Directed technical change as a response to natural-resource scarcity

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We develop quantitative macroeconomic theory of input-saving technical change to analyze how markets economize on scarce natural resources, with an application to fossil fuel. We find that aggregate U.S. data calls for a very low short-run substitution elasticity between energy and the capital/labor inputs. Our estimates imply that energy-saving technical change took off when the oil shocks hit in the 1970s. This response implies significant substitutability with the other inputs in the long run: even under every-rising energy prices, long-run consumption growth is still possible, along with a modest factor share of energy.

1 Introduction

What is the future of our dependence on natural resources in finite supply? How will consumption growth be affected by scarcity? We develop quantitative theory to answer these questions and apply it to the case of fossil fuel-based energy as an input into production. The market's first response to scarcity is a rise in the price of the scarce resource, with curbed use as a result. In this paper we focus on an implication of a higher price: endogenous resource-saving technical change, in the form of new techniques and products allowing us to save on the scarce inputs. We use the theory to interpret the postwar U.S. data on fossil energy dependence but we also make projections into the future.

In concrete terms we formulate an aggregate production function that is a nested CES in a capital-labor composite and fossil energy. The formulation allows us to identify separate input-augmenting technology series: by applying an assumption of perfect competition in the input markets, the technology levels can be backed out from firm demand, given data on inputs and their prices. The findings are striking: (i) fossil-energy saving was dormant until the oil shocks hit but then took off and grew rapidly; (ii) capital/labor saving grew more slowly after the oil shocks; and (iii) more generally, fossil-energy prices and saving on fossil energy comove clearly even if one excludes the large oil shocks in the 1970s. These observations suggest an extended theory of endogenous, directed technology choice, along the lines of Kennedy (1964) and Acemoglu (2002).[1] We develop such a theory, allowing us to capture the natural notion that there is very low short-run substitutability between energy and other inputs, once the technology factors at a point in time have been chosen, but significantly higher substitutability over longer periods when these factors are endogenous. Our theory also makes quantitative predictions for the long-run values of the income shares of inputs.

We begin the paper by interpreting U.S. data from the perspective of a CES function. A key parameter for us to estimate in this function is the elasticity of substitution between the capital/labor composite and fossil energy, but even a cursory look at the data suggests that the elasticity must be very low: the fossil price and its income share comove very strongly. Thus, it appears that ex post, on an annual level, the finite resource is very hard to substitute by increasing capital or labor. At the same time, when evaluated with a near-Leontief elasticity, the two implied input-saving technology series show negative comovement, so that the elasticity from a longer-run perspective is higher. Thus, a permanent price increase for fossil fuel would generate endogenous energy saving, mitigating a long-run increase in its income share.

To implement these ideas step by step, we first formulate a theory with exogenous technical change that has the production-function structure just discussed. We show that, in the long run, such a theory implies a fossil income share that goes to one, unless energy-saving technical change is fast enough—in which case it goes to zero. In fact, only under a knife-edge condition will the income shares of inputs be balanced under exogenous technical change, but we then go on to show how, under endogenous technical change, the only balanced-growth outcome is precisely that satisfying the knife-edge condition. That is, the model with endogenous energy saving implies a long-run fossil income share strictly between zero

[1] See also Hicks (1932) and Drandakis and Phelps (1966).

and one. This share, moreover, will depend critically on the "technology menu" available, i.e., the technological possibilities for trading off energy saving against capital/labor-saving. We then, finally, return to U.S. data and formally estimate both the key ex-post elasticity parameter and key parameters of the technology menu. This estimation is conducted structurally and does deliver an elasticity close to zero.[2] The implied long-run income share for fossil fuel is 8%, but we also point out that the use of lower-frequency movements in the data would suggest a somewhat larger share. An auxiliary implication is that there is a small reduction in consumption growth.

Section 2 briefly discusses some important connections to the literature and Section 3 then takes a preliminary look at the data. Section 4 goes through the models with exogenous and endogenous technical change and Section 5 covers the formal estimation and our central results. Section 6 concludes.

2 Some connections to the literature

Relative to the existing literature on limited resources, we see our key contribution as an applied, quantitative one: we formulate a theory of endogenous, directed technical change that builds straight onto the workhorse macroeconomic model used for growth and business-cycle analysis and use it to assess the time path for fossil-energy saving in the aggregate economy. The variables in our framework have direct counterparts in national income and product accounts data and our structural model, once estimated, allows us to make rather striking quantitative observations. In particular, there appears to be endogenous technical change directed toward fossil-energy saving as a function of market conditions—as measured by a rise in the aggregate series for energy-augmenting technology during the period of persistently rising fossil prices. Through the lens of the theory, we can then also learn about the technological trade-offs that the economy has faced and what they might imply for the future.

To be sure, it is not surprising that a mechanism of the qualitative sort we study can be identified in the aggregate data: there are both micro-empirical and theoretical studies to

[2] A substitution elasticity between energy and capital/labor much above zero would require very large short-run changes in the production technology parameters, which is challenging to rationalize and indeed is ruled out in our econometric estimates. Relatedly, electricity demand on short horizons are even viewed to be good instruments for aggregate economic activity; see Jorgenson and Griliches (1967) and the many studies following it. Applications in the literature of the type of production function we employ also use estimates consistent with what we find here; e.g., Manne, Mendelsohn, and Richels (1995) use an elasticity of 0.4 for a model with a ten-year time period.

suggest it. On the microeconomic level, the phenomenon we model in the aggregate has been studied (and found to be relevant) by, among others, Popp (2002) and Aghion et al. (2016) for the application to "clean" and "dirty" technologies in the case of autos. Our quantitative macroeconomic findings provide further support for, as well as complement, these studies.

The previous theoretical work on the topic is clearly also suggestive, as it explores mechanisms within frameworks with some of the ingredients we have here. First, a number of papers have looked at endogenous technology growth in the presence of limited resources; early contributions include Aghion and Howitt (1998), Barbier (1999), Scholz and Ziemes (1999), Grimaud and Rouge (2003), and Groth and Shou (2007).[3] Their main focus is on whether output growth is bound to stop, or even reverse, and whether market outcomes are optimal. In the present paper, or goal is not to study the overall resources spent on innovation—we keep them fixed—but to examine its direction.[4] To be sure, there is a theoretical literature with this aim too; key early papers include Smulders and de Nooij (2003), Groth (2007), and Di Maria and Valente (2008). These studies have different aims, ranging from an interest in the long-run nature of how technological change occurs to specific policy issues, and they use different formulations for how the different factors of production enter the production of output and of R&D. Our formulation of endogenous directed technical change is similar but also distinct in that it is, in its core, a neoclassical macroeconomic framework with capital and labor, built so that it can be calibrated to standard data on income shares, along with a decentralized setup that is not focusing on explicit R&D but on externality-based learning by doing. Our focus is also a little different in that we compare exogenous to endogenous technology and focus on the long-run income shares of inputs. However, most importantly our main purpose is to assess the extent of observed energy-saving technical change in the data, which we accomplish by estimating the key model parameters of our structural model.

Of course, all of the above studies build further on an earlier literature on the depletion of natural resources that prominently include papers in a 1974 volume of the *Review of Economic Studies*, featuring papers by Dasgupta and Heal (1974), Solow (1974), and Stiglitz (1974). The ensuing literature has important quantitative contributions, such as Jorgenson et al. (2013), and more recently Casey (2019), who uses our setup. From these perspectives, our present setting should be seen as a quantitative version of the Dasgupta-Heal model that includes endogenous directed technical change, implying input substitutability that differs

[3]Bovenberg and Smulders (1995) address a closely related phenomenon—pollution (and "natural capital")—and how endogenous growth interacts with it.

[4]We do obtain effects on the aggregate growth rate in the same direction as these authors find, but they are quite modest.

depending on the time horizon.

Our aggregate focus resembles that in the literature on directed technical change toward high- vs. low-skilled labor (or products intensive in these respective inputs). Beginning with Katz and Murphy's (1992) paper, an argument was put forth—also using an aggregate CES technology—that there has been skill-biased technical change since the late 1970s. Acemoglu (1998) later looked at how changes in the sizes of college-graduating cohorts could have explained this fact. We conduct these exercises jointly and include an estimation of both the CES elasticity and the technology available for choosing factor-augmenting technologies.[5]

The large literature following Acemoglu (1998) exploits modern endogenous-growth techniques to formulate models of endogenous directed technical change. We stand on the shoulders of all this work and, relative to it, our purpose is very applied: we look at the natural resource case, with a focus on fossil energy, where a stock is depleted over time, and we restrict attention to cases where the natural resource is "hard" to replace in the short run: we assume an input substitution elasticity less than unity. We do consider a competitive-equilibrium version but mostly focus on a planning problem, in large part because our setting implies both positive and negative externalities (which can even cancel exactly).[6]

Another important reference, and in fact a motivation behind the present paper, is the recent literature on climate change, where fossil fuels are in focus. Acemoglu et al. (2012) considers the direction of technological progress with respect to clean and dirty energy, but does not examine the saving on energy vis-a-vis other inputs.[7] This literature also makes clear that even though there are very large remaining deposits of fossil fuel in the world, we must limit its use significantly in order to contain global warming. Thus, the use of fossil fuel is, in practice, likely to be constrained beyond available supplies.

Our approach for modeling ex-ante/ex-post distinctions between input elasticities is related to the earlier theoretical literature on the topic building on vintage structures. For the latter, see Atkeson and Kehoe (1999) or Rausch and Schwerin (2018); for a closely related application to capital vs. labor, see Léon-Ledesma and Satchi (2019), whose formulation we follow. As for empirical estimates of short- vs. long-run elasticities, we find lower short-run values than in Berndt and Wood (1975) and long-run values roughly in line with those found by Griffin and Gregory (1976), who both use translog cost functions and data from 1947 to 1971. Our estimate is also broadly consistent with those used in the recent applied theoreti-

[5] Note also that a very similar interpretation of the data is contained in Jones (2002).

[6] More general technologies and market settings are studied, e.g., in Acemoglu (2002, 2003, and 2007).

[7] See also Hémous (2016) that uses an endogenous-technology setting based on Aghion and Howitt (1992).

cal literature on climate change; e.g., a similar functional form to the one here, calibrated to five-year periods, uses elasticities of the order of 0.5; see Manne, Mendelsohn, and Richels (1995) for the MERGE model, Bosetti et al. (2006) for the WITCH model, and van der Werf (2008), as well as with a broad range of econometric estimates (see, e.g., Dahl and Sterner, 1991).[8]

3 Empirical motivation

Our focus is on the U.S. economy. We look at a broad measure of fossil-fuel energy that includes oil, coal, and natural gas and use an accompanying price index (see the Appendix for details). We interpret our fossil energy measure as an input into domestic production and define the (annual) fossil income share ep/y, where e is quantity, p is price (in chained 2005 dollars), and y is production; both e and y are measured net of the net export of fossil fuel.[9] Figure 1 shows the movements over time in this share and in the fossil price.

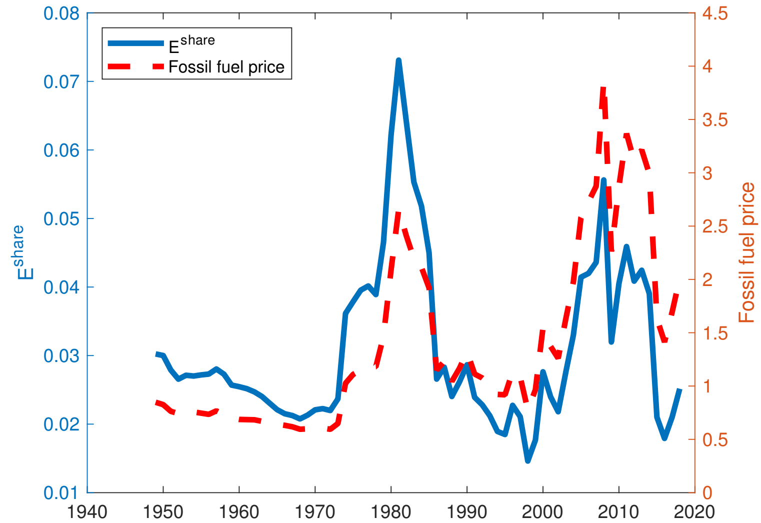

Figure 1: Fossil prices (in chained 2005 U.S. dollars) and the share of income fossil energy

[8]A related elasticity is the percentage response of fossil energy use to a one-percent increase in its price, which some studies argue is large; see, e.g., Kilian (2008). Such an estimate is still consistent with our production function parametrization if all inputs can be varied in the short run due to movements in capacity utilization in response to fossil-energy prices.

[9]Our variable definitions are motivated by the production-function assumptions we make later in this section and we discuss them there.

The figure reveals a strong positive comovement between the price and the share. Specifically, the share starts out around three percent in 1949 and then decreases somewhat up to the first oil price shock when it increases dramatically. The share then falls drastically between 1981 and the second half of the nineties and then finally increases again. The share does not seem to have an obvious long-run trend.

Taken together, these facts suggest a theory that has strong complementarity between energy and other inputs. We now propose a structure of this sort, one that can be viewed as a straightforward extension of macroeconomic frameworks used for quantitative analysis. Thus we posit a production function F for aggregate domestic output, y, that has three inputs: aggregate capital, k, aggregate labor, l, and aggregate fossil energy, e. We assume that the production of fossil energy requires no inputs, and hence delivers pure rents. This means that our production function, which is gross in nature as the input fossil energy is an intermediate input, can also be interpreted as GDP minus the value of energy use outside of domestic production; this outside energy use equals net export of (fossil) energy plus the household use of fossil energy as a final good (which largely consists of auto fuel, which amounts to about 10% of the total).[10] The theory is greatly simplified by not having to consider the allocation of capital, labor, and energy across sectors. So is the empirical analysis, as we would otherwise need to separately track input use (capital, labor, and energy) over time in both the final good and the energy sector.[11] Our appendix briefly describes the data sources and construction.

For the production function, we use a nested CES:

$$y_t = F\left(A_t k_t^{\alpha} l_t^{1-\alpha}, A_{et} e_t\right) = \left[(1-\gamma)\left(A_t k_t^{\alpha} l_t^{1-\alpha}\right)^{\frac{\varepsilon-1}{\varepsilon}} + \gamma\left(A_{et} e_t\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad (1)$$

where $\alpha \in (0,1)$ and where ε is the elasticity of substitution between capital/labor and fossil energy. γ is a share parameter.[12] Note that when $\varepsilon = \infty$, the Cobb-Douglas composite

[10] We set the latter to zero for lack of a consistent time series on it. Energy used for heating homes is different: it is considered an intermediary good in the production of housing services, which is a(n imputed) part of GDP.

[11] For robustness, we elaborated with alternatives and found that, because the energy sector is small relative to the total, they deliver only very marginal changes to our quantitative results (both in terms of the basic plots, such as Figure 1, and the estimations below). These robustness checks included assuming (i) that the energy-producing sector has the same isoquant shapes as in the non-energy-producing sector and (ii) that the energy-sector production function is Cobb-Douglas.

[12] A similar production function is considered by Stern and Kander (2012). Using a capital-labor composite is a somewhat more attractive nesting than the alternatives—a structure where either capital or labor forms a composite with energy would imply significant changes in the capital or labor income shares in response to the oil shocks in the 1970s, which we did not observe—but does not materially affect our analysis of energy

and fossil energy are perfect substitutes, when $\varepsilon = 1$, the production function collapses to being Cobb-Douglas in all input arguments; and when $\varepsilon = 0$ the Cobb-Douglas composite and energy are perfect complements, implying a Leontief function in the capital-labor composite and energy. The two variables A_t and A_{et} are the *input-saving* technology levels for capital/labor and energy, respectively; these are well-defined so long as ε is not equal to 1.

It is now possible, conditional on a value for the substitution elasticity, to use this production function, along with data on inputs and outputs, to back out the two input-saving technology series. One would expect these series to behave "like technology", i.e., be rather smooth and increasing. In addition, we gain insight into the prevalence of input saving by looking at how the series change as input prices change. To this end, under the assumption of perfect competition in input markets, it is possible to solve explicitly for the two technology trends A_t and A_{et} in terms of observables (and production-function parameters). This delivers, with $l^{share} \equiv wl/y$ and $e^{share} = pe/y$,

$$A_t = \frac{y_t}{k_t^{\alpha} l_t^{1-\alpha}} \left[\frac{l_t^{share}}{(1-\alpha)(1-\gamma)} \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{2}$$

and

$$A_{et} = \frac{y_t}{e_t} \left[\frac{e_t^{share}}{\gamma} \right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{3}$$

The key unknown parameter is ε.[13]

In Section 5 of the paper we will estimate ε formally. However, the striking positive comovement between the fossil energy income share and its price already suggests an ε close to zero: with values of ε above 0.5, the implied movements in A_e are extremely volatile, challenging the interpretation of this variable as technology. In Figure 2 below, we thus use an elasticity close to zero ($\varepsilon = 0.02$) for a preliminary examination.

The figure shows the path for fossil energy-saving technology A_e. Two observations are noteworthy. First, we observe a weakly increasing, and overall reasonable-looking graph for fossil energy-specific technology. The mean growth rate is 1.52 percent and the standard deviation is 2.13 percent. Second, as illustrated by plotting separate trends lines before and after the first oil-price shocks—1949–1973 and 1973–2018—we see the energy-saving technology series appears to have a kink around the time of the first oil price shock; the

saving. The restriction to a Cobb-Douglas nesting of capital and labor is restrictive from the perspective of Karabarbounis and Neiman (2013); we maintain it for simplicity.

[13]The parameter γ is a mere shifter of these time series and will not play a role in the subsequent analysis.

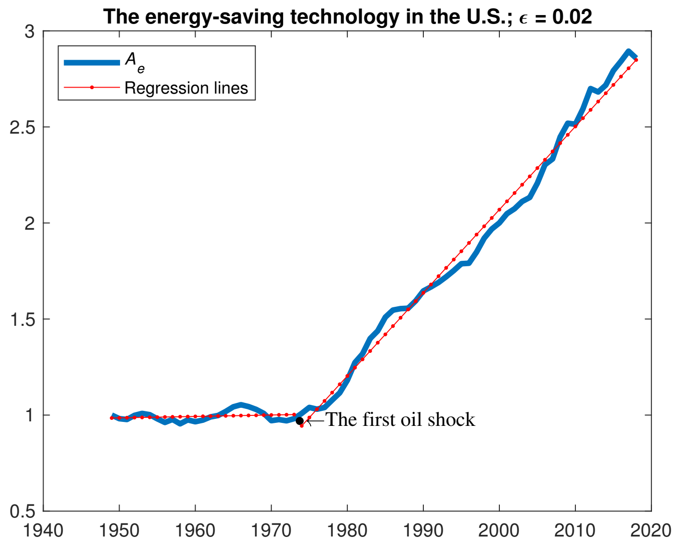

Figure 2: Energy-saving technology

growth rate is 0.15 percent per year up to 1973 and 3.66 percent per year after 1973. In the left panel of Figure 3 we see the relationship between the fossil-fuel price and the average growth rate of energy saving. Both variables remain low up to 1973, after which they increase fast and eventually peak in the early 1980s. As the price comes down, so does the average energy-saving growth rate. Finally, both variables start increasing again around the early 2000s. The higher average growth rate for energy efficiency after 1973 thus also coincides with a substantially higher—more than two times—average fuel price relative to before 1973.

What does a low substitution elasticity imply for the evolution of the capital/labor-augmenting technology? The series for A is plotted in the right panel of Figure 3, alongside the A_e series. A, like A_e, is rather smooth and increasing and very much looks like the conventional total-factor productivity (TFP) series. The mean growth rate in A is 1.39 percent and the standard deviation is 1.62 percent. The figure also allows us to see that the two technology series comove negatively. Roughly when the energy price takes off, energy saving responds positively at the same time as input saving in capital/labor slows down.

As another preliminary check on our setup, we estimate equation (3) with OLS, using the log income share of energy as dependent variable, the fossil price as an independent variable, and letting a linear time trend capture the evolution of A_e.[14] In line with the

[14]This is very similar to the approach in Katz and Murphy (1992), except that here we take prices and not quantities to be exogenous. For details, see our online appendix.

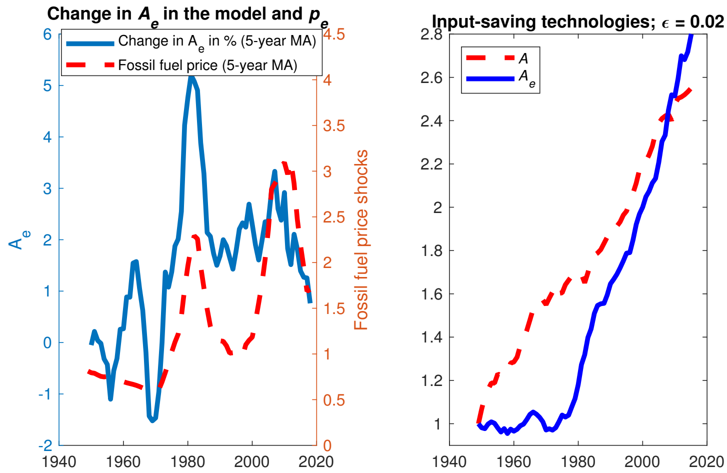

Figure 3: The fossil price and input-saving technologies

ocular inspection of Figure 1 above, this regression yields an estimate of ε of 0.13 over the whole time period; based on data only after the first oil shock the estimate falls to 0.04.

In sum, a strong comovement of the fossil energy price and the fossil energy income share suggests a low elasticity of substitution between capital/labor and fossil energy, which in turn delivers input-saving series that (i) look like technology series in that they are rather smooth and non-decreasing; (ii) comove negatively; and (iii) comove with the fossil price. These observations suggest a theory where technical change directs itself toward the input on which it is profitable to save. We develop such a theory in the next section of the paper and we then estimate the fuller model formally on the U.S. data presented here.

4 The model

In this section, we formulate a model with the aim of allowing technical change to respond endogenously to changes in the economic environment, allowing us in particular to evaluate the predictions for future energy dependence. The model will have the features suggested by the above data: a low short-run elasticity between energy and other inputs ($\varepsilon < 1$) but a significantly higher one over longer horizons, engineered by directed technical change that saves on expensive inputs. We take a general-equilibrium perspective here, thus allowing us to solve endogenously for fossil fuel prices, long-run growth rates of technology, and the income shares of inputs. In this sense, it is a global model and of course, as such, stylized. In the ensuing section, in contrast, we estimate the model in an application to the United

States, and then we take fossil-fuel prices as given.

The model will be constructed in steps. First, in Section 4.1, we specify a standard neoclassical dynamic macroeconomic model with an energy input but with exogenous technology growth. In Section 4.2 we then endogenize technology. Whether technology is exogenous or endogenous will, as we will see, make a significant difference for outcomes. In the former case, the long-run input income shares go to either zero or one, unless the technology growth rates satisfy a knife-edge condition, whereas in the latter case the economy endogenously picks out said knife-edge case: the income shares will be strictly between zero and one.

4.1 Exogenous technical change

In our model, there is one consumption good each period and we consider an infinitely lived household with the following utility function.[15]

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma}, \tag{4}$$

where β is the discount factor and σ determines risk aversion. Output, y, is produced with capital, k, labor, l, and a natural resource, e, as inputs, exactly as in the specification in the empirical section above: y_t is given by $F(A_t k_t^\alpha, A_{et} e_t)$, where F is defined in equation (1) above; the one difference is that we keep labor input equal to 1 here. In this section, we assume constant growth rates for both of the input-saving technologies:

$$A_t = g_A^t \quad \text{and} \quad A_{et} = g_{A_e}^t,$$

where g_A and g_{A_e} are both strictly greater than 0 and finite.

The period resource constraint, as in standard one-sector growth models, is given by

$$c_t + k_{t+1} = y_t + (1-\delta) k_t, \tag{5}$$

with the depreciation rate $\delta \in (0, 1)$.

The size of the fossil stock at time zero is R_0 and the following constraint must be

[15] We assume that β times the growth factor for consumption raised to $1-\sigma$ is less than one; consumption's growth rate is a nontrivial—but easy-to-determine—factor of the underlying parameters, as we will see below.

satisfied:

$$\sum_{t=0}^{\infty} e_t \leq R_0, \tag{6}$$

where R_t is the remaining stock of resource in ground in the beginning of time t; we could equivalently write $R_{t+1} = R_t - e_t$ for all t. We impose that e and R be non-negative at all times; these constraints will not bind for any cases of interest so they are omitted for brevity. We assume, for simplicity, that fossil fuel is costless to extract.[16]

We define an *Asymptotic Constant Growth Path* (ACGP) as a limit solution to the planner's optimality conditions—including transversality conditions—where all variables grow at constant, though possibly different, rates. An *Exact Balanced Growth Path* (EBGP) is a solution to the same conditions that, by appropriate choice of initial conditions, features exact balanced growth at all points in time with (i) identical growth rates for capital, consumption, and output and (ii) constant positive income shares for the two inputs, i.e., the energy share $s_{et} \equiv A_{et}e_t F_2(A_t k_t^\alpha, A_{et}e_t)/F(A_t k_t^\alpha, A_{et}e_t)$ is strictly between zero and one. Hence, an EBGP is a special case of an ACGP, and an ACGP does not necessarily deliver balanced income shares nor can it necessarily feature exact constant growth. For either case, here and in the analysis below on endogenous technical change, we do not consider the possibility of limit behavior that is not asymptotically of the constant-growth variety (such as cycles, chaos, or exploding paths).

The following theorem communicates how, under exogenous growth, relative scarcity—captured by $\tilde{\beta}\tilde{g}$ below—has drastic implications for income shares when the elasticity of input substitution is less than one.

Theorem 1 *Suppose* $\varepsilon < 1$ *and define* $\tilde{\beta} = \beta g_A^{\frac{1-\sigma}{1-\alpha}}$ *and* $\tilde{g} = g_{A_e}/g_A^{\frac{1}{1-\alpha}}$.

1. *If* $\tilde{\beta}\tilde{g} > 1$, *then there is no EBGP and there is a unique ACGP where the energy income share is zero and where output, consumption, and capital all grow at the same rate and where the ratio* $Ak^\alpha/(A_e e)$ *goes to zero.*

2. *If* $\tilde{\beta}\tilde{g} = 1$, *then there is a unique ACGP and it is an EBGP where output, consumption, and capital all grow at an equal, unique rate and where the ratio* $Ak^\alpha/(A_e e)$, *along with*

[16] Oil is available from different sources, each associated with a specific (non-zero) unit cost of extraction. Although the marginal cost of most oil in Saudi Arabia is low, it is not close to zero in the North Sea. Moreover, the extraction costs can also be affected by R&D and may be stochastic (shale gas and tar sand are examples of recent innovations of this nature). A full quantitative treatment of oil hence needs a richer structure. It also needs the inclusion of other energy sources, fossil as well as non-fossil. Our simplifying assumptions allow us to uncover mechanisms rather clearly and to characterize long-run outcomes without resorting to numerical analysis.

the energy income share, are finite and positive and determined by the initial condition on $A_eR/(Ak^\alpha)$.

3. *If $\tilde{\beta}\tilde{g} < 1$, then there is no EBGP and there is a unique ACGP where the energy income share is one and where output and consumption grow at the same rate but capital grows at a lower rate and where the ratio $Ak^\alpha/(A_e e)$ goes to infinity.*

The theorem says that balanced income shares will only result under a knife-edge condition. The key parameter expression $\tilde{\beta}\tilde{g}$ captures the roles of technology growth and energy scarcity: if its value is high, although energy is scarce, it will not go to zero fast enough, relative to the growth of the factor-augmenting technologies, to prevent the long-run energy income share from going to zero. The growth rates of the factor-augmenting technologies are, of course, key, because they gauge the relative scarcities of the inputs. The discount factor, β, appears because it is a key determinant of how fast energy use is chosen to go to zero (under logarithmic preferences, energy goes to zero at rate β exactly).[17]

The proof of this theorem, along with the proofs of our other formal propositions, can be found in our online appendix. It is straightforward and relies on a transformation of variables and then working out the different cases of the key parameter inequalities. In the proof, one sees that in the case where the energy income share goes to zero, output asymptotically becomes linear in the first production input; here, capital is the "bottleneck", because energy-saving technical change grows so fast. When the energy income share goes to one, instead, output becomes linear in the energy input and although Ak^α grows faster than $A_e e$, capital grows more slowly than output and the asymptotic capital-output ratio is actually zero.

Theorem 1 also says that when an exact balanced growth path exists, although its growth rates are pinned down uniquely by $\tilde{\beta}$ and $\tilde{g}$, the long-run energy income share depends on initial conditions.[18] This is in sharp contrast to the results below under endogenous technology, where the long-run share is always uniquely determined, independently of initial conditions.

The requirement that ε be less than one of course captures "sufficient complementarity" and is critical: it implies unique ACGEs with the energy income share going to 100% unless

[17] A high σ raises curvature in consumption and hence slows down growth and the rate of depletion of the resource; $g_A^{\frac{1}{1-\alpha}}$, as in a standard macroeconomic model, is consumption growth.

[18] Uzawa (1961) features a similar knife-edge result. Under his knife-edge case, the long-run income shares of inputs are uniquely pinned down, independently of initial conditions, whereas we show that the long-run energy share will depend on initial conditions.

the energy-saving technology grows fast enough relative to the capital/labor-augmenting technology. This outcome points to strong incentives to improve on energy efficiency (or similarly for the case where the long-run energy income share goes to zero, to improve on capital/labor efficiency). In the next section, we allow for such a channel: endogenous factor-augmenting technical change.

4.2 Endogenous technical change

Let us now consider technology that can be directed, which would be of particular interest when applied to scarce inputs. We will show below that under relatively mild conditions, and unlike in the case of exogenous technical change, an EBGP will exist and be the only ACGP. I.e., g_A and g_{A_e} will adjust endogenously to values satisfying the knife-edge condition of Theorem 1. We first look at a static model of technology choice in Section 4.2.1 and then incorporate it into the dynamic setting studied above in Section 4.2.2.

4.2.1 Static model

We assume the same technology for producing output as in the previous section and add a menu from which (A, A_e) can be chosen. In the economy as a whole, the amounts of capital and energy—k and R, respectively—are given. We first study the planner's problem and then a perfectly competitive decentralization with joint input and technology choice on the firm level.

The planner The problem here is thus to maximize

$$\left[(1-\gamma)\left(Ak^{\alpha}\right)^{\frac{\varepsilon-1}{\varepsilon}}+\gamma\left(A_e R\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

by choice of A and A^e subject to the technology menu:

$$G(A, A_e)=0.$$

Here, G is strictly increasing in both arguments and hence the choice to select a high level of one of the input-saving technologies comes at the expense of the other. We will assume that G has constant returns to scale and is quasi-concave and twice differentiable. Different assumptions on its curvature will then, as we shall see, deliver qualitatively different outcomes. In the dynamic section we will present a straightforward extension to the case where the technology levels evolve dynamically over time.

Our aim is to consider cases where there is an active tradeoff between the two forms of input saving. To make sure that the first-order condition for the technology choice is satisfied with equality, we first establish a result to that effect.

Proposition 1 *When $\varepsilon < 1$, planner's technology choice is unique and interior.*

The proposition is straightforwardly proven, given that G is quasiconcave. We now turn to the competitive equilibrium version of the static model.

Competitive equilibrium We consider a representative firm choosing (k, R, A, A_e) taking prices for the inputs, r and p, respectively, as given. We have:

Proposition 2 *Suppose $\varepsilon < 1$. Then the equilibrium allocation coincides with that of the efficient solution and the energy income share, e^{share} is given by*

$$\frac{1-e^{share}}{e^{share}} = \frac{AG_1(A, A_e)}{A_e G_2(A, A_e)}.$$

Since the firm can choose both inputs and the technology levels—which multiply the inputs—the proof is not immediate. However, given that the production function is homogeneous in (k, l, R) and that the scale of research is given—it is only its "direction" that can be chosen—it is sufficient to consider the firm's cost minimization problem. The cost function can be derived in closed form, given our CES formulation, and using this closed form it is straightforward to show that the isoquants in (A, A_e) space are strictly concave. Hence, given a quasiconcave G, the problem is well-behaved. It is also easy to derive the expression for the energy income share. The relative shares of the capital/labor composite and energy is given by $AF_1 k^\alpha/(A_e F_2 R)$. The first-order condition for A and A_e in the profit-maximization problem, on the other hand, equalizes, per research unit, the benefits of the two ways of directing it:

$$\frac{k^\alpha F_1}{G_1} = \frac{RF_2}{G_2}.$$

Hence the share becomes the object stated in the proposition.

Specialization Violations of the assumptions underlying the theorem can lead to corner solutions and specialization. For example, assume that production is entirely symmetric, with $\alpha = 1$ and a linear F ($\varepsilon = \infty$): output is then proportional to $Ak + A_e e$. Moreover, let G be symmetric and linear. Then one obtains full specialization and which factor is chosen depends on the available amounts of capital and energy (or alternatively, from the firm's perspective, on the prices of these two inputs). The key is to note that the technology levels are endogenous and multiply the input levels in production. In the dynamic model, specialization is of course also possible, but we will focus on the cases where specialization does not occur.

4.2.2 Dynamic model

A straightforward extension of our static technology choice to a dynamic one is the following:

$$G(A_{t+1}/A_t, A_{e,t+1}/A_{et}) = 0. \tag{7}$$

I.e., we consider the same function G only with growth factors, not levels, as arguments.

Thus, the planning problem here reads

$$\max_{\{c_t, k_{t+1}, e_t, A_{t+1}, A_{e,t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma} \tag{8}$$

subject to

$$c_t + k_{t+1} = F(A_t k_t^{\alpha}, A_{et} e_t) + (1-\delta) k_t,$$

where F again is given by (1), and

$$G(A_{t+1}/A_t, A_{e,t+1}/A_{et}) = 0$$

for all t and

$$\sum_{t=0}^{\infty} e_t \leq R_0.$$

Under our assumptions, one can conveniently define an intermediate variable n and functions f and f_e such that the following equation system describes the same technology:

$$A_{t+1}/A_t \equiv g_{A,t} = f(n_t) \tag{9}$$

$$A_{e,t+1}/A_{et} \equiv g_{A_e,t} = f_e(1-n_t). \tag{10}$$

We interpret n as the share of a fixed amount of R&D resources that is allocated to enhancing the efficiency of the capital/labor bundle; $1-n$ is the fraction allocated toward energy-saving.

As an example, consider the case where F is Leontief. Then A and A_e just define input requirement coefficients. By changing n, these can be changed over time. Hence, we obtain a source of factor substitutability over time even though there is none in the very short run. For our estimated system below, f and f_e are functional forms such that the long-run elasticity is endogenous. Our estimates there imply a local elasticity of a little below one.[19]

The long-run income share of energy Theorem 2 below shows that the long-run energy income share in an EBGP—defined as in Section 4.1—of our economy depends exclusively, through the R&D tradeoff, on a small set of model parameters. Thus, if we restrict attention to exact balanced growth, the energy income share does not depend directly on the elasticity of substitution between capital/labor and energy, nor on the stock of fossil energy. The theorem also determines the rate at which energy use is chosen to go to zero on the EBGP.

Theorem 2 *On an exact balanced growth path (EBGP) with an interior choice for technology, the following features must hold:*

1. *The two arguments of the aggregate production function, $A_t k_t^\alpha$ and $A_{et} e_t$, both grow at the rate of output g.*
2. *Energy use falls at a constant rate: $\frac{e_{t+1}}{e_t} = \beta g^{1-\sigma}$.*
3. *Technology effort n and the consumption growth factor g are determined by $f_e(1-n)\beta = f(n)^{\frac{\sigma}{1-\alpha}} = g^\sigma$.*
4. *Energy's share of income is exclusively determined by how costly it is to enhance energy efficiency in terms of lost capital/labor efficiency. Specifically, the long-run energy income share is implicitly given by equation (11):*

$$\frac{1 - e^{share}}{e^{share}} = -\frac{dg_{A_e}}{dg_A} \frac{g_A}{g_{A_e}}. \tag{11}$$

The proof simply involves working out implications of first-order conditions and constraints under the assumption of exact balanced growth. The first statement follows straightforwardly from imposing exact balanced growth. The second statement, asserting that energy use has to fall at a rate determined by preference parameters only is connected to the

[19] For a related study with constant ex-ante elasticities, see Farajpour Bibalan and Hinkelmann (2021).

Hotelling (1931) theorem: the marginal value of a finite resource should rise at the real interest rate. The gross interest rate will thus both equal $\frac{g^\sigma}{\beta}$, from the Euler equation, and $A_{e,t+1}F_{2,t+1}/(A_{et}F_{2t})$, from the Hotelling equation. The latter must, because of the theorem's first statement, equal g/g_e on a balanced path; hence the second statement follows.

The third statement of the theorem follows directly from the first and second statements: in growth factors, using the functions determining the technology growth rates, the first statement now reads $f_e(1-n)\beta g^{1-\sigma} = f(n)g^\alpha = g$. Rearranged, this delivers the third statement and it allows us to solve for n and g, and thus the key remaining growth rates.[20] The fourth statement, finally, is a dynamic extension of Proposition 2.

Notice that the theorem implies that the steady-state income share of energy is unique and determined independently of initial conditions—unlike in the corresponding EBGP case under exogenous growth. Key for this result is the constant-returns assumption on F, which means that the two arguments of the production function need to grow at the same rate on any exact balanced growth path. This is also a reason why the elasticity between inputs is not relevant in the theorem or in the determination of n and the energy income share.

Theorem 2 focuses on EBGPs but leaves open whether there may be long-run constant-growth paths, ABGPs, that do not feature exact balanced growth. The answer is no, provided that the technology menu for growth rates is such that it is possible to rule out corner solutions, for which a sufficient condition is

Assumption 1 $\beta f_e(0)/f(1)^{\frac{1}{1-\alpha}} < 1$ *and* $\beta f_e(1)/f(0)^{\frac{1}{1-\alpha}} > 1$,

where we recall that f and f_e describe the growth factors of A and A_e, respectively.

We now have the following for the dynamic model with endogenous technology.

Theorem 3 *Suppose that $\varepsilon < 1$ and that Assumption 1 holds. Then there is a unique ACGP and it is an EBGP.*

The theorem, which is proved in our online appendix, says that with endogenous technology, any asymptotic constant-growth path has income shares of inputs strictly between zero and one.[21] Our result is reminiscent of Acemoglu's work on technical change (see, e.g.,

[20] The share, $-\frac{dg_{A_e}}{dg_A}\frac{g_A}{g_{A_e}}$ from the theorem, only depends on n, since we can write $g_{A_e} = f_e(1-f^{-1}(g_A))$. If f and f_e are such that $g_{A_e} = f_e(1-f^{-1}(g_A))$ describes a concave function in the positive orthant, a higher n (i.e., g_A) implies a higher derivative $\frac{dg_{A_e}}{dg_A}$ in absolute value as well as a higher g_A/g_{A_e}, and hence a higher $-\frac{dg_{A_e}}{dg_A}\frac{g_A}{g_{A_e}}$.

[21] The proof is straightforward, though somewhat tedious. The assumptions stated in Assumption 1 ensure ruling out corner solutions where all of the technical change is concentrated on one of the inputs.

Acemoglu, 2003 and how technical change in a capital vs. labor context will be chosen to be labor-augmenting), though of course here both inputs are changing at endogenous rates.

We now briefly discuss decentralized equilibria for our economy.

Competitive equilibrium with dynamic externalities Recall from our analysis of the static model in Section 4.2.1 that firms were depicted as both acquiring inputs in perfectly competitive markets and choosing their two input-saving technologies so as to maximize profits. In the present section, we formulate a dynamic version of this setting. The firm will still have a static choice, and equilibrium profits will still be zero, but due to the presence of externalities here the equilibrium may not be efficient.

The dynamic externality works as follows. Firms solve static problems. A firm operating at $t+1$ (buying capital, labor, and energy at $t+1$) will choose $(A_{t+1}, A_{e,t+1})$ from the menu $G(A_{t+1}/A_t, A_{e,t+1}/A_{et}) = 0$, with (A_t, A_{et}) given by the choices of firms (on *average*) at t. Thus, firms do not internalize any dynamic spillovers of its technology choice today on its future choices. This static problem is thus identical to that in Section 4.2.1. Our equilibrium definition, formally, is that used in Romer (1986), but it operates as a dynamic externality here and it is directed: by choosing more of one type of input-saving today, firms improve the possibilities of saving on the same kind tomorrow (a positive spillover), while at the same time worsening the possibilities for saving on the other input (a negative spillover).

We will omit the formal equilibrium definition here but now use it in a special case.[22]

4.2.3 Example: a log-linear technology menu

Suppose G is log-linear, so that $G(x, y) = 0$ can be written $\log y = a - b \log x$, where a and b are positive constants. We do not go through the equations in detail but merely summarize the main conclusions from the analysis.[23] This case generates a closed-form solution for the production function, after the technology levels have been maximized out. More precisely, when the firm chooses A_{t+1} and $A_{e,t+1}$, the reduced-form production function is a Cobb-Douglas function in k_{t+1} and e_{t+1} with the exogenous starting levels A_t and A_{et} appearing only in the form of a TFP factor (which itself is Cobb-Douglas).[24] Thus, the model of

[22] Another interesting special case is studied in our online appendix: the ex-post production function is Leontief, in which case one can obtain analytical solutions for most objects of interest. For an application of this model, see Casey (2019). We solve a more general specification of the model numerically in the section below on estimation.

[23] An online appendix contains full derivations for the case where the utility function is logarithmic and there is full depreciation.

[24] A version of this case is studied in Jones (2005).

endogenous technology choice reduces to one of exogenous technical change with well-known and well-behaved properties.

Interestingly, in the log-linear technology case the competitive equilibrium with dynamic externalities is optimal. This is not a complete surprise, as a positive spillover for saving on one input always comes along with a negative spillover for the other and $\varepsilon < 1$ (so that technology specialization is not optimal). In this special case, thus, these spillovers cancel each other exactly: a social planner would make the same choice. The takeaway here is that as far as the *direction* of research, the net spillover of firms' actions may not be major.[25]

4.2.4 Alternative energy sources

The general production function posited here—$F\left(Ak^{\alpha}l^{1-\alpha}, A_e e\right)$, where F has constant returns to scale—can be thought of more generally than from our limited-resource example. In particular, "e" can be any source of energy, or it could be a composite of multiple energy sources. So what if we consider an alternative to fossil fuel: what are the implications then for the future energy income share?

The question can be straightforwardly analyzed through the lens of our theory. Because we focus on balanced growth paths where *both* the capital-labor composite and energy are actively used and command constant (positive) income shares, we know from Theorem 2 that $g = g_A g^{\alpha} = g_{A_E} g_e$, with the same notation as before and where g_e is now the long-run growth factor for the energy input. Together with the technology menu assumptions, we then see that

$$f(n)^{\frac{1}{1-\alpha}} = f_e(1-n)g_e$$

must hold. That is, given a growth rate for energy, this equation determines how the research input (n) must be allocated. Clearly, for given functions f and f_e, capturing the potentials for saving on capital/labor vs. energy, an increase in g_e will increase n, i.e., less energy saving, and the energy income share will then decrease. The extent of the decrease depends on the global properties of f and f_e.

In our focus on fossil fuel, an energy source in finite supply, we derived g_e endogenously. When it comes to various alternatives, one might imagine prolonged periods of growth. Both solar and windpower options are developing gradually, and new types of nuclear power, like thorium breeders and fusion power reactors, may also have the potential to imply $g_e > 1$ for a period at least as long as the time over which we have seen increasing fossil fuel use.

[25]This conclusion contrasts that regarding the overall *scale* of research, which we do not study here.

Our discussion here makes clear that the key is how g_e evolves; in our fossil-fuel case, g_e is endogenous, and it can be for other energy sources as well. The discussion also indicates that the particular way in which we model the price formation for the fossil fuel in our decentralized model—through Hotelling theory—is not important for our key characterizations.[26] In fact, none of the main results would change qualitatively if, instead of our optimal-extraction assumption, we used an exogenous path for the rate at which the resource is depleted: Theorem 1 would apply again (with the exogenous extraction factor g_e appearing in place of preference parameters), as would Theorem 2 and Theorem 3. The value of g_e of course matters in the quantitative analysis, and absent a firm view on it, we find the optimal-extraction assumption use above useful.

5 Estimation

Our theory developments above are predicated on the assumption of less than unitary substitution between the capital/labor composite and energy in the production of annual output. Section 3 indicated that such an assumption appears to be consistent with the data at a first glance, and we now revisit this issue formally and draw out implications of our resulting estimates for future energy dependence.

Our informal initial look at the data, as captured in Figure 1, used the argument that A and A_e are technology variables and, hence, should be expected to be smooth. As a result, any values for ε other than those close to zero could be ruled out, since they imply drastic fluctuations in technology.[27] Now, in contrast, we introduce shocks explicitly. Our focus is still on the United States and, in line with our initial look at the data, we use a version of the model in the previous section where the prices of fuel are exogenous—based on the notion that world markets determine them. Thus, there is world trade in fuel, but the estimation does not involve these trade flows.[28] We consider fossil-fuel-price shocks, shocks to the price

[26] The Hotelling (1931) model of extraction predicts that if the resource is extracted at both t or $t+1$, the resource producer has to be indifferent between producing in the two periods, implying that the marginal revenue from extraction—price minus marginal cost—will rise at the real rate of interest. It is not clear that this prediction is borne out in data—for an early discussion, see Smith (1981)—and a number of suggestions have been made that could potentially yield different predictions. In our view, however, there is of yet no fully satisfactory alternative for understanding the long-run price implications. For intermediate-run predictions, the recent work in Bornstein, Krusell, and Rebelo (2021), arguing for a high short-run price elasticity of demand and high costs of adjusting quantities, can be helpful.

[27] A formal version of this procedure defines a metric that penalizes fluctuations in technology. With a straightforward implementation of such an approach, the estimated ε lands very close to zero.

[28] We also estimated a model where we treated the U.S. as a closed economy with fully endogenous prices.

of investment goods, and shocks to the input-saving technologies. We also need to provide a parametric specification of the technology menu G, and we use, with the notation above in equations (9–10),

$$f\left(z_{At}, n_t\right) = \exp\left(z_{At}\right)\left(1 + B n_t^{\phi}\right), \tag{12}$$

$$f_e\left(z_{A_e t}, 1 - n_t\right) = \exp\left(z_{A_e t}\right)\left(1 + B_e\left(1 - n_t\right)^{\phi}\right), \tag{13}$$

where B, B_e, and ϕ are parameters and z_A is a growth-rate shock to general technology (TFP) whereas z_{A_e} is specific to energy saving.

To estimate the model, it must be rendered stationary. To this end, define $x_t \equiv k_t^{\alpha} A_t$ to be the first argument of the aggregate production function and then transform into stationary variables by dividing through by it: $\hat{c}_t = c_t/x_t$ and $\hat{k}_t = k_t/x_t$. Also, $a_{et} = A_{et}/p_0\gamma_p^t$ is stationary, where γ_p is the exogenous growth factor for the fossil-fuel price.[29] We also define $\hat{e}_t = e_t p_0 \gamma_p^t / x_t$. The dynamic problem can now be compactly restated as

$$\max_{\left\{\hat{c}_t, \hat{k}_{t+1}, x_{t+1}, a_{e,t+1}, \hat{e}_t\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{\left(\hat{c}_t x_t\right)^{1-\sigma}}{1-\sigma}$$

subject to

$$\hat{c}_t + \frac{\hat{k}_{t+1}}{\exp\left(q_t\right)} \frac{x_{t+1}}{x_t} = F(1, a_{et}\hat{e}_t) + (1-\delta)\hat{k} - \exp\left(z_{pt}\right)\hat{e}_t, \tag{14}$$

and

$$\frac{a_{e,t+1}}{a_{et}} \gamma_p \frac{1}{\exp(z_{A_e t})} - 1 - B_e\left(1 - \left[\left(\left(\frac{x_{t+1}}{x_t}\right)^{1-\alpha}\left(\frac{\hat{k}_{t+1}}{\hat{k}_t}\right)^{-\alpha}\frac{1}{\exp(z_{At})} - 1\right)\frac{1}{B}\right]^{\frac{1}{\phi}}\right)^{\phi} = 0 \quad . \tag{15}$$

Here, $1/\exp(q)$ denotes the relative price of investment, where q is assumed to be stationary, z_p is a shock to the oil price, which is modeled as trend-stationary with a trend growth factor γ_p, and z_A and z_{A_e} (mentioned above) are iid growth-rate shocks. To complete the specification of the stochastics of our system, we assume that the innovations are all normal and mutually uncorrelated; the zs are iid and q and z_p have AR(1) specifications.

As described in our technical appendix, in the implied recursive system, x_t can be fac-

In such a model, fitting the price volatility is very difficult but the estimated parameter values still ended up very close to those found here. As discussed in Barsky and Kilian (2002), reality is surely somewhere in between: developments in the U.S. have had an impact on prices.

[29] $A_{et}e_t$ and $\gamma_p^t e_t$ must both grow at the rate of output and hence A_{et} will grow at γ_p on a balanced path.

torized out, with the state vector becoming $(\hat{k}_t, a_{et}, q_t, z_{pt})$ and the control $(\frac{x_{t+1}}{x_t}, \hat{e}_t, \hat{\lambda}_t)$. The model parameters are estimated using Bayesian methods. Specifically, we employ a Kalman filter on a linearized version of the model. In the online appendix, the model is also estimated with the generalized method of moments (GMM) using a pruned perturbation approximation with similar results.[30] The model to be estimated consists of first-order conditions and resource constraints from the above maximization problem; the first-order conditions are laid out in the online appendix.

Five parameters are calibrated in the estimation: α, β, δ, σ, and γ. The first four of these parameters are standard in the macroeconomic literature; α is set to 0.2632 in order to match the relative capital/labor shares, β is set to 0.985, the depreciation rate is set to 0.05 (per annum), and σ is set to 1. The parameter γ plays very little role in the estimation so long as the substitution elasticity ε is not near 1. We set it to 0.05 so as to match energy's share of income when $\varepsilon = 1$—then energy's income share in the theory is a constant equal to γ. The shock process for fossil prices is used as an observable in the estimation and the properties of this process are therefore estimated separately using a trend growth rate γ_p, an autocorrelation ρ_p, and a variance σ_p. The results from the separate estimation yield $\gamma_p = 1.02$, $\rho_p = 0.92$, and $\sigma_p = 0.18$. We impose these values in the main estimation.

The parameters ε, B, B_e, ϕ, and ρ_q are then jointly estimated, along with the shock variances. The first column of Table 1 below presents the priors for the coefficients: means and standard deviations. We specify Beta distributions for the parameters of the deterministic version of the model; inverse Gamma distributions are then used for the parameters of the stochastic representation. For ε, the mean of the distribution is 0.20, which is based on previous studies on the (short-run) elasticity of substitution between capital and energy.[31] The parameter B, a parameter pinning down an average growth rate of the A technology the prior is set to 0.0150 to match the average TFP growth rate. The mean of parameter B_e is set at 0.20 which allows for a relatively low energy income share, as observed historically. The prior mean of ϕ, is set at just below 1. We set a low prior for ρ_q, but we allow for a relatively large standard deviation for this parameter.

We use four data series as observables—the growth rate of output, the growth rate of fossil-fuel use, energy's share of income, and the fossil-fuel price.[32] Figure 4 shows fossil use, which reaches a peak in the early 1970s and then falls significantly, with a gradual

[30] In so doing, we follow Andreasen et al. (2017).
[31] See, for instance, Berndt and Wood (1975).
[32] Output and fossil-fuel use are expressed in per-worker terms consistent with the model specification.

flattening out. From the perspective of production, the observed decrease in the fossil input would amount to a significant drag on output, were it not for counteracting fossil-saving technological change. Our estimation formalizes and confirms this interpretation.

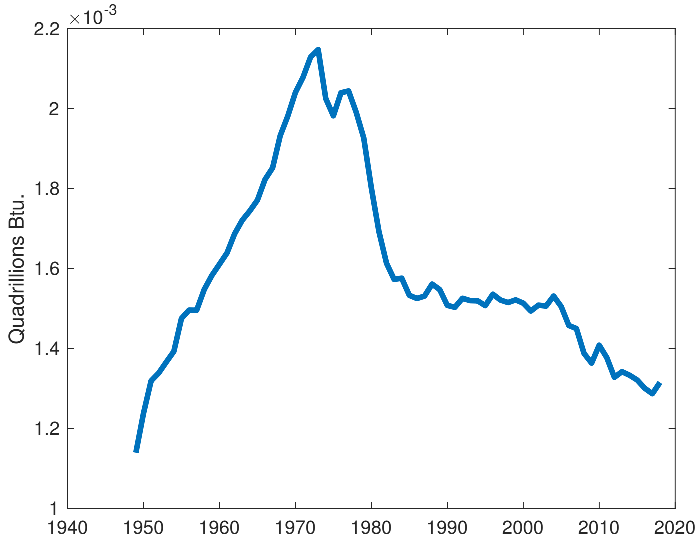

Figure 4: Fossil use per employed worker

The time period in the benchmark estimation is 1949 to 2018, but alternative time periods are considered in the online appendix, which also features several robustness checks with respect to data usage and parameter values. The data is described in Section 3 and is further discussed in detail in the online appendix.

Our main results can be found in Table 1. Consistent with the intuition in Section 3, the posterior value for ε is low. As shown in the sensitivity analysis that is carried out in the online appendix, the estimates are robust to a large number of changes. In particular, the low value for ε does not depend on the oil-price hikes of the 1970s: the estimate is similar for different periods that excludes the 1970s. The estimation results are also similar if growth rates of total output and capital are used as observables instead of per-capita measures, and if the model is estimated only with data on the manufacturing sector.

The inferred shocks to energy- capital/labor-saving technologies and investment are plotted in Figure 5.[33] Note that the estimated shocks are fairly stable over the period, with the

[33]The means of the average growth rates of the A and the A_e technologies are, respectively, 1.24 and 1.60 percent. The shocks to the fossil price match the data by construction; our AR(1) formulation for this price is very simple and, of course, associated with a large standard deviation.

Table 1: Estimation results, 1949–2018

		Prior		Posterior		
Coefficient	Prior density	Mean	Sd	Mean	Sd	$[10, 90]$
ε	Beta	0.2000	0.1600	0.0207	0.0167	$[0.0006, 0.0445]$
B_e	Beta	0.2000	0.0300	0.1760	0.0211	$[0.1416, 0.2106]$
B	Beta	0.0150	0.0300	0.0164	0.0017	$[0.0135, 0.0193]$
ϕ	Beta	0.9000	0.0150	0.9234	0.0129	$[0.9027, 0.9447]$
ρ_q	Inv Gamma	0.2000	0.1000	0.1946	0.0953	$[0.0427, 0.3393]$
Shocks						
$std\,(z_{Ae})$	Inv Gamma	0.0500	0.0200	0.0251	0.0023	$[0.0213, 0.0288]$
$std\,(z_A)$	Inv Gamma	0.0500	0.0200	0.0183	0.0018	$[0.0155, 0.0212]$
$std\,(\chi_q)$	Inv Gamma	0.0500	0.0200	0.0291	0.0048	$[0.0216, 0.0367]$

The posterior estimates are from 5 chains with 100,000 draws, where we discard the initial 50,000 data points. The acceptance ratio is approximately 30.

exception of the shock to energy-saving technology, which does dip before 1973 and then rise. Thus, from the perspective of our estimated model, the movements in aggregate energy saving are not just a function of the fossil price, but also to some extent a result of unexpected exogenous shocks. This should not come as a surprise: our structural approach focuses on a particular mechanism and leaves out other determinants of rising energy saving, including changes in environmental regulation, subsidies, trade policy, comparative advantages, and preferences, as per capita income rises. However, the mechanism we focus on here appears powerful, as illustrated in Figure 6.

Here, we plot changes in energy saving over time in a simulation of our estimated model when all other drivers than the fossil price are shut down (i.e., z_A, z_{A_e}, and q do not vary around their trends). The figure resembles Figure 3 above; the difference is that here changes in energy saving are not those observed but those simulated from our model. The model predicts that price shocks and changes in energy-saving technology, A_e, move in lockstep: as prices rise, firms invest in energy-saving. This is not a foregone conclusion even qualitatively: if price shocks were iid, energy-saving would not respond. The simulation shows that the mechanism proposed here has quantitative power when a realistic price series is used.

5.1 Long-run implications of energy-saving technical change

In this section we move to the long-run implications for fossil energy dependence, which depend heavily on the presence of directed technical change in the saving on inputs. Our estimates of the technology menu for the growth rates of A and A_e are not as precise as

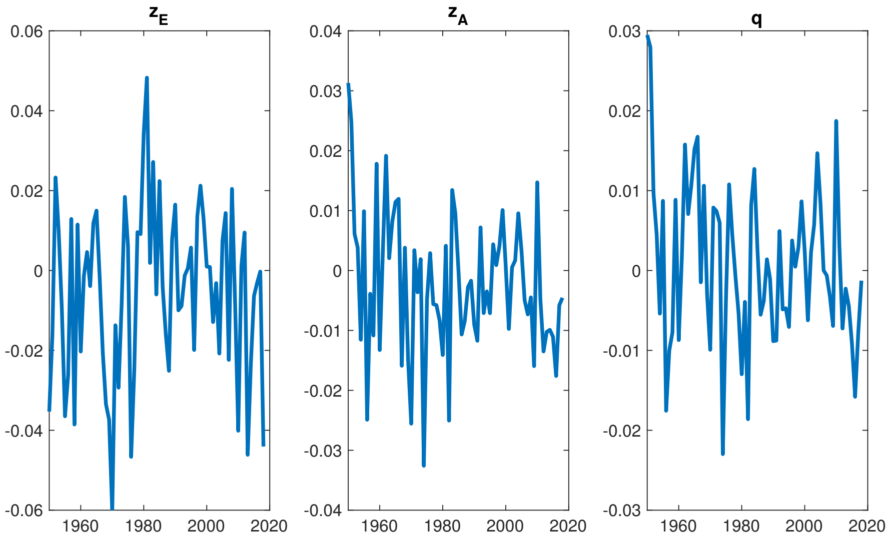

Figure 5: The estimated shocks

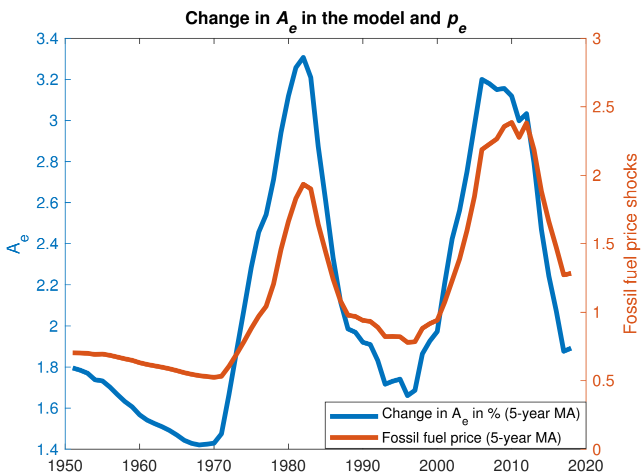

Figure 6: Fossil prices and model-induced energy saving

that for the short-run elasticity parameter ε in terms of magnitudes but they show the clear presence of a tradeoff. First recall Figure 3, which shows the A and A_e series based on a substitution elasticity very close zero, i.e., close to that just estimated: the two series nearly mirror each other, indicating that there is a tradeoff in the direction of technology choice. Thus, early on, the capital/labor-augmenting technology series grows at a relatively fast rate, whereas the growth rate for the energy-augmenting technology is relatively slow.

This goes on until around 1970, i.e., somewhat just before the first oil-price shock. After 1970, the energy-augmenting technology grows at a faster rate and the growth rate for the capital/labor-augmenting technology slows down. This continues up to the mid-1980s. Hence, the much-discussed productivity slowdown coincides with a faster growth in the energy-saving technology. Note also that this interpretation indicates that there are substantial costs associated with improving energy efficiency, since a higher energy efficiency seems to come at the cost of protracted lower growth of capital/labor-efficiency.

As for the parameters driving this technology tradeoff (B, B_e, ϕ), the formulation in our estimated structure is

$$g_{A_e} = 1 + B_e \left[1 - \left(\frac{g_A - 1}{B} \right)^{\frac{1}{\phi}} \right]^{\phi}. \tag{16}$$

The parameter point estimates imply the line plotted in Figure 7.

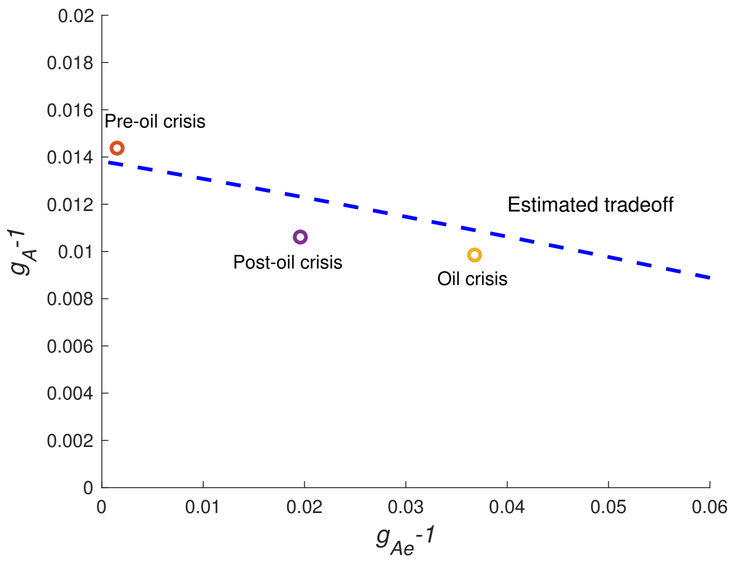

Figure 7: Medium-run growth trade-offs

It is possible to arrive at measures of the tradeoffs given by the technology menu with other, less structural methods. First, one can use medium-run averages. To this end, the figure also plots the average growth rates for the two technologies during three specific time periods: the pre-oil crisis (1949–1973), the oil crisis (1973–1985), and the post-oil crisis (1985–2018). As can be seen, these points also display a negative relation and they are scattered around the estimated relationship. The implied slope would be higher than that coming from our structural estimates: the scope for energy-saving technical change would be smaller, implying stronger energy dependence. Second, a different way of capturing a medium-run tradeoff is to apply an HP filter to the two series estimated growth series, take out the cyclical component, and then regressing the trend growth of energy-augmenting technology on the trend growth of the capital-augmenting technology. This procedure gives even less scope for energy-saving technical change (in terms of the figure, it gives a higher slope). From the perspective of using lower-frequency movements in the technology series, therefore, our baseline estimate of the scope for energy-saving technical change should be viewed as an upper bound.[34]

As for the long-run input income share implications, we know from the theory developed in Section 4.2.2 that the energy share can be computed directly from the slope $-\frac{dg_{A_e}}{dg_A}\frac{g_A}{g_{A_e}}$ evaluated on the balanced long-run growth path. The relation between these growth factors is not exactly log-linear, and hence one needs to know at which point to evaluate the derivative. To this end, we find the intersection between the technology tradeoff line with that characterizing balanced growth: on a balanced path, the two production inputs (the capital/labor composite and energy) need to grow at the same rate. This implies $g_{A_e} = g_e^{-1} g_A^{\frac{1}{1-\alpha}} = \gamma_p$ in our economy with exogenous fossil prices.[35] Thus, the long-run equilibrium is found by evaluating the line with a negative slope at $g_A = 1.0123$ and $g_{A_e} = 1.02$. This implies a long-run growth rate of consumption of 1.67 percent per year.

We now compute the required slope by differentiation of equation (16) and evaluation using the obtained long-run growth rates. This implies that $-\frac{dg_{A_e}}{dg_A}\frac{g_A}{g_{A_e}} = 11.5$, which in turn delivers a long-run energy share of income $e^{share} = \frac{1}{11.5+1} = 0.08$. Hence, our findings suggest that energy will earn a higher scarcity rent in the future than now. Note also that resource scarcity and the higher energy income share do not appear very harmful for economic growth, which will be somewhat lower than historically but not by a large amount.

Finally, we also computed transition dynamics numerically for our model.[36] Convergence in technology space, and in the energy income share, is rather slow for the estimated parameters, but capital and the interest rate converge quickly (as in the standard growth model). Interestingly, our model can predict protracted, rising use of the scarce resource when the initial condition is such that capital/labor-saving technology is low in relative terms.[37]

[34]Our findings of a negative relation adds macroeconomic support to the findings in Popp (2002), who uses patent data from 1970–1994 to estimate a long run price elasticity between energy prices and energy patents of 0.35. Even though Popp's findings have implications for the impact of factor prices on the direction R&D will take, he does not explicitly compute the tradeoff between the two growth rates for the technologies.

[35]See footnote 29.

[36]Results are available upon request.

[37]Standard Hotelling models robustly predict falling use.

6 Concluding remarks

In this paper we propose a parsimonious framework for thinking about technological change as an economy's response to the finiteness of natural resources. Using U.S. data, we estimate an aggregate production function in capital, labor, and fossil energy, along with a menu for input-saving technology choice. We find strong evidence that the economy actively directs its efforts at input-saving so as to economize on expensive, or scarce, inputs, as captured here by shocks to the fossil-energy price. The evidence for our mechanism stretches over the whole time period—not just as represented by a reaction to the oil-price shocks in the 1970s. It is possible that it should be complemented with a behavioral channel, representing these shocks as a "wake-up call"; we remain agnostic on this and view it as an interesting complementary channel.

Through the lens of our theory, we also make projections regarding future energy use and sustainability, implying a significant rise in the fossil income share going forward; our projection suggests around seven percent, but higher values are not to be ruled out since our estimate relies on unchanged overall R&D efforts. From the perspective of future consumption growth, our estimates imply a long-run growth rate of consumption that is reduced to a somewhat lower number than in the past: around 1.7% per year. Our framework lends itself to many natural extensions and we are pursuing such extensions in follow-up work.[38]

An important question in the area of natural resource management and exhaustibility of resources is whether there is a need for government regulation. Our model features no major market failures; in its decentralized version R&D features spillovers but given its directed nature, market outcomes are close to optimal (and exactly optimal in simple versions of the model): the externalities are proportional to both private costs and private benefits and hence nearly cancel.[39] We think this finding is rather robust. We do not include options to increase overall R&D resources in our model; such a (standard, to the endogenous-growth literature) formulation would imply a need to subsidize overall research.

We base our analysis on aggregate data, which might mask a possible alternative explanation for the kink around 1973 in input-saving behavior: structural transformation. The expansion of the service sector relative to manufacturing would deliver something like a kink in aggregate energy saving if the production of services requires relatively less energy.[40] Fig-

[38] In Hassler, Krusell, Olovsson, and Reiter (2019), we apply the present analysis in a climate-economy model. Another application is to examine a fossil fuel-specific "environmental Kuznets curve".

[39] Had we included a climate externality in the model, a Pigouvian carbon tax would have been called for.

[40] Of course, one can imagine reverse causality here: that the oil-price shock pushed structural change to

ure 8 in Appendix A.2 shows the energy-saving technology in the manufacturing sector, and it is qualitatively very similar to that in the aggregate; the kink is somewhat less pronounced, but the interpretation is that there appears to have been a drastic increase in growth rate of energy efficiency in both the manufacturing and the service sectors.

Finally, a motivating factor behind this work has been an interest in climate change, where fossil-fuel use of course is critical. Thus, higher taxes on carbon emissions would trigger technical change in the form of energy saving just like we have described it to play out in response to mere price shocks. Thus, the reasons for the increased cost of using fossil fuel—whether or not it is because of taxes, and regardless of what explains price changes—are immaterial: our analysis here can be directly merged into integrated assessment models of economics and the climate.[41]

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occur faster than it otherwise would have.

[41]Whether the supply side is, or has been, characterized by monopoly or oligopoly, e.g., through the Texas Railroad Commission or OPEC, is not of immediate consequence. For discussions of these issues, in the case of oil, see Barsky and Kilian (2002), Kilian (2009), and Bornstein, Krusell, and Rebelo (2021). For an application to climate change, see Hassler, Krusell, Olovsson, and Reiter (2019).

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A Appendix

A.1 Data sources and construction of our variables

In the model, y is a final good used for consumption and capital investment. The inputs are capital, labor, and fossil fuel. By "fossil fuel" here we mean its energy equivalent and we take this measure to equal the fossil energy index (in Btus) from the U.S. Energy Information Agency. We take all the other data from the National Income and Product Accounts and all data sources are described in detail in the online appendix.

We follow the EIA and take into account that the average price per Btu of coal is on average 3.82 times higher for oil and 1.63 times higher for natural gas over the whole considered period. The fossil-fuel composite, E_t, is then computed as $E_t = E_t^c + 3.82E_t^o + 1.63E_t^g$, and the fossil-fuel composite price is computed as $P_t = \left(P_t^c E_t^c + P_t^o E_t^o + P_t^g E_t^g\right)/E_t$. Details on these measures are provided in the online appendix.

From the assumption that y is produced from a constant-returns function F, we obtain that the income shares of these inputs sum to unity in the model. The production of fossil energy is assumed to be at zero cost and is hence treated as a pure rent—a part of capital income. Hence, abstracting from the fact that not all fossil energy produced is used as an intermediate good in domestic production, y will equal GDP, the sum of the payments to labor and capital plus a pure rent (which too can be thought of as capital income). Because fossil energy is also used as a final good by consumers and because some of it is net exported, y will not exactly equal GDP. Denoting these uses by e_c and e_x, respectively, GDP is equal to $y + p(e_c + e_x)$, where p is the price of fossil fuel. The energy income share in data we would ideally use is the one corresponding to the share in producing y. So it should be $p\frac{e-e_c}{GDP-p(e_c+e_x)}$, where e is total domestic fuel use. However, given that we do not have data on e_c (except for a shorter period of time), we set $e_c = 0$ in the previous expression for the energy income share. The data we do have suggests that the omission of e_c only has a minor level effect on the energy income share and no effect on its movements.

A.2 Energy-saving in the manufacturing sector

Figure 8 plots the evolution of the level of the energy-saving technology for the manufacturing sector. The kink is less pronounced, but the energy-saving technology is clearly growing at a faster rate after the oil shocks than before.

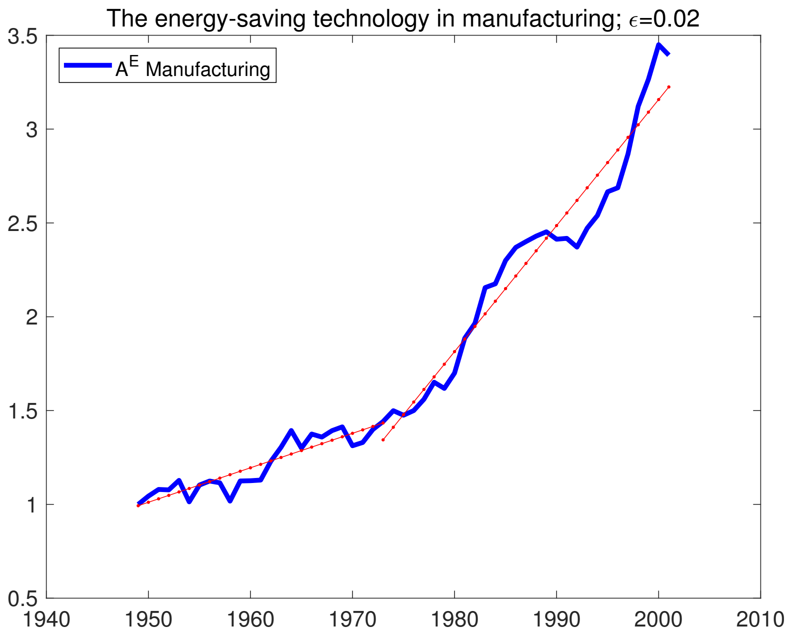

Figure 8: The energy-saving technology in manufacturing

We thank the editor and three anonymous referees for invaluable comments and suggestions and numerous seminar participants for feedback. Hassler and Krusell thank Mistra-Swecia, the Swedish Energy Agency, the Swedish Research Council, the Knut and Alice Wallenberg Foundation, and the Riksbankens Jubileumsfond for financial support. The opinions expressed in this article are the sole responsibility of the authors and should not be interpreted as reflecting the views of Sveriges Riksbank.

Directed technical change as a response to natural-resource scarcity
Online Appendix

John Hassler*, Per Krusell†, and Conny Olovsson‡§

In this appendix, we describe the data in detail, prove the theorems and propositions that are stated in the paper, as well as supply some additional material.

1 Data

The data is annual and the considered period is 1949–2018.

1.1 Energy

All data related to energy is taken from the Energy Information Administration (EIA). This includes data on energy use, net import of fossil fuel, and energy prices. The data was originally published in Annual Energy Review (2011), but it is also available online from the EIA Total Energy database (TED) at https://www.eia.gov/totalenergy/data/annual/. The tables that we refer to in this section can all be found on that specific web page. Because some data sources provided by the EIA ends in 2011, our paper originally only included data up to 2011. In the latest version of the paper, we have collected the required data to extend the period to 1949 to 2018. The labelling of the data tables are consistent between the Annual Energy Review (2011) and online, but two things should be noted. First, some formats provided by the EIA only give values for selected years. Second, the numbers in the AER (2011) and those provided online differ marginally in some cases.

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§The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Executive Board of Sveriges Riksbank.

Data on energy use, e, between 1949 and 2018 is taken from Table 1.3: "Primary Energy Consumption by Source" that contains information about usage of oil, coal, and natural gas measured in Quadrillion Btu:s. Net import of fossil fuel between 1949 and 2018 is taken from table 1.4.

The EIA Total Energy database only provides data on fossil fuel prices between 1949 and 2011. Hence to get data on prices up to 2018, we combine the fuel price data from the TED with additional sources from the EIA.

For the oil price, we use the "domestic Crude Oil First Purchase Prices by Area" for the whole period. This data is found at https://www.eia.gov/dnav/pet/pet_pri_dfp1_k_a.htm. For natural gas, we combine the "Natural Gas Wellhead Price" and the "Electric Power Price". Specifically, we use the Wellhead price up to 2012 where the series end, and the use the Electric Power Price for the period 2013 to 2018. These sources are available at the EIA web page at https://www.eia.gov/dnav/ng/hist/n9190us3a.htm and https://www.eia.gov/dnav/ng/ng_pri_sum_a_EPG0_PEU_DMcf_a.htm. For coal, finally, we use the coal price from the TED for the period 1949 and 2009 and the variable "Coal shipments to the electric power sector: price, by plant state: all coal 2018" for the period 2010 to 2018. This variable is available from the Coal Data Browser. All prices are expressed in thousands of 2005 dollars per million Btu (deflated with the GDP deflator described in Section 1.2).

Our extended prices series (blue solid line) along with the fuel prices from the TED (red dashed line) are plotted in Figure 1. As can be seen, the two series are basically identical for each fuel. The EIA also provides a fossil-fuel-composite price in Table 3.1: "Fossil Fuel Production Prices". This price is adjusted for the different energy contents of the different fuels. Specifically, the composite price is: "[d]erived by multiplying the price per Btu of each fossil fuel by the total Btu content of the production of each fossil fuel and dividing this accumulated value of total fossil fuel production by the accumulated Btu content of total fossil fuel production".[1] We follow the same approach when computing the composite fuel price, i.e., each fuel price is adjusted for the energy content of the fuel.

The average price per Btu of coal is on average 3.82 times higher for oil and 1.63 times higher for natural gas over the whole considered period. The underlying assumption is that coal is priced at marginal extraction costs and that this determines the price of oil and gas. The latter two are priced so that it compensates for the higher efficiency, providing rents

[1] See the footnote under Table 3.1.

to extractors due to its lower extraction cost per unit of efficiency.[2] Denoting the period-t consumption of coal, oil and gas respectively by E_t^c, E_t^o and E_t^g, the sum of all fossil fuel consumption is E_t^c+ $E_t^o + E_t^g$. We then calculate the fossil fuel composite to be

$$E_t = E_t^c + 3.82E_t^o + 1.63E_t^g. \tag{1}$$

The fossil-fuel composite price is then

$$P_t = \frac{P^c E_t^c + P^o E_t^o + P^g E_t^g}{E_t}. \tag{2}$$

As a robustness check, we have verified that all results are robust to not adjusting for the fact that the prices of oil and gas are higher than for coal. This, instead, implies that $E_t = E_t^c + E_t^o + E_t^g$, with the price still given by (2).

Data on net import of fossil fuel measured in Btu:s is taken from Table 1.4: "Primary energy trade by source", and the value of this net import measured in thousands of 2005 dollars is from Table 3.9.

1.2 Output, labor and capital

Data on nominal GDP is taken from the FRED database and then converted to billions of 2005 dollars with the GDP deflator that also is from the FRED database. The output measure employed in the paper, y, is then computed as GDP minus net export of fossil fuel in chained (2005) dollars, i.e.,

$$y = GDP - (\text{export of fossil fuel-import of fossil fuel}).$$

Data on the labor force, L, is taken from the Bureau of Labor Statistics: "Employed, 16 years and over". This data is available at https://www.bls.gov/webapps/legacy/cpsatab10.htm. The labor share of income is computed as *Total compensation of employees*/y, where *Total compensation of employees* is from the BEA, Table 1.13, available at https://apps.bea.gov /iTable/iTable.cfm?reqid=19&step=2. The compensation is denoted in Billions of dollars and was converted to Billions of 2005 dollars with the GDP deflator.

The capital stock is taken from the FRED database, and the variable is denoted "RK-

[2]The supply of oil and gas will depend on how the supply side is organized, which we abstract from here.

NANPUSA666NRUG_20190411".

2 Motivational regressions

We here run regressions in the style of Katz and Murphy (1992) as a first evaluation of the elasticity of substitution between capital/labor and energy. The following definitions are used

$$REL\_Share_t \equiv \log\left(\frac{e_t^{share}}{1-e_t^{share}}\right), \text{ and}$$
$$REL\_Price_t = \log\left(\frac{p_t}{r_t^{\alpha} w_t^{1-\alpha}}\right),$$

where p_t, r_t, and w_t respectively denote factor prices for fossil energy, capital, and labor. The interest rate is computed from the relation $r_t = k_t^{share}/Y_t$, where $k_t^{share} = 1 - e_t^{share} - l_t^{share}$. We then run the following regression.

$$REL\_Share_t = const + \gamma t + \beta REL\_Price_t + \epsilon_t,$$

where ϵ_t is the error term.

The results are reported in Tables (1) and (2). With ε given by 1 minus the regression coefficient on REL_Price, we have elasticities of 0.1265 and 0.0443 for the full period and the post 1973-period respectively.

Table 1: Katz and Murphy regression, 1949–2017

				Number of obs	69
				F(2, 42)	196.59
				Prob > F	0.0000
				R-squared	0.8563
				Adj R-squared	0.8519
				Root MSE	0.13811
REL_Share	$Coef.$	$Std.Err.$	t	$P > \|t\|$	[95% Conf. Interval]
REL_Price	0.8734682	0.0443158	19.71	0.0000	[.7849888.9619477]
γ	0.0050818	0.0008512	5.97	0.0000	[.0033824.0067812]
$const$	0.48336	0.20962	2.31	0.0024	[.0648402.9018798]

Table 2: Katz and Murphy regression, 1973–2017

				Number of obs	45
				F(2, 42)	1158.32
				Prob > F	0.0000
				R-squared	0.9822
				Adj R-squared	0.9813
				Root MSE	0.05634
REL_Share	*Coef.*	*Std.Err.*	t	$P > \|t\|$	[95% Conf. Interval]
REL_Price	0.955701	0.0206103	46.37	0.0000	[0.91410770.9972944]
γ	-0.00317	0.0006562	-4.83	0.0000	$[-0.0044943 - 0.0018457]$
const	1.219482	0.0979918	12.44	0.0000	[1.0217271.417238]

3 The main model

For transparency, we here restate the model in the paper. The representative consumer derives utility from a discounted sum of a power function of consumption at different dates:

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma}. \tag{3}$$

The period resource constraint is given by

$$c_t + k_{t+1} = y_t + (1-\delta)k_t, \tag{4}$$

as in standard one-good models.

Energy comes from a fossil-fuel source which we think of as oil. Oil is a finite resource, which implies that the following constraint must be respected:

$$\sum_{t=0}^{\infty} e_t \leq R_0, \tag{5}$$

where R_t is the remaining stock of oil in ground in the beginning of time t. We furthermore assume that oil is costless to extract.

Output, y, is produced with capital (k), labor (l), and fossil fuel (e) according to the following function

$$y_t \equiv F\left(A_t k_t^{\alpha} l_t^{1-\alpha}, A_{e,t} e_t\right) = \left[(1-\gamma)\left(A_t k_t^{\alpha} l_t^{1-\alpha}\right)^{\frac{\varepsilon-1}{\varepsilon}} + \gamma\left(A_{e,t} e_t\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{6}$$

For all the results in this appendix labor is exogenous and can, without loss of generality, be normalized to one for all periods. For notational simplicity, we therefore omit labor from the analysis where this is possible. In addition, unless otherwise stated we only focus on the case where $\varepsilon < 1$.

4 Proof of Theorem 1

Theorem 1 refers to a model with exogenous constant growth rates for both of the input-saving technologies. These technologies, respectively, are assumed to grow at the following rates:

$$A_t = g_A^t$$

and

$$A_{e,t} = g_{A_e}^t.$$

We look at cases where g_A and g_{A_e} are both greater than or equal to 1.

To prove the theorem, let us transform this problem into one that, at least potentially, is stationary: define new variables $\tilde{k}_t = k_t/g_A^{\frac{t}{1-\alpha}}$, $\tilde{c}_t = c_t/g_A^{\frac{t}{1-\alpha}}$, $\tilde{A}_{et} = A_{et}/g_A^{\frac{t}{1-\alpha}}$, and $\tilde{e}_t = e_t\tilde{A}_{e,t}$.

The problem is then to maximize

$$\sum_{t=0}^{\infty} \tilde{\beta}^t \frac{\tilde{c}_t^{1-\sigma} - 1}{1-\sigma}, \tag{7}$$

where $\tilde{\beta} = \beta g_A^{\frac{1-\sigma}{1-\alpha}}$, subject to

$$\tilde{c}_t + \tilde{k}_{t+1} g_A^{\frac{1}{1-\alpha}} = F\left(\tilde{k}_t^{\alpha}, \tilde{e}_t\right) + (1-\delta)\tilde{k}_t, \tag{8}$$

and

$$\sum_{t=0}^{\infty} \frac{\tilde{e}_t}{\tilde{A}_{et}} \le R_0. \tag{9}$$

This problem looks stationary except for the factor $\tilde{A}_{e,t}$ in the energy resource constraint.

This factor grows exponentially: its growth rate is $\tilde{g} = g_{A_e}/g_A^{\frac{1}{1-\alpha}}$.

Let us now drop tildes for convenience, except for the case of the growth rate $\tilde{g}$. The marginal products of production with respect to capital and energy are given by

$$F_1\left(k_{t+1}^{\alpha}, e_{t+1}\right)\alpha k_{t+1}^{\alpha-1} = \left[1-\gamma+\gamma\left(\frac{e_{t+1}}{k_{t+1}^{\alpha}}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{1}{\varepsilon-1}}(1-\gamma)\,\alpha k_{t+1}^{\alpha-1}; \tag{10}$$

$$F_2\left(k_t^{\alpha}, e_t\right) = \left[(1-\gamma)\left(\frac{k_t^{\alpha}}{e_t}\right)^{\frac{\varepsilon-1}{\varepsilon}}+\gamma\right]^{\frac{1}{\varepsilon-1}}\gamma. \tag{11}$$

The first-order conditions w.r.t. k_{t+1} and e_t are given by the usual Euler equation,

$$\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = \frac{\beta}{g_A^{\frac{1}{1-\alpha}}}\left[F_1\left(k_{t+1}^{\alpha}, e_{t+1}\right)\alpha k_{t+1}^{\alpha-1}+1-\delta\right], \tag{12}$$

and the Hotelling equation

$$F_2\left(k_t^{\alpha}, e_t\right)\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = \beta F_2\left(k_{t+1}^{\alpha}, e_{t+1}\right)\tilde{g}, \tag{13}$$

respectively. We will now look at three different cases: we will distinguish whether $\tilde{g}$ is equal to, above, or below 1.

4.1 $\tilde{g} = 1$

This case becomes similar in nature to a case without any technical change at all. In a case without technical change, energy needs to go to zero, as do capital and output; here these features refer to the transformed variables so depending on the rate at which the variables go to zero one obtains either positive or negative growth for the untransformed economy. To analyze this case, note first from (10) that if the ratio $\frac{e_{t+1}}{k_{t+1}^{\alpha}}$ becomes constant (asymptotically; if not otherwise stated, all statements henceforth refer to asymptotics), the marginal product of capital goes to infinity as k_{t+1} goes to zero. Hence, e and k^{α} cannot grow at the same rate.

We then have two remaining possibilities: either $g_e > g_k^{\alpha}$ or $g_e < g_k^{\alpha}$.

First, if $g_e > g_k^\alpha$, then $\lim_{t\to\infty} \frac{e_t}{k_t^\alpha} = \infty$. With $\varepsilon < 1$, we then obtain

$$\lim_{t\to\infty} \left[(1-\gamma) + \gamma\left(\frac{e_t}{k_t^\alpha}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{1}{\varepsilon-1}} = (1-\gamma)^{\frac{1}{\varepsilon-1}}.$$

The marginal product of capital is now asymptotically given by

$$\lim_{t\to\infty} \alpha k_t^{\alpha-1} F_1\left(k_t^\alpha, e_t\right) = (1-\gamma)^{\frac{\varepsilon}{\varepsilon-1}} \alpha k_t^{\alpha-1} = \infty.$$

We conclude that this case is not asymptotically balanced either.

If, instead, $g_e < g_k^\alpha$, then $\lim_{t\to\infty} \frac{e_t}{k_t^\alpha} = 0$. We then obtain $\lim_{t\to\infty} \left[(1-\gamma) + \gamma\left(\frac{e_t}{k_t^\alpha}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{1}{\varepsilon-1}} = \gamma^{\frac{1}{\varepsilon-1}} \left(\frac{e_t}{k_t^\alpha}\right)^{\frac{1}{\varepsilon}}$. The marginal product of capital is then asymptotically given by

$$\lim_{t\to\infty} \alpha k_t^{\alpha-1} F_1\left(k_t^\alpha, e_t\right) = e_t^{\frac{1}{\varepsilon}} k_t^{\alpha-1-\frac{\alpha}{\varepsilon}} (1-\gamma)\, \alpha\gamma^{\frac{1}{\varepsilon-1}}.$$

We obtain, in order for the interest rate not to go to infinity or zero, it will have to be the case that

$$g_e^{\frac{1}{\varepsilon}} g_k^{\alpha\frac{\varepsilon-1}{\varepsilon}-1} = 1. \tag{14}$$

The asymptotic marginal product of energy is given by

$$\lim_{t\to\infty} F_2\left(k_t, e_t\right) = \gamma^{\frac{\varepsilon}{\varepsilon-1}},$$

which is a finite constant. Equation (13) then gives that

$$g_c\sigma = \beta. \tag{15}$$

Equations (12) and (15) together implies that the gross interest rate is equal to $g_A^{\frac{1}{1-\alpha}}$. The value of $e_t^{\frac{1}{\varepsilon}} k_t^{\alpha\frac{\varepsilon-1}{\varepsilon}-1}$ needs to adjust so as to deliver this interest rate.

Aggregate output can be written as

$$\lim_{t\to\infty} y_t = e_t\gamma^{\frac{\varepsilon}{\varepsilon-1}},$$

which must grow at the same rate as e_t, i.e., $g_y = g_e$.

We now show that $g_k < g_e$. From the resource constraint this implies that $g_c = g_y = g_e$; capital grows at a lower rate than output and, hence, the investment-output ratio goes to zero (the case where it goes to infinity is not possible). Equation (14) can be written as

$$g_k = g_e^{\frac{1}{\alpha+\varepsilon(1-\alpha)}},$$

and with $g_e < 1$, we must have $g_k < g_e$ (as well as $g_e < g_k^\alpha$). In conclusion, this case is asymptotically balanced.

4.2 $\tilde{g} < 1$

The previous analysis for the case $g_e \geq g_k^\alpha$ holds without changes: this case is not consistent with asymptotically balanced growth. The case $g_e < g_k^\alpha$ is very similar to before; here $g_c\sigma = \beta$ is replaced by $g_c\sigma = \beta\tilde{g}$, so that the gross interest rate, when read off the Euler equation, must become $g_A^{\frac{1}{1-\alpha}}\tilde{g} = g_{A_e}$.

4.3 $\tilde{g} > 1$

This case is different in nature, as it is evident that a constant value for energy now is resource-feasible: this value becomes $\frac{\tilde{g}}{\tilde{g}-1}R_0$. An exponentially growing path for energy use is also feasible, if the growth rate is lower than $\tilde{g}$. What asymptotic paths will now satisfy the system of equations at hand?

Let us start with the case $g_e = g_k^\alpha$. For the gross interest rate to be finite and above $1-\delta$ it would have to be that $g_k = 1$. This implies $g_e = 1$, i.e., the case just alluded to. Here, output does not grow. On the other hand, the Hotelling equation implies that $g_c = (\beta\tilde{g})^{\frac{1}{\sigma}}$. Thus, unless $\beta\tilde{g} = 1$, this case is not consistent with balanced growth, as consumption would have to grow at the rate of output.

Under $g_e > g_k^\alpha$, given that F_1 would become constant asymptotically, one would have to have $g_k = 1$. In such a case, we obtain that F_2 grows at the rate $g_e^{-\frac{1}{\varepsilon}}$, leading equation (13) to asymptotically read

$$g_c\sigma = \beta\tilde{g}g_e^{-\frac{1}{\varepsilon}}.$$

Thus, the interest rate becomes $g_{A_e}g_e^{-\frac{1}{\varepsilon}}$.

Output becomes proportional to k_t^α in this case, which means output is constant and consumption will therefore be constant too. This implies, from the equation above, a value for g_e that equals $(\beta\tilde{g})^{\varepsilon}$. So long as $\tilde{g}$ is large enough as to make this expression greater than one, this is a feasible asymptotic solution: energy grows at some rate and output is constant (we need $\tilde{g} > 1/\beta$). Now given that $g_e = (\beta\tilde{g})^{\varepsilon}$, we see that the interest rate becomes $g_{A_e}(\beta\tilde{g})^{-1} = (1/\beta)g_A^{\frac{1}{1-\alpha}}$, so that the long-run level of output is given by $(1-\gamma)^{\frac{\varepsilon}{\varepsilon-1}}k^{\alpha}$, where k is given by

$$\alpha(1-\gamma)^{\frac{\varepsilon}{\varepsilon-1}}k^{\alpha-1} + 1 - \delta = (1/\beta)g_A^{\frac{1}{1-\alpha}}.$$

It is noteworthy, here, that g_{A_e} does not affect long-run capital or output (or consumption): a higher rate of energy-saving technical change raises $\tilde{g}$ but this parameter does not appear in the equation. Hence more energy-saving in this case really lets us save on capital in the short run, allowing higher consumption early on along the transition path.

Under $g_e < g_k^{\alpha}$, finally, the Euler equation again requires us to satisfy equation (14), as F_1 goes to infinity in this case. The Hotelling equation, since F_2 becomes constant here, delivers

$$g_c^{\sigma} = \beta\tilde{g}.$$

Output is growing at rate g_e, so $g_c = g_e = (\beta\tilde{g})^{\frac{1}{\sigma}}$. We also need to verify that capital grows at a rate lower than this rate. Since it grows at $g_e^{\frac{1}{\alpha+\varepsilon(1-\alpha)}}$, where the exponent is strictly greater than one, we conclude that it does whenever $\beta\tilde{g} < 1$.

We summarize and compare our three cases, all of which admit asymptotic growth under certain parameter restrictions, as follows: (i) if $\beta\tilde{g} > 1$, there is a unique asymptotically balanced path and it has $g_k^{\alpha} = g_k = g = 1 < g_e = (\beta\tilde{g})^{\varepsilon}$; (ii) if $\beta\tilde{g} = 1$, there is a unique exact balanced path and it has $g_k = g = 1 = g_e = g_k^{\alpha}$; and (iii) if $\beta\tilde{g} < 1$, there is a unique asymptotically balanced path and it has $g_k < g = (\beta\tilde{g})^{\frac{1}{\sigma}} = g_e < g_k^{\alpha}$.

Let us also calculate the growth rates of the shares of energy in these three cases. In case (i) the marginal product of capital is constant and capital grows at the rate of output, whereas F_2 grows at rate $(\beta\tilde{g})^{-\frac{1}{\varepsilon}}$ (i.e., it goes to zero) and e grows at $(\beta\tilde{g})^{\varepsilon}$, the product of which is negative and hence the energy share goes to zero here. In case (ii), both marginal products are constant and both e and k grow at the rate of output, so here we obtain balanced shares. In case (iii), finally, F_2 is constant and so is the marginal product of capital (F_1 goes to zero but F_1 times $k^{\alpha-1}$ remains constant), whereas e grows at the rate of output whereas capital grows at a lower rate; hence the energy share is 100% in the limit.

This completes the proof of Theorem 1.

5 Proof of Proposition 1

The problem is to maximize

$$\left[(1-\gamma)\left(Ak^{\alpha}\right)^{\frac{\varepsilon-1}{\varepsilon}}+\gamma\left(A_{e}R\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

by choice of A and A_e subject to the technology constraint:

$$G(A, A_e) = 0. \tag{16}$$

The first-order conditions are as follows. For k,

$$(1-\gamma)\,\alpha Ak^{\alpha-1}\left[1-\gamma+\gamma\left(\frac{A_{e}R}{Ak^{\alpha}}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{1}{\varepsilon-1}}=1; \tag{17}$$

for A,

$$(1-\gamma)\,k^{\alpha}\left[1-\gamma+\gamma\left(\frac{A_{e}R}{Ak^{\alpha}}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{1}{\varepsilon-1}}=\mu G_{A}\left(A,A_{e}\right); \tag{18}$$

and for A_e,

$$\gamma R\left[(1-\gamma)\left(\frac{Ak^{\alpha}}{A_{e}R}\right)^{\frac{\varepsilon-1}{\varepsilon}}+\gamma\right]^{\frac{1}{\varepsilon-1}}=\mu G_{A_{e}}\left(A,A_{e}\right). \tag{19}$$

Divide (18) by (19) and multiply by A/A_e to get

$$\frac{(1-\gamma)\,Ak^{\alpha}\left[1-\gamma+\gamma\left(\frac{A_{e}R}{Ak^{\alpha}}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{1}{\varepsilon-1}}}{\gamma A_{e}R\left[(1-\gamma)\left(\frac{Ak^{\alpha}}{A_{e}R}\right)^{\frac{\varepsilon-1}{\varepsilon}}+\gamma\right]^{\frac{1}{\varepsilon-1}}}=\frac{AG_{A}\left(A,A_{e}\right)}{A_{e}G_{A_{e}}\left(A,A_{e}\right)}.$$

Rearranging the above expression delivers the following expression:

$$\frac{1-\gamma}{\gamma}\left(\frac{Ak^{\alpha}}{A_{e}R}\right)^{\frac{\epsilon-1}{\epsilon}}=\frac{AG_{1}(A,A_{e})}{A_{e}G_{1}(A,A_{e})}. \tag{20}$$

We now have three equations (16), (17), and (20). Now suppose that G is such that, once appropriately substituted into the right-hand side of (20), this right-hand side depends on A/A_e alone and is *increasing* in it. (I.e., $d\log A_e/d\log A$ is increasing in A/A_e along the curve defined by (16).) Then we conclude from (20) that (i) Ak^α/A_eR is decreasing in A/A_e (so long as $\epsilon < 1$). It also follows, by inspecting this same equation, that (ii) A/A_e is decreasing in k^α/R (if $\varepsilon < 1$). From (i) and (ii) we conclude that (iii) Ak^α/A_eR is increasing in k^α/R.[3]

Now consider (17): let us look at the three factors containing endogenous variables and express these exogenous variables as functions solely of k^α/R. We will show that each of these functions of k^α/R is strictly decreasing and hence that the equation has a unique solution.

First, given the nature of G (being increasing in both arguments), if A goes up, A/A_e increases. Then from fact (ii) we conclude that the first endogenous factor, A, is decreasing in k^α/R. The second endogenous factor is $k^{\alpha-1}$. It is (trivially) decreasing in k^α/R. The third endogenous factor is decreasing in Ak^α/A_eR. But fact (iii) then shows that the third endogenous factor is also decreasing in k^α/R. This completes the proof of Proposition 1.

6 Proof of Proposition 2

The goal is to ensure that the minimization problem

$$\min_{k,l,R,A,A_e} rk + wl + pR$$

subject to $F(Ak^\alpha l^{1-\alpha}, A_eR) \geq y$ and $G(A, A_e) \leq 0$ is well-defined and has a unique solution for all y. Conceptually, one way to ascertain that first-order conditions are sufficient for a global minimum is to assume quasiconcavity of the objective in the whole vector of choice variables. However, it is also possible to proceed sequentially. The idea is that a maximization problem $\max_{x,y} f(x,y)$ can equivalently be written $\max_x (\max_y f(x,y))$. Though in general $f(x, y(x))$, where $y(x)$ is a maximizer at x, could be quite a complicated function of x, here it turns out not to be and it is possible to show quasiconcavity of this function in x. We therefore proceed by rewriting the problem as

$$\min_{(A,A_e):G(A,A_e)\leq 0} \left\{ \min_{k,l,R} rk + wl + pR \quad \text{s.t.} \quad F(Ak^\alpha l^{1-\alpha}, A_eR) \geq y \right\}$$

[3]Notice that (iii) follows despite $Ak^\alpha/A_eR = (A/A_e)\cdot(k^\alpha/R)$ where the first factor is decreasing in k^α/R.

and note that, since F is CES and homogeneous of degree 1, it is well-known (and easy to verify) that the last minimization problem results in a value $yC(A, A_e; r, w, p)$, where

$$C(r, w, p, y; A, A_e) = y\left(\left(\frac{\left(\frac{r}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}}{(1-\gamma)^{\frac{\epsilon}{\epsilon-1}}A}\right)^{1-\epsilon} + \left(\frac{p}{\gamma^{\frac{\epsilon}{\epsilon-1}}A_e}\right)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}.$$

Thus, our minimization problem now reads

$$y \cdot \min_{(A,A_e):G(A,A_e)\leq 0} C(A, A_e; r, w, p).$$

Since G is quasiconcave, it now suffices to show that C is quasiconcave in (A, A_e). Thus let $\bar{C}$ be a level associated with an iso-cost curve:

$$\bar{C} = \left((1-\gamma)^{\epsilon}\left(\frac{A}{\left(\frac{r}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}}\right)^{\epsilon-1} + \gamma^{\epsilon}\left(\frac{A_e}{p}\right)^{\epsilon-1}\right)^{-\frac{1}{\epsilon-1}}.$$

It suffices to demonstrate that A_e, solved as a function of A from this equation, is (downward-sloping and) concave in A given the isocost level $\bar{C}$. Now

$$\bar{C}^{1-\epsilon} = (1-\gamma)^{\epsilon}\left(\frac{A}{\left(\frac{r}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}}\right)^{\epsilon-1} + \gamma^{\epsilon}\left(\frac{A_e}{p}\right)^{\epsilon-1}.$$

Thus

$$A_e = \frac{1}{b}\left[\bar{C}^{1-\epsilon} - aA^{\epsilon-1}\right]^{\frac{1}{\epsilon-1}}.$$

where a and b involve r, w, p, γ and ϵ, all held constant here. Now

$$A_e = -\frac{a}{b}A^{\epsilon-2}\left[\bar{C}^{1-\epsilon} - aA^{\epsilon-1}\right]^{\frac{1}{\epsilon-1}-1},$$

where we note that the bracket exponent equals $\frac{2-\epsilon}{\epsilon-1}$ and that $A^{\epsilon-2} = (A^{1-\epsilon})^{\frac{2-\epsilon}{\epsilon-1}}$. Thus

$$A_e = -\frac{a}{b}\left[\left(\bar{C}A\right)^{1-\epsilon} - a\right]^{\frac{2-\epsilon}{\epsilon-1}}.$$

Clearly A_e is an increasing function of A when $\epsilon < 1$. Thus, the isocost curve A_e of A is decreasing and convex (C is quasiconcave in (A, A_e)).

Proposition 2 also states that e^{share} is given by

$$\frac{1 - e^{share}}{e^{share}} = \frac{AG_1(A, A_e)}{A_e G_2(A, A_e)}.$$

The relative share of the capital/labor composite and energy is given by $AF_1 k^\alpha / A_e F_2 R$. The first-order condition for A and A^e in the profit-maximization problem, implies

$$\frac{k^\alpha F_1}{G_1(A, A_e)} = \frac{RF_2}{G_2(A, A_e)}. \tag{21}$$

Multiplying both sides of (21) by A/A_e and rearranging delivers the relative share. This completes the proof of Proposition 2.

7 Proof of Theorem 2

We now add the following definitions and constraint on the technology technology:

$$A_{t+1}/A_t \equiv g_{A,t} = f(n_t) \tag{22}$$

$$A_{e,t+1}/A_{et} \equiv g_{Ae,t} = f_e(1 - n_t), \tag{23}$$

where $n_t \in [0, 1]$.

Theorem 2 then reads as follows.

Theorem 1 *On an exact balanced growth path (EBGP) with an interior choice for technology, the following features must hold:*

1. *The two arguments of the aggregate production function, $A_t k_t^\alpha$ and $A_{e,t} e_t$, both grow at the rate of output g.*
2. *Energy use falls at rate $\beta g^{1-\sigma}$.*
3. *Technology effort n and the consumption growth rate g are determined by $f_e(1-n)\beta = f(n)^{\frac{\sigma}{1-\alpha}} = g\sigma$.*

4. *Energy's share of income is exclusively determined by how costly it is to enhance energy efficiency in terms of lost capital/labor efficiency. Specifically, the long-run energy share is implicitly given by equation (24):*

$$\frac{1-e^{share}}{e^{share}} = -\frac{\partial g_{A_e}/g_{A_e}}{\partial g_A/g_A}. \tag{24}$$

The proof of the theorem has several parts. To derive (24), start by maximizing (3) subject to constraints (4), (5), (22), and (23). Let $\lambda_t\beta^t$, κ, $\mu_t\beta^t$, and $\mu_{et}\beta^t$ denote the multipliers on these four constraints. The first-order conditions with respect to c_t, k_{t+1}, e_t, n_t, A_{t+1}, and A_{et+1} are then given by

$$\lambda_t = c_t^{-\sigma} \tag{25}$$
$$\lambda_t = \beta\lambda_{t+1}F_1(t+1)\left(\alpha A_{t+1}k_{t+1}^{\alpha-1}+1-\delta\right) \tag{26}$$
$$\kappa = \beta^t\lambda_t F_2(t)\,A_{e,t} \tag{27}$$
$$\frac{\mu_t}{\mu_{et}} = \frac{A_{e,t}f_e'(1-n_t)}{A_t f'(n_t)} \tag{28}$$
$$\mu_t = \beta\left[\lambda_{t+1}F_1(t+1)\,k_{t+1}^{\alpha}+\mu_{t+1}f(n_{t+1})\right] \tag{29}$$
$$\mu_{et} = \beta\left[\lambda_{t+1}F_2(t+1)\,e_{t+1}+\mu_{et+1}f_e(1-n_{t+1})\right], \tag{30}$$

where $F_i(t+1)$ refers to the derivative of F with respect to its ith argument, evaluated in period $t+1$.

On an EBGP, the resource constraint dictates that c, k, and y all grow at the rate g. Both the arguments of production then have to grow at the rate of output, since the production function is homogeneous of degree one (stated feature 1). It follows from Euler's theorem that both $F_1(t)$ and $F_2(t)$ have to be constant on the EBGP. From (25), it then follows that the multiplier λ must grow at rate $g^{-\sigma}$. Using this fact in combination with (29) reveals that the balanced-growth rate for μ is $g^{\alpha-\sigma}$. Equation (27) shows that the balanced growth rate for A_e is $g_{A_e} = g^{\sigma}/\beta$, which implies $g_e = \beta g^{1-\sigma}$: feature 2. Equation (30) can then be used to infer that μ_e will grow at rate $\beta g^{1-2\sigma}$ on the EBGP. The first feature stated in the theorem implies $f(n)g^{\alpha} = f_e(1-n)g_e = g$. This equation and the determination of g_e delivers stated feature 3.

Combining (29) and (30) and dividing through by $F(t+1)$ delivers the following expression

$$\frac{F_2(t+1)A_{e,t+1}e_{t+1}}{F(t+1)} = \frac{F_1(t+1)A_{t+1}k_{t+1}^{\alpha}}{F(t+1)}\frac{\mu_{et}/\beta - \mu_{e,t+1}f_e(1-n_{t+1})}{\mu_t/\beta - \mu_{t+1}f(n_{t+1})}\frac{A_{e,t+1}}{A_{t+1}}. \tag{31}$$

Using (28) in the above equation gives

$$\frac{1-e_{t+1}^{share}}{e_{t+1}^{share}} = \frac{\frac{\mu_t}{\mu_{t+1}\beta f(n_{t+1})}-1}{\frac{\mu_{et}}{\mu_{e,t+1}\beta f(1-n_{t+1})}-1}\frac{f_e'(1-n_{t+1})/f_e(1-n_{t+1})}{f'(n_{t+1})/f(n_{t+1})}. \tag{32}$$

Inserting the balanced growth rates for μ and μ_e into (32) reveals that the first ratio on the right-hand side of this equation on an EBGP equals one. Finally, substituting the fact that $-\frac{\partial g_{A_e,t}/g_{A_e,t}}{\partial g_{A,t}/g_{A,t}} = \frac{f_e(1-n_t)/f_e(1-n_t)}{f'(n_t)/f(n_t)}$ into (32), and recognizing that e^{share} and n are both constant on an EBGP delivers (24).

This completes the proof of Theorem 2.

8 Proof of Theorem 3

To prove Theorem 3, let us rewrite (29) and (30) as follows:

$$\frac{\mu_t A_t}{\lambda_t}\frac{\lambda_t}{\lambda_{t+1}}\frac{A_{t+1}}{A_t} = \beta\left[F_{1,t+1}A_{t+1}k_{t+1}^{\alpha} + \frac{\mu_{t+1}A_{t+1}}{\lambda_{t+1}}f(n_{t+1})\right] \tag{33}$$

$$\frac{\mu_{et}A_{et}}{\lambda_t}\frac{\lambda_t}{\lambda_{t+1}}\frac{A_{e,t+1}}{A_{et}} = \beta\left[F_{2,t+1}A_{e,t+1}e_{t+1} + \frac{\mu_{e,t+1}A_{e,t+1}}{\lambda_{t+1}}f_e(1-n_{t+1})\right]. \tag{34}$$

We can then define $\tilde{\mu}_t \equiv \frac{\mu_t A_t f(n_t)}{\lambda_t}$ and $\tilde{\mu}_{et} \equiv \frac{\mu_{et}A_{et}f_e(1-n_t)}{\lambda_t}$ and obtain the equation system

$$\tilde{\mu}_t = \beta\frac{\lambda_{t+1}}{\lambda_t}\left[F_{1,t+1}A_{t+1}k_{t+1}^{\alpha} + \tilde{\mu}_{t+1}\right] \tag{35}$$

$$\tilde{\mu}_{et} = \beta\frac{\lambda_{t+1}}{\lambda_t}\left[F_{2,t+1}A_{e,t+1}e_{t+1} + \tilde{\mu}_{e,t+1}\right]. \tag{36}$$

Let us now consider asymptotic balanced growth paths. On such paths, n has to be constant so equation (28) then means that the $\tilde{\mu}$s are proportional to each other in the long run—they grow at the same rate.

There are now two possibilities here: the first terms on the right-hand side of the two equations grow at the same rate as the $\tilde{\mu}$s or they do not. If they do not, then they cannot grow at a faster rate—then the equations would not be met in the limit—so the relevant case to contemplate is that they grow at a slower rate. If they do, then the limit growth rate of each of the $\tilde{\mu}$s equals $\frac{1}{\beta}g_c^\sigma$, which must also equal the long-run interest rate (from the Euler equation). This would violate the transversality condition for the μs (one is then accumulating "too much" of the As).

If the two terms on the right-hand sides instead grow at the same rate in the limit, then this means that $F_{1,t+1}A_{t+1}k_{t+1}^\alpha$ and $F_{2,t+1}A_{e,t+1}e_{t+1}$ grow at the same rate. This is clearly true if $A_{t+1}k_{t+1}^\alpha$ and $A_{e,t+1}e_{t+1}$ grow at the same rate—the main case in our theorem, which is based on the construction of an exact balanced growth path. Let us now in contrast consider the case in which they do not. We are, essentially, looking at whether $\frac{xF_x(x,y)}{yF_y(x,y)}$ can converge to a finite positive value when x/y goes to either zero or infinity. Using the fact that F is CES, we see that the ratio becomes

$$\frac{x}{y}\frac{\left[1-\gamma+\gamma\left(\frac{y}{x}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{1}{\varepsilon}}}{\left[\gamma+(1-\gamma)\left(\frac{x}{y}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{1}{\varepsilon}}}.$$

When x/y goes to infinity, this expression goes to zero; when y/x goes to infinity, the expression goes to infinity. Hence, this ratio cannot yield a finite positive value in the limit. We therefore conclude that $A_{t+1}k_{t+1}^\alpha$ and $A_{e,t+1}e_{t+1}$ must grow at a common rate, i.e., be part of an exact balanced growth path.

What is not considered above is the possibility of corners, with n_∞ being either 0 or 1. This can be ruled out with Inada conditions on f and f_e (that both are zero when evaluated at zero, with an infinite local derivative there). However, let us instead consider the possibility of corner solutions with finite derivatives. Given

$$A_{t+1} = A_t f(n_t) \tag{37}$$

$$A_{e,t+1} = A_{e,t} f_e(1-n_t), \tag{38}$$

the first-order condition for R&D, i.e., for n_t, can equivalently to equation (28) be written

$$\mu_t A_t f'(n_t) - \mu_{et} A_{e,t} f_e'(1-n_t) - \nu_{1t} + \nu_{2t} = 0. \tag{39}$$

Here, $\nu_{1t} \geq 0$ is a Kuhn-Tucker (KT) multiplier for $n_t = 1$ and $\nu_{0t} \geq 0$ is a KT multiplier for $n_t = 0$. If the constraint $n_t = 1$ binds, ν_{1t} is positive: intuitively, the equation without multipliers says "> 0", so that the marginal value of increasing the growth in A at the expense of A_e is strictly positive. Notice that we can rewrite the binding $n_t = 1$ condition as

$$\tilde{\mu}_t \frac{f'(1)}{f(1)} - \tilde{\mu}_{et} \frac{f_e'(0)}{f_e(0)} > 0, \tag{40}$$

where we also assume that the levels and derivatives of f and f_e are positive and bounded. Hence we can conclude that

$$\infty > \frac{\tilde{\mu}_t}{\tilde{\mu}_{et}} > \frac{\frac{f_e'(0)}{f_e(0)}}{\frac{f'(1)}{f(1)}} > 0. \tag{41}$$

Now let us consider the long-run implications of $n_t = 1$. Suppose $f(1) \equiv \bar{g}_A$ and $f_e(0) \equiv \underline{g}_{A_e}$ are such that $\beta \underline{\tilde{g}} < 1$, where $\underline{\tilde{g}} = \underline{g}_{A_e} / \bar{g}_A^{\frac{1}{1-\alpha}}$. Then we can conclude, from the analysis in Section 4, that we are in case (iii), i.e., the case where eA_e is chosen to grow at a lower rate than that of Ak^α. In that case, we know that the cost share of energy must go to 1. This means that $\frac{eA_e}{Ak^\alpha}$ will go to infinity. Let us use this fact to study the behavior of $\tilde{\mu}/\tilde{\mu}_e$ using equations (35)–(36). These equations can be combined into

$$\frac{\tilde{\mu}_t - \beta \frac{\lambda_{t+1}}{\lambda_t} \tilde{\mu}_{t+1}}{\tilde{\mu}_{et} - \beta \frac{\lambda_{t+1}}{\lambda_t} \tilde{\mu}_{e,t+1}} = \frac{F_{1,t+1} A_{t+1} k_{t+1}^\alpha}{F_{2,t+1} A_{e,t+1} e_{t+1}}. \tag{42}$$

This expression can be rewritten as

$$\frac{\tilde{\mu}_t}{\tilde{\mu}_{et}} \frac{1 - \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{\tilde{\mu}_{t+1}}{\tilde{\mu}_t}}{1 - \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{\tilde{\mu}_{e,t+1}}{\tilde{\mu}_{et}}} = \frac{F_{1,t+1} A_{t+1} k_{t+1}^\alpha}{F_{2,t+1} A_{e,t+1} e_{t+1}}. \tag{43}$$

We know from the law of motion for $\tilde{\mu}$ that its asymptotic growth rate $g_{\tilde{\mu}}$ is less than or equal to $1/(\beta g_\lambda)$. The same holds true for $\tilde{\mu}_e$: $g_{\tilde{\mu}_e} \leq 1/(\beta g_\lambda)$. Moreover, from (41) we know that $\tilde{\mu}$ and $\tilde{\mu}_e$ have to grow at the same rate asymptotically: $g_{\tilde{\mu}} = g_{\tilde{\mu}_e}$. If both of these quantities are below $1/(\beta g_\lambda)$, we obtain, for an asymptotic constant-growth path,

$$\frac{\tilde{\mu}_t}{\tilde{\mu}_{et}} \frac{1 - \beta g_\lambda g_{\tilde{\mu}}}{1 - \beta g_\lambda g_{\tilde{\mu}_e}} = \frac{F_{1,t+1} A_{t+1} k_{t+1}^\alpha}{F_{2,t+1} A_{e,t+1} e_{t+1}}. \tag{44}$$

On the left-hand side of this equation, the second factor is positive and constant. On the right-hand side, we note from above that $\frac{A_t k_t^\alpha}{e_t A_{et}}$ must go to zero. Hence $\frac{\mu_t}{\mu_{et}}$ will go to zero. This contradicts that $\frac{\tilde{\mu}_t}{\tilde{\mu}_{et}}$ is strictly bounded below by zero. If $g_{\tilde{\mu}} = g_{\tilde{\mu}_e} = 1/(\beta g_\lambda)$, we have a bubble solution to both of the forward-looking equations; these violate transversality. Hence, we conclude that the corner case with $n = 1$ cannot satisfy the set of necessary conditions for an optimum.

Similarly, considering the possible corner $n = 0$, we assume that $f(0) \equiv \underline{g}_A$ and $f_e(1) \equiv \bar{g}_{A_e}$ are such that $\beta\tilde{\tilde{g}} \equiv \frac{\bar{g}_{A_e}}{\underline{g}_A^{1-\alpha}} > 1$ and are able to arrive at a contradiction of the first-order conditions with parallel arguments. In conclusion, by making $\tilde{g}$ have a high enough maximum and a low enough minimum, we can rule out corner solutions.

9 A dynamic decentralized economy with a log-linear $G-$function

Assume that the trade-off between $A_{e,t}/A_{e,t-1}$ and A_t/A_{t-1} is log-linear and given by

$$\frac{A_{e,t}}{A_{e,t-1}} = \exp(a)\left(\frac{A_t}{A_{t-1}}\right)^{-b}. \tag{45}$$

We now divide the full problem into two sub-problems. Within every period, the growth rates for the technology levels are chosen in the first step, whereas the factor inputs of capital, labor, and energy are chosen in the second step.

9.1 Choosing technology levels

The problem of choosing technology levels is, in period t, formally given by

$$\begin{array}{c} \max\limits_{A_t, A_{et}} \left[(1-\gamma)\left(A_t k_t^\alpha l^{1-\alpha}\right)^{\frac{\varepsilon-1}{\varepsilon}} + \gamma\left(A_{e,t} e_t\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \\ -\mu_t\left[b\log A_t + \log\left(A_{e,t}\right) - b\log\left(A_{t-1}\right) - \log\left(A_{e,t-1}\right) - a\right] \end{array}. \tag{46}$$

The first-order conditions are as follows: for A_t,

$$\left[(1-\gamma)\left(A_t k_t^\alpha l^{1-\alpha}\right)^{\frac{\varepsilon-1}{\varepsilon}} + \gamma\left(A_{e,t} e_t\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}-1}(1-\gamma)\left(A_t k_t^\alpha l^{1-\alpha}\right)^{\frac{\varepsilon-1}{\varepsilon}} = b\mu_t;$$

and foc $A_{e,t}$,

$$\left[(1-\gamma)\left(A_t k_t^{\alpha} l^{1-\alpha}\right)^{\frac{\varepsilon-1}{\varepsilon}} + \gamma\left(A_{e,t}e_t\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}-1} \gamma\left(A_{e,t}e_t\right)^{\frac{\varepsilon-1}{\varepsilon}} = \mu_t.$$

Dividing the first of these equations with the second delivers

$$\frac{1-\gamma}{\gamma}\left(\frac{A_t k_t^{\alpha} l^{1-\alpha}}{A_{e,t}e_t}\right)^{\frac{\varepsilon-1}{\varepsilon}} = b. \tag{47}$$

Now, combine (47) with the technology constraint (45) to obtain

$$A_t k_t^{\alpha} l^{1-\alpha} = e_t^{\frac{1}{1+b}} \left(k_t^{\alpha} l^{1-\alpha}\right)^{\frac{b}{1+b}} \left(A_{e,t-1}A_{t-1}^{b}\right)^{\frac{1}{1+b}} \left[\left(b\frac{\gamma}{1-\gamma}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right]^{\frac{1}{1+b}} \left(\exp(a)\right)^{\frac{1}{1+b}}; \tag{48}$$

and

$$A_{e,t}e_t = \left(A_{e,t-1}A_{t-1}^{b}\right)^{\frac{1}{1+b}} e_t^{\frac{1}{1+b}} \left(k_t^{\alpha} l^{1-\alpha}\right)^{\frac{b}{1+b}} \left[\left(b\frac{\gamma}{1-\gamma}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right]^{\frac{-b}{1+b}} \left(\exp(a)\right)^{\frac{1}{1+b}}. \tag{49}$$

Inserting the two above expressions into the production function, we arrive at

$$y_t = \left(k_t^{\alpha} l^{1-\alpha}\right)^{\frac{b}{1+b}} e_t^{\frac{1}{1+b}} \Theta_{t-1},$$

where Θ_{t-1} only depends on parameters and is given by

$$\Theta_{t-1} \equiv$$

$$\left(A_{t-1}A_{e,t-1}\right)^{\frac{1}{1+b}} \left[(1-\gamma)\left(\left[\left(b\frac{\gamma}{1-\gamma}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right]^{\frac{1}{1+b}}\right)^{\frac{\varepsilon-1}{\varepsilon}} + \gamma\left(\left[\left(b\frac{\gamma}{1-\gamma}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right]^{\frac{-b}{1+b}}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \left(\exp(a)\right)^{\frac{1}{1+b}}.$$

Hence, in any period t, the production function is effectively Cobb-Douglas in $k_t^{\alpha} l^{1-\alpha}$ and e_t.

9.2 Choosing factor inputs

The representative firm now takes the levels for A_t and $A_{e,t}$ as given and chooses levels for k_t, l, and e_t. Formally, this problem is

$$\pi_t = \max_{k_t,l,e_t} \left(k_t^{\alpha} l^{1-\alpha}\right)^{\frac{b}{1+b}} e_t^{\frac{1}{1+b}} \Theta_{t-1} - r_t k_t - w_t l - p_t e_t.$$

The first-order conditions with respect to k_t, l, and e_t, are respectively given by

$$r_t = \frac{b}{1+b} \left(k_t^{\alpha} l^{1-\alpha}\right)^{\frac{b}{1+b}} e_t^{\frac{1}{1+b}} \Theta_{t-1} \frac{\alpha}{k_t}; \quad (50)$$

$$w_t = \frac{b}{1+b} \left(k_t^{\alpha} l^{1-\alpha}\right)^{\frac{b}{1+b}} e_t^{\frac{1}{1+b}} \Theta_{t-1} \frac{1-\alpha}{l}; \quad (51)$$

and

$$p_t = \frac{1}{1+b} \left(k_t^{\alpha} l^{1-\alpha}\right)^{\frac{b}{1+b}} e_t^{\frac{1}{1+b}} \Theta_{t-1} \frac{1}{e_t}. \quad (52)$$

9.3 Consumers

A representative household derives utility from a stream of consumption units, c_t. It owns a depletable resource R, and it supplies one unit of labor, l, inelastically each period. The problem for the households is to maximize

$$U = \sum_{t=0}^{\infty} \beta^t \log\left(c_t\right),$$

subject to the resource constraint

$$\sum_{t=0}^{\infty} e_t = R_0, \quad (53)$$

and the budget constraint

$$c_t + k_{t+1} = w_t l + r_t k_t + p_t e_t. \quad (54)$$

Denoting the multiplier on (53) by κ, the first-order conditions with respect to k_{t+1}, e_t, and e_{t+1} are respectively given by

$$\frac{c_{t+1}}{c_t} = \beta r_{t+1}; \quad (55)$$

$$\beta^t \frac{p_t}{c_t} = \kappa;$$

and

$$\beta^{t+1} \frac{p_{t+1}}{c_{t+1}} = \kappa.$$

Combining the two first-order conditions for e gives

$$\frac{c_{t+1}}{c_t} = \beta \frac{p_{t+1}}{p_t}. \tag{56}$$

Combining (55) and (56) delivers the Hotelling equation

$$\frac{p_{t+1}}{p_t} = r_{t+1}. \tag{57}$$

9.4 Equilibrium

Combining (54), (55), and the period-$t+1$-version of (50) gives the following Euler equation:

$$\frac{w_{t+1}l + r_{t+1}k_{t+1} + p_{t+1}e_{t+1} - k_{t+2}}{w_t l + r_t k_t + p_t e_t - k_{t+1}} = \beta \frac{b}{1+b} \left(k_{t+1}^{\alpha} l^{1-\alpha}\right)^{\frac{b}{1+b}} e_{t+1}^{\frac{1}{1+b}} \Theta_t \frac{\alpha}{k_{t+1}}.$$

Since (50)-(52) imply zero profits, the following condition must hold.

$$\left(k_t^{\alpha} l^{1-\alpha}\right)^{\frac{b}{1+b}} e_t^{\frac{1}{1+b}} \Theta_{t-1} = r_{t+1}k_{t+1} + w_{t+1}l + p_{t+1}e_{t+1}.$$

The Euler equation can then be written as

$$\frac{\left(k_{t+1}^{\alpha} l^{1-\alpha}\right)^{\frac{b}{1+b}} e_{t+1}^{\frac{1}{1+b}} \Theta_t - k_{t+2}}{\left(k_t^{\alpha} l^{1-\alpha}\right)^{\frac{b}{1+b}} e_t^{\frac{1}{1+b}} \Theta_{t-1} - k_{t+1}} = \beta \frac{b}{1+b} \left(k_{t+1}^{\alpha} l^{1-\alpha}\right)^{\frac{b}{1+b}} e_{t+1}^{\frac{1}{1+b}} \Theta_t \frac{\alpha}{k_{t+1}}.$$

Guessing on a constant savings rate, i.e., $k_{t+1} = s y_t$ delivers

$$k_{t+1} = \alpha\beta \frac{b}{1+b} y_t, \tag{58}$$

which verifies the constant savings rate.

Now take the period-$t+1$-version of (50), and the period-t and period-$t+1$-versions of

(52) and insert them into the Hotelling equation to arrive at

$$\frac{e_{t+1}}{e_t} = \beta, \tag{59}$$

or, equivalently, that

$$e_t = (1-\beta)\, R_t. \tag{60}$$

Using (58) and (60) into (48) and (49) and imposing $l = 1$, we obtain the evolution for A_{t+1} and $A_{e,t+1}$ as functions of the state variables (k_t, A_t, $A_{e,t}$, A_{t-1}, and $A_{e,t-1}$):

$$A_{t+1} = \left(\frac{\beta\left((1-\beta)\,R_t\right)^{\frac{1+b-\alpha}{1+b}}}{\left(\frac{\alpha\beta b}{1+b}\right)^{\alpha} k_t^{\frac{\alpha^2 b}{1+b}} \Theta_{t-1}^{\alpha}} \right)^{\frac{1}{1+b}} \left[\left(b\frac{\gamma}{1-\gamma} \right)^{\frac{\varepsilon}{\varepsilon-1}} \right]^{\frac{1}{1+b}} (\exp(a))^{\frac{1}{1+b}} \left(A_t A_{e,t}\right)^{\frac{1}{1+b}};$$

$$y_t = \left(k_t^{\alpha} l^{1-\alpha}\right)^{\frac{b}{1+b}} e_t^{\frac{1}{1+b}} \Theta_{t-1};$$

and

$$A_{e,t+1} = \left(\frac{\left(\frac{\alpha\beta b}{1+b}\right)^{\alpha} k_t^{\frac{\alpha^2 b}{1+b}} \Theta_{t-1}^{\alpha}}{\beta\left((1-\beta)\,R_t\right)^{\frac{1+b-\alpha}{1+b}}} \right)^{\frac{b}{1+b}} \left[\left(b\frac{\gamma}{1-\gamma} \right)^{\frac{\varepsilon}{\varepsilon-1}} \right]^{\frac{-b}{1+b}} (\exp(a))^{\frac{1}{1+b}} \left(A_t A_{e,t}\right)^{\frac{1}{1+b}}.$$

The model has now been fully solved. It is straightforward to verify that the derived allocation is optimal.

10 A Leontief economy

In our estimation section, we find that the substitutability parameter ε is close to zero. Thus, a special case that turns out to be of empirical relevance is case with a Leontief production function. This case also turns out to allow for significant analytical tractability, so we will briefly show this case here.[4] In presenting this example, we will also make some additional assumptions that simplify the analysis: logarithmic preferences and full depreciation

[4]For a follow-up paper that employs this formulation, see Casey (2017).

of capital.[5]

If the production function is Leontief, $F(A_t k_t^{\alpha}, A_t^e e_t) = \min\{A_t k_t^{\alpha}, A_t^e e_t\}$. We focus on interior solutions such that capital is fully utilized. This requires initial conditions where capital is not too large, in which case it could be optimal to let some capital be idle for some time. In a deterministic model with full depreciation and forward-looking behavior, less than full utilization can only occur in the first period.[6] Due to solutions being interior, we replace the Leontief production function by the equality

$$A_t k_t^{\alpha} = A_t^e e_t \tag{61}$$

and let the planner maximize

$$\sum_{t=0}^{\infty} \beta^t \log(A_t k_t^{\alpha} - k_{t+1})$$

subject to condition (61) for all t, which will be referred to as the *Leontief condition*, $A_{t+1} = A_t f(n_t)$ and $A_{t+1}^e = A_t^e f_e(1-n_t)$ for all t, and $\sum_{t=0}^{\infty} e_t = R_0$, by choice of $\left\{k_{t+1}, A_{t+1}, A_{t+1}^e, n_t, e_t\right\}_{t=0}^{\infty}$.

It is straightforward to use the first-order conditions of this problem, as a special case of the more general proof of Theorem 1 above, to derive some properties of optimal behavior. We first define $\hat{e}_t \equiv \beta^{-t} e_t$ and $\hat{s}_t \equiv s_t/(1-s_t)$. Manipulation of the Euler equation then delivers

$$\hat{s}_t = \frac{\alpha\beta}{1-\alpha\beta} - \kappa \sum_{k=0}^{\infty} (\alpha\beta)^{k+1} \hat{e}_{t+k+1}. \tag{62}$$

Thus, given a sequence $\{\hat{e}_t\}$ and a value of κ—the Lagrange multiplier of the natural-resource constraint—this equation uniquely, and in closed form, delivers the full sequence of saving rates. We can see from this equation that the more of the natural resource is used in the future (in relative terms), the lower is current $\hat{s}$, implying a lower current saving rate. The intuitive reason for this is that more capital requires more of the natural resource and/or higher technological efficiency of the natural resource. This limits the value of accumulating capital and more so, the more scarce is the natural resource. If the natural resource were not scarce, we would have $\kappa = 0$ and $\hat{s} = \frac{\alpha\beta}{1-\alpha\beta} \Rightarrow s = \alpha\beta$; in that case, the model would thus

[5] There is a tension between full depreciation and the Leontief assumption: the former is better the longer is one time period, whereas the reverse is true for the latter.

[6] In a model with shocks, one can imagine recurring periods of less than full capital utilization.

look just like the Cobb-Douglas model (and the textbook Solow model).

Let us now look at the optimal use of the natural resource. By further manipulation of the first-order conditions (details can be found in an online appendix), one can derive the following closed form:

$$\frac{f_e'(1-n_t)f(n_t)}{f_e(1-n_t)f'(n_t)} = \frac{\frac{\beta}{\kappa(1-\beta)(1-\alpha\beta)} - \frac{1}{\alpha}\sum_{k=0}^{\infty}\beta^k\sum_{j=0}^{\infty}(\alpha\beta)^{j+1}\hat{e}_{t+1+k+j}}{\sum_{k=0}^{\infty}\beta^{k+1}\hat{e}_{t+k+1}}. \tag{63}$$

This equation enables us to solve for the current direction of technological development, n_t, directly as a function of the future values of $\hat{e}$ and the shadow value of the resource—just like in the case of the saving rate. We see that the more energy is used in the future, the lower is current n, i.e., the more is R&D labor allocated toward natural resource-saving today, and that this effect is larger the scarcer is the resource.

It is straightforward to solve equation (63) numerically for transition dynamics.[7] The long-run outcome will be a balanced growth path. Its features are, in part, already given by parts of the theorem. For example, n is pinned down by the condition that $f_e(1-n)\beta = f(n)^{\frac{1}{1-\alpha}}$ and the resource is extracted at the rate of discount ($\sigma = 1$ given logarithmic utility). In terms of this special case, this means that $\hat{e}$ will be constant (it is given by $(1-\beta)R_0$) and that equation (63) can be used to solve for the $\kappa\hat{e}$, and hence for κ. Clearly, κ will respond one-to-one in percentage terms to R_0: a doubling of the resource stock cuts its shadow value in half. The determination of $\kappa\hat{e}$ also delivers the saving rate from equation (62). Moreover, it is easy to see that the long-run cost share of the resource must satisfy $e^{share} = 1 - \frac{s}{\alpha\beta}$ or, put, differently, that $s = \alpha\beta(1-e^{share})$: the resource share is directly tied to how much lower the saving rate for physical capital is given the presence of the resource (recall that the saving rate would be $\alpha\beta$ otherwise).

[7] Guess on the shadow value of the resource, κ, and on a natural-resource sequence, $\{\hat{e}_t\}_{t=0}^{\infty}$. Then obtain the sequences $\{(k_t, n_t)\}_{t=0}^{\infty}$ from the above two equations and, using equation (61) repeatedly, obtain an update for the natural-resource sequence. Unless the guess is correct, it will not meet the natural-resource constraint with equality. Adjust κ and the natural resource sequence accordingly and repeat.

11 Estimation

We state the original problem in an environment of certainty, as we will linearize anyway (and, hence, use certainty equivalence). We then have

$$\max_{\left\{c_t,k_{t+1},A_{t+1},A^e_{t+1},e_t\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

subject to

$$c_t + \frac{k_{t+1}}{\exp(q_t)} = F(k_t^{\alpha} A_t, A_t^e e_t) + (1-\delta)k_t - \exp(z_{pt})\, p_0 \gamma_p^t e_t,$$

and

$$G\left(\frac{1}{\exp(z_{At})}\frac{A_{t+1}}{A_t}, \frac{1}{\exp(z_{A_e t})}\frac{A^e_{t+1}}{A^e_t}\right) = \bar{G},$$

and where the shocks obey the following laws of motions

$$\begin{aligned} z_{At} &\sim N\left(0, \sigma_A^2\right), \\ z_{Ae,t} &\sim N\left(0, \sigma_{Ae}^2\right), \\ q_{t+1} &= \rho_q q_t + \chi_{q,t+1},\ \chi_q \sim N\left(0, \sigma_q^2\right), \text{ and} \\ z_{pt+1} &= \rho_p z_{pt} + \chi_{p,t+1},\ \chi_p \sim N\left(0, \sigma_p^2\right). \end{aligned}$$

11.1 Transformation

Define $x_t \equiv k_t^{\alpha} A_t$, $\hat{c}_t = \frac{c_t}{x_t}$, $\hat{k}_t = \frac{k_t}{x_t}$, $a_{et} = \frac{A_t^e}{p_0 \gamma_p^t}$, and $\hat{e}_t = \frac{e_t p_0 \gamma_p^t}{x_t}$. Here, note that x and a_e are state variables in a true sense: all their components are predetermined.

$$\max_{\left\{\hat{c}_t,\hat{k}_{t+1},x_{t+1},a_{e,t+1},\hat{e}_t\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{(\hat{c}_t x_t)^{1-\sigma}}{1-\sigma}$$

subject to

$$\hat{c}_t + \frac{\hat{k}_{t+1}}{\exp(q_t)}\frac{x_{t+1}}{x_t} = F(1, a_{et}\hat{e}_t) + (1-\delta)\hat{k} - \exp(z_{pt})\,\hat{e}_t$$

and

$$G\left(\frac{1}{\exp(z_{At})}\left(\frac{x_{t+1}}{x_t}\right)^{1-\alpha}\left(\frac{\hat{k}_{t+1}}{\hat{k}_t}\right)^{-\alpha}, \frac{1}{\exp(z_{A_e t})}\gamma_p \frac{a_{e,t+1}}{a_{et}}\right) = \bar{G},$$

where we have used $A_t = x_t k_t^{-\alpha} = x_t \left(\hat{k}_t x_t\right)^{-\alpha} = x_t^{1-\alpha}\hat{k}_t^{-\alpha}$ and $A_t^e = a_{et} p_0 \gamma_p^t$.

This can be written as a dynamic programming problem as follows.

$$v(x, a_e, \hat{k}, z_A, z_{A_e}, z_p, q) =$$
$$\max_{g_x, g_{a_e}, \hat{k}', \hat{e}} \frac{\left((F(1, a_e\hat{e}) + (1-\delta)\hat{k} - \hat{k}' g_x - \exp(z_p)\hat{e})x\right)^{1-\sigma}}{1-\sigma} + \beta v(g_x x, g_{a_e} a_e, \hat{k}', z_A', z_{A_e}', z_p', q')$$

subject to

$$G\left(\frac{1}{\exp(z_A)} g_x^{1-\alpha}\left(\frac{\hat{k}'}{\hat{k}}\right)^{-\alpha}, \frac{1}{\exp(z_{A_e})}\gamma_p g_{a_e}\right). \tag{64}$$

Here we guess and verify that $v(x, a_e, \hat{k}, z_A, z_{A_e}, z_p, q) = x^{1-\sigma}\hat{v}(a_e, \hat{k}, z_A, z_{A_e}, z_p, q)$. Substituting in we obtain

$$x^{1-\sigma}\hat{v}(a_e, \hat{k}, z_A, z_{A_e}, z_p, q) =$$
$$\max_{g_x, g_{a_e}, \hat{k}', \hat{e}} \frac{\left((F(1, a_e\hat{e}) + (1-\delta)\hat{k} - \hat{k}' g_x - \exp(z_p)\hat{e})x\right)^{1-\sigma}}{1-\sigma} + (g_x x)^{1-\sigma}\beta\hat{v}(g_{a_e} a_e, \hat{k}', z_A', z_{A_e}', z_p', q')$$

subject to the same constraint. We can cancel the part involving x, and hence the functional equation is satisfied for all x since x no longer appears in the equation:

$$\hat{v}(a_e, \hat{k}, z_A, z_{A_e}, z_p, q) =$$
$$\max_{g_x, g_{a_e}, \hat{k}', \hat{e}} \frac{\left((F(1, a_e\hat{e}) + (1-\delta)\hat{k} - \hat{k}' g_x - \exp(z_p)\hat{e})\right)^{1-\sigma}}{1-\sigma} + \beta g_x^{1-\sigma}\hat{v}(g_{a_e} a_e, \hat{k}', z_A', z_{A_e}', z_p', q')$$

subject to (64).

The first-order conditions, now stated sequentially, are given below.

$$-\frac{\hat{c}_t^{-\sigma}}{\exp(q_t)}\frac{x_{t+1}}{x_t} + \frac{1}{\exp(z_{At})}\hat{\lambda}_t G_{1t}\left(\frac{x_{t+1}}{x_t}\right)^{1-\alpha}\alpha\hat{k}_t^{\alpha}\hat{k}_{t+1}^{-\alpha-1} + \tag{65}$$

$$\beta\left(\hat{c}_{t+1}\frac{x_{t+1}}{x_t}\right)^{-\sigma}\frac{x_{t+1}}{x_t}(1-\delta) - \beta\frac{1}{\exp(z_{A,t+1})}\hat{\lambda}_{t+1}G_{1,t+1}\left(\frac{x_{t+1}}{x_t}\right)^{1-\sigma}\left(\frac{x_{t+2}}{x_{t+1}}\right)^{1-\alpha}\alpha\hat{k}_{t+2}^{-\alpha}\hat{k}_{t+1}^{\alpha-1} = 0.$$

$$0 = -\frac{\hat{c}_t^{-\sigma}}{\exp(q_t)}\hat{k}_{t+1} - \frac{1}{\exp(z_{At})}\hat{\lambda}_t G_{1t}(1-\alpha)\left(\frac{x_{t+1}}{x_t}\right)^{-\alpha}\left(\frac{\hat{k}_{t+1}}{\hat{k}_t}\right)^{-\alpha} + \tag{66}$$

$$\beta\left(\hat{c}_{t+1}\frac{x_{t+1}}{x_t}\right)^{-\sigma}\left(\hat{c}_{t+1} + \frac{\hat{k}_{t+2}}{q_{t+1}}\frac{x_{t+2}}{x_{t+1}}\right) + \beta\frac{1}{\exp(z_{A,t+1})}\hat{\lambda}_{t+1}G_{1,t+1}(1-\alpha)\left(\frac{x_{t+1}}{x_t}\right)^{-\sigma}\left(\frac{x_{t+2}}{x_{t+1}}\right)^{1-\alpha}\left(\frac{\hat{k}_{t+2}}{\hat{k}_{t+1}}\right)^{-\alpha}.$$

$$\beta\hat{c}_{t+1}^{-\sigma}F_{2,t+1}\hat{e}_{t+1} - \frac{1}{\exp(z_{A_e t})}\hat{\lambda}_t\left(\frac{x_{t+1}}{x_t}\right)^{\sigma-1}\frac{\gamma_p G_{2t}}{a_{et}} + \beta\frac{1}{\exp(z_{A_e,t+1})}\hat{\lambda}_{t+1}\gamma_p G_{2,t+1}\frac{a_{e,t+2}}{a_{e,t+1}^2} = 0. \tag{67}$$

$$\hat{c}_t = F(1, a_{et}\hat{e}_t) + \hat{k}_t(1-\delta) - \frac{\hat{k}_{t+1}}{q_t}\frac{x_{t+1}}{x_t} - \exp(z_{pt})\,\hat{e}_t \tag{68}$$

$$F_2(1, a_{et}\hat{e}_t)a_{et} = \exp(z_{pt})\,. \tag{69}$$

$$G\left(\frac{1}{\exp(z_{At})}\left(\frac{x_{t+1}}{x_t}\right)^{1-\alpha}\left(\frac{\hat{k}_{t+1}}{\hat{k}_t}\right)^{-\alpha}, \frac{1}{\exp(z_{A_e t})}\gamma_p\frac{a_{e,t+1}}{a_{et}}\right) = \bar{G}, \tag{70}$$

The observables are then given by

$$e^{share} = a_{et}\hat{e}_t\frac{F_2(1, a_{et}\hat{e}_t)}{F(1, a_{et}\hat{e}_t)}, \text{ and} \tag{71}$$

$$g_{e,t+1} \equiv \frac{e_{t+1}}{e_t} = \frac{1}{\gamma_p}\frac{\hat{e}_{t+1}}{\hat{e}_t}\frac{x_{t+1}}{x_t}, \tag{72}$$

$$g_{a_e,t+1} \equiv \frac{a_{e,t+1}}{a_{et}} = \frac{F(1, a_{e,t+1}\hat{e}_{t+1})}{F(1, a_{et}\hat{e}_t)}\frac{x_{t+1}}{x_t} \tag{73}$$

$$p \equiv \exp(z_{pt}) = F_2(1, a_{et}\hat{e}_t)a_{et}. \tag{74}$$

11.2 Robustness

In this section, we evaluate the robustness of the empirical results derived in Section 5 in the manuscript. In all cases below, the acceptance ratio is around 30.[8]

First, we consider different time periods. Specifically, we evaluate to what extent the results are driven by the oil-price hikes in the 1970s by estimating the model on two sub periods: 1985–2018 and 1949–1985. The results that are presented in Tables 3–4 show that all posterior estimates are relatively close to the values from the estimation over the full period. Hence, the results are not driven by the oil-price hikes of the 1970s.

Table 3: Robustness: 1985–2018

		Prior		Posterior		
Coefficient	Prior density	Mean	Sd	Mean	Sd	[10, 90]
ε	Beta	0.2000	0.1600	0.0379	0.0233	[0.0007, 0.0781]
B_e	Beta	0.2000	0.030	0.1903	0.0234	[0.1511, 0.2288]
B	Beta	0.0150	0.0300	0.0160	0.0021	[0.0124, 0.0197]
ϕ	Beta	0.9000	0.0150	0.8996	0.0147	[0.8754, 0.9234]
ρ_q	Inv Gamma	0.2000	0.1000	0.1909	0.0951	[0.0404, 0.3332]
Shocks						
$std\,(z_{Ae})$	Inv Gamma	0.0500	0.0200	0.0370	0.0041	[0.0293, 0.0444]
$std\,(z_A)$	Inv Gamma	0.0500	0.0200	0.0199	0.0022	[0.0158, 0.0237]
$std\,(\chi_q)$	Inv Gamma	0.0500	0.0020	0.0312	0.0052	[0.0217, 0.0399]

The posterior estimates are from 5 chains with 100,000 draws, where we discard the initial 50,000. Parameters γ_P and ρ_p are estimated separately and their respective values are 1.0358 and 0.79.

Table 4: Robustness: 1949–1985

		Prior		Posterior		
Coefficient	Prior density	Mean	Sd	Mean	Sd	[10, 90]
ε	Beta	0.2000	0.1600	0.0514	0.0144	[0.0007, 0.1137]
B_e	Beta	0.1500	0.0200	0.1501	0.0183	[0.1164, 0.1854]
B	Beta	0.0120	0.0200	0.0259	0.0077	[0.0140, 0.0376]
ϕ	Beta	0.9000	0.0200	0.8554	0.0209	[0.8196, 0.8909]
ρ_q	Inv Gamma	0.2000	0.1000	0.1952	0.0972	[0.0440, 0.3411]
Shocks						
$std\,(z_{Ae})$	Inv Gamma	0.0500	0.0100	0.1036	0.0112	[0.0822, 0.1272]
$std\,(z_A)$	Inv Gamma	0.0500	0.0100	0.0319	0.0032	[0.0261, 0.0375]
$std\,(\chi_q)$	Inv Gamma	0.0500	0.0100	0.0435	0.0060	[0.0328, 0.0540]

The posterior estimates are from 5 chains with 100,000 draws, where we discard the initial 50,000. Parameters γ_P and ρ_p are estimated separately and their respective values are 1.069 and 0.92.

[8] Note that the priors differ marginally for some estimations.

We here also consider the alternative sub period 1973–2018. The results from this exercises is presented in Table 5.

Table 5: Robustness: 1973–2018

		Prior		Posterior		
Coefficient	Prior density	Mean	Sd	Mean	Sd	$[10, 90]$
ε	Beta	0.2000	0.1600	0.0318	0.0187	$[0.0004, 0.0679]$
B_e	Beta	0.2000	0.020	0.1776	0.0013	$[0.1569, 0.1970]$
B	Beta	0.0150	0.0200	0.0130	0.0007	$[0.0111, 0.0149]$
ϕ	Beta	0.9000	0.0150	0.8990	0.0079	$[0.8763, 0.9234]$
ρ_q	Inv Gamma	0.2000	0.1000	0.1883	0.1507	$[0.0382, 0.3295]$
Shocks						
$std\,(z_{Ae})$	Inv Gamma	0.0500	0.0200	0.035	0.0038	$[0.0271, 0.0392]$
$std\,(z_A)$	Inv Gamma	0.0500	0.0200	0.0192	0.0022	$[0.0157, 0.0226]$
$std\,(\chi_q)$	Inv Gamma	0.0500	0.0200	0.0307	0.0051	$[0.0219, 0.0394]$

The posterior estimates are from 5 chains with 100,000 draws, where we discard the initial 50,000. Parameters γ_P and ρ_p are estimated separately and their respective values are 1.0124 and 0.84.

As second robustness check, the model is estimated by using the growth rate of *total* output, and the growth rate of *total* fossil-fuel use instead of per-capita measures as in the benchmark estimation. The results are presented in Table 6 and they verify that the estimated parameters are not sensitive to whether total or per-capita variables are used as observables.

Table 6: Robustness: total quantities

		Prior		Posterior		
Coefficient	Prior density	Mean	Sd	Mean	Sd	$[10, 90]$
ε	Beta	0.2000	0.1600	0.0180	0.0026	$[0.0006, 0.0271]$
B_e	Beta	0.2000	0.0200	0.2115	0.0182	$[0.1476, 0.1982]$
B	Beta	0.0150	0.0200	0.0239	0.0022	$[0.0118, 0.0212]$
ϕ	Beta	0.9000	0.0150	0.9187	0.0125	$[0.8557, 0.9099]$
ρ_q	Inv Gamma	0.2000	0.1000	0.1967	0.1119	$[0.0423, 0.3486]$
Shocks						
$std\,(z_{Ae})$	Inv Gamma	0.0500	0.0200	0.0259	0.0021	$[0.0286, 0.0391]$
$std\,(z_A)$	Inv Gamma	0.0500	0.0200	0.0240	0.0023	$[0.0221, 0.0313]$
$std\,(\chi_q)$	Inv Gamma	0.0500	0.0200	0.0368	0.0073	$[0.0245, 0.0485]$

The posterior estimates are from 5 chains with 100,000 draws, where we discard the initial 50,000. Parameters γ_P and ρ_ξ are estimated separately and their respective values are 1.02 and 0.92.

A third robustness check is carried out by only using observables from the manufacturing sector. Data on employment in the manufacturing sector is taken from the FRED database.[9]

[9] The data series is denoted MANEMP and includes all employees in manufacturing. The monthly series

Energy consumption in the industrial sector is from EIA, Annual Energy Review, table 2.1d.[10] As described in Section 1, the energy composite is computed with equation (1).

Data on value added in manufacturing from 1949 to 1997 is from the Bureau of Economic Analysis (BEA). The BEA points out that the quality of the manufacturing data "is significantly less than that of the higher level aggregates in which they are included. Compared to these aggregates, the more detailed estimates are more likely to be either based on judgmental trends, on trends in the higher level aggregate, or on less reliable source data."[11] The data on value added in manufacturing from 1997 is also from the BEA.[12]

The results for the manufacturing sector are displayed in Table 7.

Table 7: Robustness: data only for the manufacturing sector

		Prior		Posterior		
Coefficient	Prior density	Mean	Sd	Mean	Sd	$[10, 90]$
ε	Beta	0.2000	0.1600	0.0688	0.0209	$[0.0005, 0.1487]$
B_e	Beta	0.2000	0.0300	0.1835	0.0250	$[0.1430, 0.2225]$
B	Beta	0.0120	0.0300	0.0158	0.0024	$[0.0117, 0.0196]$
ϕ	Beta	0.9000	0.0150	0.9054	0.0144	$[0.8826, 0.9294]$
ρ_q	Inv Gamma	0.2000	0.1000	0.1891	0.0944	$[0.0422, 0.3312]$
Shocks						
$std\,(z_{Ae})$	Inv Gamma	0.0500	0.0200	0.0530	0.0046	$[0.0424, 0.0630]$
$std\,(z_A)$	Inv Gamma	0.0500	0.0200	0.0260	0.0027	$[0.0214, 0.0306]$
$std\,(\chi_q)$	Inv Gamma	0.0500	0.0020	0.0380	0.0076	$[0.0245, 0.0509]$

The posterior estimates are from 5 chains with 100,000 draws, where we discard the initial 50,000. Parameters γ_P and ρ_p are estimated separately and their respective values are 1.014 and 0.93.

Table 8 shows the sensitivity of the results with respect to parameter γ. As argued in the paper, this parameter is of little importance for the posterior estimates. The table verifies this by setting $\gamma = 0.50$, i.e., a value that is ten times higher than in the benchmark estimation.

We also evaluate the sensitivity of the results with respect to using an unweighted fossil-fuel composite instead of computing E from (1). Here, we instead take the opposite approach and just sum the Btu content for each fuel: $E_t = E_t^c + E_t^o + E_t^g$. Table 9 presents the results from this exercise, and verifies that this has little effect on the posterior estimates.

was converted to an annual series by computing averages over 12 months.

[10] Available at https://www.eia.gov/totalenergy/data/annual.

[11] The data is available at https://www.bea.gov/industry/io-histannual.

[12] Data available at https://www.bea.gov/data/gdp/gdp-industry.

Table 8: Robustness: Alternative calibration

		Prior		Posterior		
Coefficient	Prior density	Mean	Sd	Mean	Sd	$[10, 90]$
ε	Beta	0.2000	0.1600	0.0193	0.0054	$[0.0007, 0.0413]$
B_e	Beta	0.2000	0.0300	0.1836	0.0215	$[0.1479, 0.2189]$
B	Beta	0.0150	0.0300	0.0163	0.0017	$[0.0133, 0.0191]$
ϕ	Beta	0.9000	0.0150	0.9242	0.0127	$[0.9022, 0.9448]$
ρ_q	Inv Gamma	0.2000	0.1000	0.1939	0.0967	$[0.0438, 0.3392]$
Shocks						
$std\,(z_{Ae})$	Inv Gamma	0.0500	0.0200	0.0260	0.0021	$[0.0221, 0.0298]$
$std\,(z_A)$	Inv Gamma	0.0500	0.0200	0.0185	0.0017	$[0.0157, 0.0213]$
$std\,(\chi_q)$	Inv Gamma	0.0500	0.0200	0.0292	0.0045	$[0.0215, 0.0367]$

The posterior estimates are from 5 chains with 100,000 draws, where we discard the initial 50,000. Parameters γ_P and ρ_p are estimated separately and their respective values are 1.02 and 0.92.

Table 9: Robustness: Unweighted energy inputs

		Prior		Posterior		
Coefficient	Prior density	Mean	Sd	Mean	Sd	$[10, 90]$
ε	Beta	0.2000	0.1600	0.0293	0.0138	$[0.0007, 0.0608]$
B_e	Beta	0.2000	0.030	0.1874	0.0217	$[0.1510, 0.2222]$
B	Beta	0.0150	0.0300	0.0158	0.0017	$[0.0129, 0.0187]$
ϕ	Beta	0.9000	0.0150	0.9132	0.0133	$[0.8919, 0.9359]$
ρ_q	Inv Gamma	0.2000	0.1000	0.1932	0.0968	$[0.0451, 0.3397]$
Shocks						
$std\,(z_{Ae})$	Inv Gamma	0.0500	0.0200	0.0261	0.0021	$[0.0223, 0.0301]$
$std\,(z_A)$	Inv Gamma	0.0500	0.0200	0.0185	0.0017	$[0.0157, 0.0214]$
$std\,(\chi_q)$	Inv Gamma	0.0500	0.0200	0.0294	0.0046	$[0.0217, 0.0370]$

The posterior estimates are from 5 chains with 100,000 draws, where we discard the initial 50,000. Parameters γ_P and ρ_p are estimated separately and their respective values are 1.02 and 0.92.

12 Alternative estimation: GMM

As an additional robustness check, the model is here estimated with the generalized method of moments (GMM) using a pruned perturbation approximation. Specifically, we use the toolbox provided by Andreasen et al. (2017). We use time series for equations (71)-(74) to construct moment conditions. The following notation is used. Let $y_t(1)$, $y_t(2)$, $y_t(3)$, and $y_t(4)$ respectively denote the first unconditional moment from equation (71)-(74), and $std\,[y_t(1)]\,, std\,[y_t(2)]\,, std\,[y_t(3)]\,,$ and $std\,[y_t(4)]$ denote the second unconditional moment from these equations.

The model is the same as that estimated in Section 11, but a difference is that we here only estimate ε, ϕ, ρ_q, and the standard deviations of the shocks because the algorithm has problems converging when also B and B_e are estimated. Parameters B and B_e are respectively calibrated to 0.017 and 0.175, i.e., values close to their estimated values in the benchmark estimation. Similarly, parameters γ_P and ρ_p are estimated separately and their respective values are 1.02 and 0.92. All remaining calibrated parameters have the same value as in the benchmark estimation. The result from the GMM estimation is presented in Table 10. The point estimate of ε is slightly higher than in the benchmark estimation.

Table 10: Robustness, GMM estimation

	Moment conditions		Estimated parameters		
	Data	Model		Mean	Sd
$E[y_t(1)]$	-3.5649	-2.7319	ε	0.0842	0.0099
$E[y_t(2)]$	0.0021	0.0004	ϕ	0.8581	0.0055
$E[y_t(3)]$	0.0176	0.0212	ρ_q	0.1870	0.3193
$E[y_t(4)]$	-0.0000	0.0000	$std(z_{Ae})$	0.0206	0.0006
$std[y_t(1)]$	0.3451	0.3962	$std(z_A)$	0.0096	0.0053
$std[y_t(2)]$	0.0277	0.0199	$std(\chi_q)$	0.0198	0.0246
$std[y_t(3)]$	0.0156	0.0117			
$std[y_t(4)]$	0.4900	0.4614			
$corr[y_t(1), y_{t-1}(1)]$	0.8597	0.9050			
$corr[y_t(2), y_{t-1}(2)]$	0.5321	0.0373			
$corr[y_t(3), y_{t-1}(3)]$	0.1632	0.0881			
$corr[y_t(4), y_{t-1}(4)]$	0.9267	0.9200			
$corr[y_t(1), y_{t-3}(1)]$	0.5916	0.7391			
$corr[y_t(2), y_{t-3}(2)]$	0.4117	0.0097			
$corr[y_t(3), y_{t-3}(3)]$	0.0280	0.0127			
$corr[y_t(4), y_{t-2}(4)]$	0.7820	0.7736			
$corr[y_t(1), y_{t-3}(1)]$	0.5922	0.3801			
$corr[y_t(2), y_{t-3}(2)]$	0.4135	-0.0068			
$corr[y_t(3), y_{t-3}(3)]$	0.0527	0.02814			
$corr[y_t(4), y_{t-3}(4)]$	0.0979	0.1313			
$corr[y_t(1), y_{t-5}(1)]$	0.2957	0.6006			
$corr[y_t(2), y_{t-5}(2)]$	0.3536	0.0065			
$corr[y_t(3), y_{t-5}(3)]$	0.0420	0.0070			
$corr[y_t(4), y_{t-5}(4)]$	0.6239	0.6620			

13 Simulating the economy

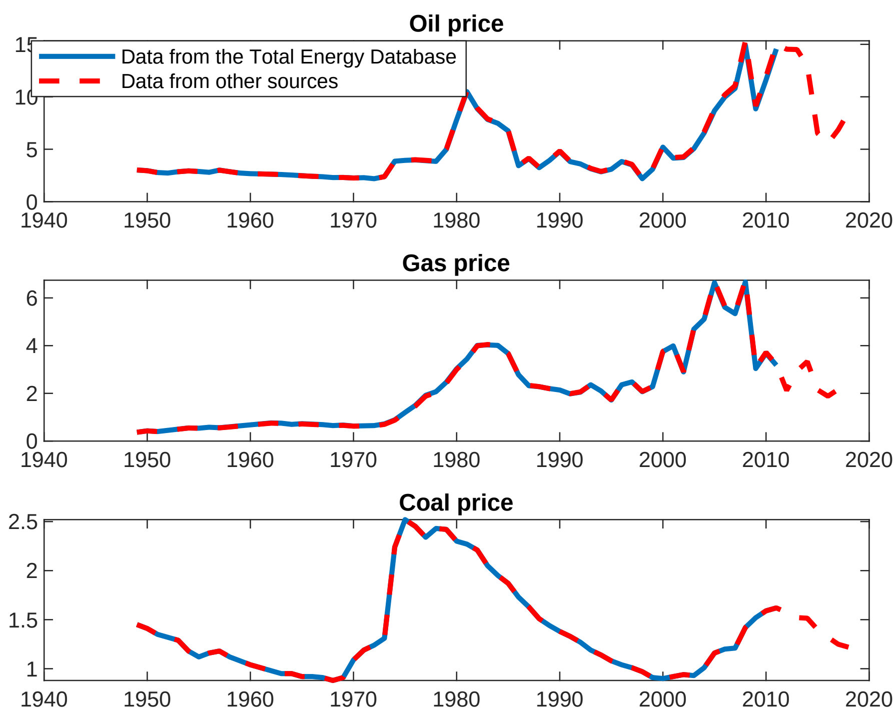
Here, we simulate the estimated economy forward under different assumptions about fossil-fuel price shocks.[13] The economy starts out in the steady state and we then consider three different shock scenarios.

[13] Recall that the fossil fuel price follows an AR(1) process in the model that we estimate.

The first scenario is the benchmark scenario and it features no fossil-fuel shocks at all. In the second scenario, a fossil-fuel-price shock of equal magnitude hits the economy for ten periods in a row. From period 11 no more shocks to the fuel-price are then realized. The third scenario, finally, instead features larger shocks for only 5 periods. The shocks in scenario three is set to make sure that the average increase in the fossil fuel price over twenty years is exactly the same as in scenario two.

The model is calibrated with the estimated parameters found in Section 5 in the paper, and only shocks to the fossil-fuel price are considered. Both shock series result in an average price growth of five percent per year for twenty years, which can be compared to 4.57 percent between 1973 and 1992 in the data.

The results show that the growth rate without shocks is roughly 0.4 percentage points higher than the growth rates with shocks. Hence, the shocks dampen the growth rate somewhat. The difference between the two scenarios with shocks is smaller and only around 0.05 percentage points. Here, the scenario with larger and less persistent fuel-price shocks is worse than that with smaller shocks over a longer period..


Oil price
Data from the Total Energy Database
Data from other sources
15
10
5
0
1940
1950
1960
1970
1980
1990
2000
2010
2020
Gas price
6
4
2
0
1940
1950
1960
1970
1980
1990
2000
2010
2020
Coal price
2.5
2
1.5
1
1940
1950
1960
1970
1980
1990
2000
2010
2020