemployment spells (13 weeks). Gomes and coauthors report that with \( \sigma_\varepsilon = 0.052 \) and \( \sigma_v = 0.085 \), the corresponding unemployment rate and duration of the model are 6.1% and 11 weeks, respectively. Again striving for comparability, we calibrate \( \sigma^2_\varepsilon \) and \( \sigma^2_v \) so as to match an unemployment rate of 5.9% and a duration of 11 weeks. In all calibrations reported in this paper, changes in \( \sigma^2_\varepsilon \) and \( \sigma^2_v \) have the same qualitative effect on the model’s unemployment rate and duration. An increase in the variability of one out of three different values. The chosen transition probabilities imply that an employed agent, who suffers no unemployment, is expected to advance from the lowest to the highest skill level in ten years. When unemployed, the rate of skill depreciation is twice as fast. (In Ljungqvist and Sargent (1998), it takes on average 7 years and 8 months for a worker to double her earnings ability. Skill depreciation is twice as fast.)
of the shocks to a worker’s productivity, \( \sigma_z^2 \), produces an increase in the unemployment rate and a decrease in the average duration of spells. Intuitively speaking, an increase in \( \sigma_z^2 \) makes it more likely that a worker’s job-specific productivity will suffer an unfavourable shock, making it more likely that agents will quit their jobs and start searching. The upshot is an increased flow of workers into unemployment. The increased volatility of wages also affect the search policy of unemployed agents, since it decreases the incentive to wait for a good wage offer. An increase in \( \sigma_z^2 \) therefore induce agents to lower their reservation productivities, thereby lowering the expected duration of each spell.

An increase in the variance of the job-offers received by searchers, \( \sigma_v^2 \), also induces an increase in the unemployment rate, but such a change also causes an increase in the average duration. This result is well known feature of search models: increasing the dispersion of the wage offers received by unemployed agents increases the benefit of continued search, inducing agents to become more picky about which jobs to accept.

While rather small changes in \( \sigma_z^2 \) and \( \sigma_v^2 \) bring about large effects on the unemployment rate, it takes considerably larger changes in these two parameters to achieve a similar percentage change in the duration of unemployment. This is the reason we target 11 weeks of unemployment duration instead of 13 weeks. If the target of the calibration exercise would be to achieve 13 weeks, the parametrization of shocks would be very different from a calibration with 11 weeks of duration.
4.2 Government Policy

A government policy consists of: 1) the transfer to the unemployed, \( b \); 2) the transfer to the retired agents, \( s \); and 3) a tax rate \( \tau \) on labor income. Given the focus on steady states with a balanced budget, it is only the level of the two transfers that will have to be calibrated; it will then be the duty of the solution algorithm to find a corresponding tax rate.

In a study of the wealth holdings of unemployed American workers, Gruber (2001) uses a sample consisting of all spells of unemployment between 1984 and 1992 that are recorded in the Survey of Income and Program Participation. Using a simulation program to proxy for UI eligibility among sample households, the author finds that the average worker receives benefits amounting to roughly 45% of the previously held wage rate. However, because workers with low educational attainment face higher unemployment rates than more educated workers, there are reasons to believe that this number overstates the replacement rate faced by the average American worker (OECD 2005). The sample used in Gruber (2001) is representative of actually unemployed workers, while here we are looking for the average replacement rate of agents in the workforce. Based on these considerations, we set the replacement rate of the benchmark calibration to 50% of the average wage rate of uneducated workers. Transfers to the retired citizens, on the other hand, are defined as a fraction of the average wage of all workers. Following Eisensee (2006), this fraction is set to 45%.^4 The benchmark

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^4Concerning both the unemployment insurance and the pension system, the calibrated replacement rates refer to the average wage rate net of taxes and rental payments to capital owners.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subjective Discount Rate</td>
<td>0.9951</td>
</tr>
<tr>
<td>Relative Risk Aversion</td>
<td>2</td>
</tr>
<tr>
<td>Elasticity of Labour Supply</td>
<td>0.10</td>
</tr>
<tr>
<td>Probability of Retirement</td>
<td>0.0025</td>
</tr>
<tr>
<td>Probability of Death</td>
<td>0.00625</td>
</tr>
<tr>
<td>Support of Permanent Productivities</td>
<td>[-0.1544 0.1544]</td>
</tr>
<tr>
<td>Support of Transitory Skills</td>
<td>[-0.2016 0.0343 0.2016]</td>
</tr>
<tr>
<td>Persistence in Shocks to Worker Prod.</td>
<td>0.9</td>
</tr>
<tr>
<td>Std of Shocks to Worker Prod.</td>
<td>0.020</td>
</tr>
<tr>
<td>Std of Searcher’s Prod. Distr.</td>
<td>0.175</td>
</tr>
<tr>
<td>Replacement Rate UI</td>
<td>50%</td>
</tr>
<tr>
<td>Replacement Rate Social Security</td>
<td>45%</td>
</tr>
</tbody>
</table>

w refers to the average wage rate, net of taxes, of all working agents, while \( w^{NC} \) refers to the average wage rate of working agents who do not have a college education.

Table 1: Calibration Summary

calibration is summarized in table 1.
Table 2: Benchmark (American) Equilibrium

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Replacement Rate UI</td>
<td>50%</td>
<td>45%\textsuperscript{1}</td>
</tr>
<tr>
<td>Social Security</td>
<td>45%</td>
<td>45%\textsuperscript{2}</td>
</tr>
<tr>
<td>Tax Rate</td>
<td>13.9%</td>
<td>22.6%\textsuperscript{3}</td>
</tr>
<tr>
<td>Interest Rate (yearly)</td>
<td>4.80%</td>
<td>4.80%</td>
</tr>
<tr>
<td>Capital-to-Output Ratio (yearly)</td>
<td>3.78</td>
<td>0.32\textsuperscript{4}</td>
</tr>
<tr>
<td>Mean-to-Median Ratio of Wealth</td>
<td>1.56</td>
<td>4.03\textsuperscript{5}</td>
</tr>
<tr>
<td>Gini Coefficient of Wealth</td>
<td>0.48</td>
<td>0.80\textsuperscript{5}</td>
</tr>
<tr>
<td>Unemployment Rate</td>
<td>5.9%</td>
<td>5.9%\textsuperscript{6}</td>
</tr>
<tr>
<td>Unemployment Rate No College</td>
<td>7.4%</td>
<td>6.9%\textsuperscript{7}</td>
</tr>
<tr>
<td>Unemployment Rate College</td>
<td>3.4%</td>
<td>3.4%\textsuperscript{7}</td>
</tr>
<tr>
<td>Average Duration (weeks)</td>
<td>11.5</td>
<td>13\textsuperscript{6}</td>
</tr>
</tbody>
</table>

\textsuperscript{1}Gruber (2000).
\textsuperscript{2}Eisensee (2006).
\textsuperscript{3}Carey and Tchilinguirian (2000).
\textsuperscript{4}Cooley and Prescott (1995).
\textsuperscript{5}Rodríguez et al. (2002).
\textsuperscript{6}Gomes et al. (2001).
\textsuperscript{7}OECD (2005).

Table 2: Benchmark (American) Equilibrium

5 Results

The main aggregate variables of the benchmark economy are presented in table 2. The model makes predictions concerning the distribution of wealth and the unemployment rates specific to uneducated and educated workers, predictions that are well suited to evaluate the model's ability to match statistics in the data. In order to investigate these predictions and to explore the economic forces at work in the model, we will take some time out to discuss, in turn, the distribution of wealth and the incentives of workers. Following that, we will present an out-of-equilibrium exercise that quantify the effect of the wealth distribution on the aggregate unemployment rate.
Table 3: Wealth Distribution by Quintiles

<table>
<thead>
<tr>
<th></th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>-0.3</td>
<td>1.3</td>
<td>5.0</td>
<td>12.2</td>
<td>81.7</td>
</tr>
<tr>
<td>Model</td>
<td>0.3</td>
<td>4.7</td>
<td>13.0</td>
<td>25.2</td>
<td>56.9</td>
</tr>
</tbody>
</table>

The first column shows the fraction of total wealth held by the poorest 20% of households, etc. The numbers in the first row are taken from table 7 in Burrida-Rodriguez et al. (2002).

5.1 The Wealth Distribution

Not surprisingly, increased heterogeneity in workers’ productivity produces increased dispersion in wealth. While the Gini coefficient of wealth in Gomes et al. (2001) is 0.38, the corresponding statistic in our benchmark economy is 0.48. Even so, the dispersion of the model economy is still far from the Gini coefficient of 0.80 that characterize the U.S. distribution of wealth. In table 3, the American and model wealth distributions are decomposed by quintiles.

Behind the statistics of the aggregate wealth distributions lurks important differences across the different educational and skill groups. The average white-collar worker in the model is twice as wealthy as the average blue-collar worker, and the difference in median wealth is even larger: the median wealth of white-collar workers is larger than that of blue-collar workers by a factor of 2.2. Differences of similar magnitudes appear when the wealth distribution is decomposed by skills. Table 4 presents the distribution of active agents across the three different skill levels, as well as a decomposition of wealth by skills.

The upper part of table 4 reveals small but non-negligible differences in skills between the two groups, which might lead one to expect that the dif-
Distribution of Skills
(Percentage of Total by Educational Attainment.)

<table>
<thead>
<tr>
<th>Skill Level</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>no college</td>
<td>15.9</td>
<td>18.6</td>
<td>65.4</td>
</tr>
<tr>
<td>college</td>
<td>14.6</td>
<td>14.3</td>
<td>71.1</td>
</tr>
</tbody>
</table>

Median Wealth Holdings by Skill Group
(Percentage of the Median Wealth of All Active Agents.)

<table>
<thead>
<tr>
<th>Skill Level</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>no college</td>
<td>14.6</td>
<td>43.3</td>
<td>95.1</td>
</tr>
<tr>
<td>college</td>
<td>19.5</td>
<td>51.4</td>
<td>223</td>
</tr>
<tr>
<td>all</td>
<td>16.3</td>
<td>45.9</td>
<td>131</td>
</tr>
</tbody>
</table>

The upper pane presents the share of workers with different skill levels, by educational attainment. The lower pane presents the median wealth of workers in different skill groups, as a percentage of the median wealth of all working agents.

Table 4: Skills and Wealth.

ference in average earnings between the two groups would be larger than the 70% targeted in the calibration. It turns out, however, that the average labor income of white-collar workers is 69.5% higher than that of blue-collar workers, indicating that the average job-specific productivity of blue-collar workers is actually higher than that of educated workers. As will become obvious in the next subsection, blue-collar workers have better average job-specific productivity because they are more picky about which jobs they accept. Why, then, do white-collar workers build twice as large assets as blue-collar workers, when the average earnings difference is only about 70%? The reason lies in the redistributive character of the government policies, which fixes the transfers to all unemployed and retired agents at the same absolute levels, b and s. Because the effective replacement rates of the two government programs are lower for educated than for non-educated workers, educated workers will have a stronger
incentive to build savings in order to smooth consumption between states of work, unemployment and retirement. For the same reasons, there are large differences in wealth holdings across workers with different skills. All workers start their working life with low skills, and then gradually accumulate skills during their employment spells. When agents find themselves in a state of high skills, they increase their saving rates in order to build a buffer stock of savings that can be used in case of unemployment, loss of skills or retirement.

5.2 Incentives to Work and to Search

As reported in table 2, the model does a good job at predicting unemployment rates specific to workers with and without education. For blue-collar workers, the benchmark calibration produces an unemployment rate of 7.4%, while that of white-collar workers is 3.4%. In 2003, the unemployment rates of American workers without and with tertiary education stood at 6.9% and 3.4%, respectively. Note that while the model was calibrated to yield an aggregate unemployment rate of 5.9%, the jobless rates specific to the two educational groups where not targeted in the calibration. Why does the model produce different unemployment rates for these two groups of workers? The answer, again, is in the design of government policy. All unemployed workers receive a per-period

\footnote{OECD (2005) reports American unemployment rates, for the year 2003, by three groups of educational attainment: lower secondary education or less, upper secondary education and tertiary education. The first two of these groups are identified as uneducated workers in the model, and the group with tertiary education is identified as educated workers. In order to obtain an unemployment rate for the whole group of uneducated workers, the unemployment rates of the first two groups of workers in OECD (2005) are weighted by the shares of the population with the corresponding educational attainments, reported for the year 2002 in OECD (2004).}
transfer of $b$, irrespective of their previous earnings history. As a consequence, educated workers face a lower effective replacement rate than do blue-collar workers, implying that the opportunity cost of search is greater for educated workers. In agents’ policy functions, this difference in opportunity costs translates into differences in reservation productivities, leading educated workers to accept jobs with lower job-specific productivity, $\bar{z}$. Because white-collar workers have higher permanent productivity, they nevertheless receive higher wages than do blue-collar workers.

Obviously, the design of government policy in the model is a simplification of the policies put in place in the United States, where the level of benefits paid to an individual worker depends on her previous income. Even so, the policy of the model economy does retain one important feature of its real world counterpart: all public unemployment schemes in the U.S. contain caps on their benefit levels, implying that benefits raise with previous income only to the level of the cap. Therefore, American workers with moderately high and high wages do face lower effective replacement rates than do low-income workers.

Summarizing the differences in behavior that pertain to educated and non-educated workers, we have seen that white-collar workers have higher saving rates and that they set lower reservation productivities than do blue-collar workers. The differences in incentives that arise because workers have different skill levels are somewhat more involved, owing to the dynamic character of the skill variable. Figure 1 displays the hazard rates out of unemployment for workers
without college education, as a function of their assets.⁶ In order to highlight the rather drastic changes in the hazard rates of highly skilled agents close to the borrowing constraint, Figure 2 reproduces the left-hand part of Figure 1, but at a different scale.

The hazard rates depicted in Figures 1 and 2 are determined jointly by

⁶The corresponding hazard rates for educated workers look very similar, with the difference that they are higher. For the sake of brevity, we therefore omit them.
Figure 2: Hazard Rate (per period) for Workers without Education (Reproduction of the left-hand part of Figure 1).
agents’ reservation productivity policies, and the distribution of job offers to the unemployed (as defined by 9). The same reservation productivity policies, combined with the distribution of shocks to the wages of employed agents (as defined by 8) determine the flow of workers into unemployment. First, let us focus on the hazard rates of a worker with a given level of assets, and see how her incentives change depending on her skill level. Disregarding the very bottom of the wealth distribution, Figure 1 reveals a v-shaped relationship between the hazard of an unemployed worker and her skill level. Low-skilled workers set the highest reservation productivities, translating into low hazard rates. Workers with intermediate skills set the lowest reservation productivities, while highly skilled workers fix their reservation productivities at an intermediate level. This non-monotonic relationship between skills and reservation wage policies was first discussed by Ljungqvist and Sargent (1998). As in their model, there are two opposing economic forces at work in our model: the incentive of workers to accumulate skills, and the incentive to find a job with a high wage rate. At the lowest skill level, workers need not fear that they lose skills, and therefore the incentive to find a job with a high productivity dominates. At the intermediate level, agents care more about preserving the skills they have and about gaining new ones, and they accordingly set very low reservation productivities. At the highest level of skills, the incentive to find a job which pays again dominates.

When agents choose how to vary their reservation productivities with their level of assets, a third economic force comes into play, namely the desire to use savings as a buffer stock against income fluctuations. In search models with no
skill dynamics, in which unemployment is the only source of income uncertainty, the desire to smooth consumption gives rise to a well known negative relationship between hazard rates and wealth. The smaller assets an agent have, the more costly it is in utility terms to forgo labor income, and when she is close to the borrowing limit, leading a hand-to-mouth existence, there is no alternative than to accept any job offer, be it at a very low wage rate. In our model, this familiar trade-off appears in the hazard rates of agents with the highest skill level, for which there are no more skills to acquire. These workers lower their reservation productivity the smaller are their assets, with rather drastic changes close to the borrowing contraint. This ‘liquidity effect’, that motivated Chetty (2006), appears clearly in Figure 2.

For workers with lower skills, the incentive to accumulate new skills enters the decision problem, giving rise to reservation wage policies that may be both increasing and decreasing in wealth. Reservation wages decrease in wealth when the effective replacement rate of the unemployment insurance and the expected returns from future skill accumulation are both high. Under such circumstances, workers may sometimes accept jobs such that the consumption net of the disutility of work effort is lower than that afforded by the unemployment benefits. Workers in this situation accept a low-paid job in the anticipation of future gains in skills. Such investment decision appear more favourable to wealthy workers, who can supplement the low wage income with her savings. This is why, in Figures 1 and 2, the hazard rates of low-skilled workers increase with wealth. In fact, low-skilled agents with no savings and no education set the high-
est reservation productivity of all workers in our calibration, with a per period hazard rate of 0.386 and an expected duration of unemployment corresponding to 15.6 weeks. These workers are effectively caught in a trap of low skills and poverty, where relatively generous UI benefits remain their best option, period after period.

5.3 The Quantitative Importance of Wealth to Aggregate Unemployment

As we have seen, the effect of wealth on an individual agent’s reservation productivities is ambiguous, and depend on where in the asset-skill distribution she finds herself. Therefore, the questions of knowing the sign and the size of the effect of the wealth distribution on the aggregate unemployment rate are both quantitative in nature. What we would like to know, more precisely, is how the equilibrium unemployment rate would change if agents’ positions in the wealth distribution did not affect their decision to work or to search. To answer this question, we perform an out-of-equilibrium simulation of the economy, where we force all agents to make this decision as if they held the median level of wealth. This simulation is performed using a large sample of agents that is representative of the distributions of skills and wealth of the benchmark equilibrium. One interpretation of this exercise is that it collapses the benchmark equilibrium to a model with a representative level of wealth. However, since our model contains two types of agents, workers with and without education, each agent will be forced to set reservation productivities as if she held the median level of wealth.
Table 5: Out-of-Equilibrium Simulation with Forced Decisions, Compared to Benchmark Equilibrium.

<table>
<thead>
<tr>
<th></th>
<th>All Active Agents</th>
<th>No College</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Benchmark Equilibrium</strong></td>
<td>5.9</td>
<td>7.4</td>
<td>3.4</td>
</tr>
<tr>
<td><strong>Forced Decisions</strong></td>
<td>5.6</td>
<td>6.9</td>
<td>3.3</td>
</tr>
<tr>
<td><strong>Change Relative to Benchmark</strong></td>
<td>-5.7%</td>
<td>-6.5%</td>
<td>-2.3%</td>
</tr>
</tbody>
</table>

Table 6: Median Wealth Holdings by Skill Group

<table>
<thead>
<tr>
<th>Skills</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>No College</td>
<td>19.1</td>
<td>56.7</td>
<td>124</td>
</tr>
<tr>
<td>College</td>
<td>11.5</td>
<td>30.3</td>
<td>131</td>
</tr>
</tbody>
</table>

(Percentage of Median of the Respective Educational Type)

of her type.

Table 5 reports the results of this simulation. The economy-wide unemployment rate falls from 5.9% to 5.6%, with the largest relative decline among blue-collar workers. The implication is that in general equilibrium, the net effect of the distribution of wealth on workers decisions to work and to search is to increase the aggregate unemployment rate. To understand this result one must look at the joint distribution of skills and wealth, and compare it with the hazard rates shown in Figures 1 and 2. Table 6 shows the median wealth holdings of agents across different skill groups, reported as a percentage of the median wealth holding of the respective educational type. As discussed earlier, there is a strong, positive correlation between skills and wealth. Among blue-collar workers, for example, the median wealth of low-skilled workers is only 19.1% of the median wealth holding of all agents of this educational type. Figure 2 showed that high-skilled workers lowered their reservation wage quite
drastically close to the borrowing constraint. However, because most high-skilled workers are relatively wealthy, this change in their reservation wage does not significantly affect aggregate outcomes. Instead, the out-of-equilibrium simulation performed here will force most workers in this group to make their decision to work or to search as if they held less savings than they actually do, with the result that they set a lower reservation productivity than they do in equilibrium.

Looking now at the very poorest workers, they are instead overrepresented in the group of low-skilled agents; these are the workers that perceive unemployment benefits to be quite generous, and who are reluctant to give them up for a low-paid job. Referring back to Figure 2, the out-of-equilibrium simulation will push these agents to the right, causing their hazard rates to increase. In summary, the interactions of the distributions of skills and wealth conspire to make the equilibrium unemployment rate higher than had it been in a representative agent framework.
References


A Solution Algorithm

The solution algorithm computes the model’s equilibria in two stages. For a given guess on the policy variables \( \{b, s, \tau\} \), an inner loop of the algorithm finds the corresponding equilibrium of the capital market as a fixed point in \( r \), the interest rate. Once approximations to the value functions and policy functions are found, the economy is simulated for a large number of periods and the excess supply of capital is computed. A new guess on the interest rate is picked, after which the process is repeated. When a capital market equilibrium is found, the program finds out whether or not the government’s budget constraint is satisfied. If this is not the case, a new guess on \( \{b, s, \tau\} \) is initiated and the process of finding an equilibrium interest rate starts over. A policy \( \{b, s, \tau\} \) is accepted as an equilibrium policy when the difference between the government’s receipts and expenses is smaller than some predetermined level of tolerance.\(^7\)

The functions \( S_j(a, \gamma) \) and \( R(a) \) are approximated by a set of one-dimensional, piecewise cubic splines, the argument of which is \( a \), the level of savings. The chosen interpolation method is shape-preserving in the sense that it preserves monotonicity.\(^8\) Given \( G \) different levels of skills \( (\gamma \in \{\gamma_1, \gamma_2, \ldots, \gamma_G\}) \) and \( P \) different levels of educational attainment \( (\phi \in \{\phi_1, \phi_2, \ldots, \phi_P\}) \), \((G \times P)\) different splines are needed to approximate \( S_j(a, \gamma) \).

In a similar fashion, \((G \times P)\) different functions are used to approximate

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7. Note that since both \( b \) and \( s \) are fractions of the average wage, net of taxes, a specific government policy is completely defined by two variables, either \( \tau \) and \( b \) or \( \tau \) and \( s \).

8. For an exhaustive explanation of this interpolation method, the interested reader is referred to Judd (1998).
the value function of a working agent, $W^j(a, \varepsilon, \gamma)$. In this case, however, the original function to be approximated has two arguments, $a$ and $\varepsilon$. The approximand chosen for this interpolation problem is a two-dimensional spline that is piecewise cubic in $a$-space and linear in $\varepsilon$-space.

The process of finding good approximands for $W^j(a, \varepsilon, \gamma)$, $S^j(a, \gamma)$ and $R(a)$ is initiated with a concave guess on each of these functions. These guesses are then used to evaluate the continuation values of the corresponding Bellman equations. For each gridpoint in $a$-space, a maximization algorithm finds the optimal level of savings to carry to the next period. The right-hand side of the Bellman equations are then evaluated at the gridpoints in $a$-space using the optimal policy, producing a new and updated guess on the functions $W^j(a, \varepsilon, \gamma)$, $S^j(a, \gamma)$ and $R(a)$. The process is repeated until convergence is achieved.